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Big04-A: External Radiation Labels and Causal-Access Certificates on Prescribed Comparison Geometries

Calibration risks, thermal interfaces, correlated receivers, and rate-bounded futures

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Abstract

We separate coefficient identifiability, causal accessibility, and finite-channel recovery on a prescribed spatially flat comparison geometry. A radiation-type prefix carries external labels $T_{\mathrm{r}} \propto U^{1/4}\kappa^{3/4}/a$ and $S_{\mathrm{r}} \propto (U/\kappa)^{3/4}$. We give their calibration inverses, exact absolute and relative minimax risks, and randomized-decision lower certificates. For a relative coefficient window of width δ, the radiation relative risk is $\tanh(\frac{3}{4}\operatorname{artanh}\delta)$, not δ; equal endpoint weights certify absolute loss, whereas relative loss requires unequal weights.

Under an explicitly stated located-circuit contract, a source with no causal path to the complete receiver interface induces a joint replacement channel, even for correlated receiver outputs. Its optimized half-diamond recovery error is $1 - d^{-2}$. We prove the robust envelope, quantify partially dark mixtures, and distinguish joint replacement certificates from products of marginal certificates. At a fixed external temperature label, identity, energy-basis dephasing, and Gibbs replacement are thermal operations with different recovery errors.

For smooth futures obeying a declared bound on $\frac{\mathrm{d}}{\mathrm{d}t}(1/H)$, we prove the exact open interval of early-junction horizon radii and its minimax consequences. General-w label dictionaries, late-label supply cases, two-prefix comparison, and spectator-resource qualifications are included. The contribution is a self-contained collection of separation and certification results. No physical dark-radiation species, host Hilbert-space dimension, temperature, entropy, Einstein dynamics, or Page curve is derived.

Version 5 adds, by insertion only, the sharp relative risk $\tanh(C_K\delta/(\kappa_0(1-\delta^2)))$, $C_K = \pi c^5/(GK^2)$, of the late dimension label at a supplied terminal rate K; a local re-proof of the two-label reduction of the hybrid-cut closed forms; and documentary separations from the energy-constrained replacement floor, the vacuum term, four-label closed sectors, and a rate-family energy minimax.

Version 6 adds, by insertion only, seven groups of results on the same prescribed host. (i) The radiation labels depend on (κ, α) only through $z = \kappa^3/\alpha$ and on no declared ordinary/dark/vacuum split; the fibre of the split-blind data together with both labels is exactly the product of the split fibre and the coefficient orbit, so a calibrated coefficient selects no split and a declared split selects no point of the orbit. (ii) A finite adaptive protocol whose initial law, selection kernels and complete accessible process are coefficient-blind has one law for every κ; it cannot calibrate either label, and on a supplied window it cannot beat the risks $\frac{1}{2}(u-\ell)$ and $\tanh(\frac{3}{4}\operatorname{artanh}\delta)$. (iii) For general w, $\partial\ln T_w/\partial\ln\kappa = 3w/(1+w)$ and S_w is constant in time on every supplied scale factor; the two-prefix comparison is the equal-exponent case of a cross-ratio certificate, and a declared change of w breaks its common-rescaling invariance. (iv) On the prefix the apparent radius Bz^2 is fixed by early data while the event radius is not; under the rate bound the event radius strictly exceeds it, and a sharp modulus bounds horizon differences. (v) The absolute and relative risks are instances of one fibre principle. (vi) The floor $1 - d^{-2}$ differs from the Shredder floor $1 - 1/d$ by $(d-1)/d^2$ and is neither a magic-fibre response nor a Page curve. (vii) A constant volume term preserves every label and selects no point of the orbit. Further inserted results, re-proved from the author's draft of version-6 additions, give the general-w quotient and epoch-change ratios, two-point bounds under approximate blindness, calibration with a supplied normalization window, the finite-settling horizon

functional, the fact that a fixed terminal rate alone leaves every positive radius $R_0 \in (0, \infty)$, and an exact clock-feedback factorization test. These results matter because they replace isolated calculations by checkable separations: none of them selects a split, a future, or a vacuum term, and κ is calibrated only by an added supply such as an exact label at fixed supplied normalization. Printed radii and risks are calculations, not measurements. A separate conditional prediction layer states six theorems under supplied closures, which are optional hypotheses outside the ledger: a radiation consistency relation with redshift drift $-H_0 z(1 + z)$ on the prefix interval; the relation $2s_H - s_T = 3$, which is equivalent to a supplied general-w closure; exact finite-time reception envelopes under the rate bound, with their attainment cases; a drift envelope; logarithmic tails with and without an event horizon; and an information-isolation threshold. It makes no observational claim and uses no measured number.

Standing ledger (verbatim from Section): Host A and Host B stay separate. Host C is not created. O.1 stays NOT_CLOSED. XV-B is not opened. XVI-N is not discharged. N-Ins.4 stays unresolved. U6-sel is not moved. Gate 0 stays NOT_SUPPLIED. ST4b stays NOT_SUPPLIED. No quantum coefficient is admitted. No quantum coefficient is derived. No Einstein dynamics, derived metric, spacetime emergence, Page curve, island, vacuum-energy computation, or quantized relational time without an external clock is claimed. A source record is not a premise. The OpenAI catalogue is not a premise. noindentCitations identify source records only; no proof transfer.

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Manuscript status. This document is a self-contained revision of the supplied *Big04-A*, version 3 [Big04v3]. The earlier source and all parent manuscripts remain unedited. For readability and correctness, the present submission text integrates the additions and corrections rather than claiming bytewise insertion-only preservation. The corrections are identified in Appendix . Authorship must be confirmed before submission. No compilation, numerical rerun, registration, or independent verification is asserted.

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Introduction and contribution

The term *dark radiation* ordinarily concerns additional relativistic degrees of freedom. For example, Buen-Abad, Chacko, Flood, Kilic, Marques-Tavares and Youn propose an interacting dark-sector model with a dark-matter subcomponent that decouples from self-interacting dark radiation [BuenAbad2025]. The present paper does not model that component, its interactions, or its observational consequences.

Instead, we study three separately specified mathematical objects:

• a prescribed classical metric and its causal relation;

• external numerical label dictionaries with an unfilled coefficient coordinate $\kappa > 0$;

• finite comparison systems, channels, and complete decoder access contracts. A relation between two of these objects is an additional declaration unless proved from hypotheses stated here.

The motivating separation appears in Cheung's *Big01-A* and *Big02-A* [Big01,Big02]: a causal-access theorem can be exact without deriving a finite Hilbert space or identifying a thermodynamic coefficient. Cheung's *Big03-A* distinguishes coefficient supply from coefficient inference [Big03]. The supplied *Big04-A*, version 3, applies this distinction to a radiation-type prefix [Big04v3]. The present edition retains that mathematical programme, strengthens its local proofs, and corrects several distinctions needed for a submission text.

The principal results are as follows.

• Sections and give radiation and general-w coefficient maps, their inverses, and exact minimax risks. The relative-risk correction is Theorem .

• Section proves exact finite replacement, dephasing, and cut certificates. Section applies them to complete causal-access loss, correlated outputs, and partially dark mixtures.

• Section separates installed channel error, unrestricted recovery error, and thermally restricted decoding. It also computes the restricted channel-identification minimax problem, including the scalar-Hamiltonian exception.

• Sections and separate the terminal rate, the early-junction horizon, and late labels, including exact rate-bounded horizon ranges.

• Sections – give a two-prefix comparison, conservation vacuity, a spectator action, and a separate approximate-Singleton resource bound.

• Version 5 adds Proposition , the relative risk of the late dimension label, and Proposition , the two-label reduction of the hybrid-cut closed forms. It also adds the documentary separations , , , , , and . The plan corrections and errata are in Appendix .

• Version 6 adds Proposition (orbit–split independence), Theorem (adaptive coefficient-blind non-calibration), Proposition (the fibre principle for the labels), Proposition and Corollary (general-w logarithmic certificates and the two-epoch special case), Theorem (event and apparent radii and a sharp modulus), Proposition (vacuum shift and orbit) and Lemma (two different finite floors), together with the documentary separations and . From the author's draft of version-6 additions it re-proves and inserts Corollaries and , Proposition , Theorem , Propositions and , Corollary , Proposition , Theorem , Proposition and Corollary . Section adds a conditional prediction layer of six theorems under supplied closures. Plan verdicts, plan corrections, the prediction-layer verdicts X6, the draft verdicts F11, the draft corrections DC3 and the crosswalk are in Appendix .

Attribution and proof dependencies

The finite-channel identities used below are standard comparison results, not new general principles of quantum error correction. They are related to the diamond-norm formulations of Watrous [WatrousSDP,WatrousBook], the error-correction framework of Knill and Laflamme [KL], and the measure-and-prepare framework of Horodecki, Shor and Ruskai [HSR]. The proofs needed for the replacement and access values are provided here.

Citations to Cheung's companion manuscripts document provenance and parallel statements; they are source record; not a premise. No companion theorem is silently imported as a proof premise. Where a standard external mathematical inequality is used rather than reproved, it is explicitly identified, as in Theorem . Unpublished manuscript identifiers are not represented as peer-reviewed articles.

Standing scope

The coefficient κ is a formal external coordinate in the place occupied by $\hbar$ in certain familiar formulas. Varying this coordinate does not assert that a physical constant varies. The gravitational coupling is written χ, never κ. The general-w energy-scale coordinate ε_s, introduced below, is another separately declared input.

Host A and Host B stay separate. Host C is not created. O.1 stays NOT_CLOSED. XV-B is not opened. XVI-N is not discharged. N-Ins.4 stays unresolved. U6-sel is not moved. Gate 0 stays NOT_SUPPLIED. ST4b stays NOT_SUPPLIED. No quantum coefficient is admitted. No quantum coefficient is derived. No Einstein dynamics, derived metric, spacetime emergence, Page curve, island, vacuum-energy computation, or quantized relational time without an external clock is claimed. A source record is not a premise. The OpenAI catalogue is not a premise.

The historical $H2$ stop belongs to its version-qualified XIV-H record. The supplied XIV-I excerpts report the narrower internal-amplitude construction at $I2$, but not the required joint map at $I3$; the supplied XIV-J excerpts retain the full-map stop at $J2$ [XIVI,XIVJ]. Nothing in this paper discharges those analytic obligations or the subsequent admission chain.

Data, decisions, and the prescribed prefix

Definition (Data maps and external supply). An account belongs to a declared parameter space $\mathscr{X}$. Its admitted datum is a map $D : \mathscr{X} \to \mathscr{Y}$. A deterministic decision is a function of D. A randomized decision has a probability law determined by D.

A parameter is *supplied* when an additional input determines it. An exact absolute label can be a supply even if that input is not named after the parameter.

A deterministic report is *coefficient-blind on a fibre* if it is constant there. A randomized collection of reports is coefficient-blind if its *joint law* is identical throughout that fibre. Equality of separate marginal laws is not sufficient.

Lemma (Fibre principle and absolute minimax risk). *Let a real target take values in a nonempty set $\mathscr{V}$ on one data fibre.*

• *The target is a function of the data exactly when it is constant on every fibre.*

• *Suppose $m = \inf \mathscr{V}$ and $M = \sup \mathscr{V}$ are finite. Every constant decision y has risk*

$$\sup_{v \in \mathscr{V}} |y - v| = \frac{M - m}{2} + \left|y - \frac{m + M}{2}\right|.$$

If $m < M$, the unique deterministic minimax decision is the midpoint and its risk is $(M - m)/2$.

• *The deterministic midpoint's worst case is a maximum exactly when $m \in \mathscr{V}$ or $M \in \mathscr{V}$. The existence of an optimal decision and the realization of its worst-case supremum are different statements.*

• *Randomization does not lower the minimax expected absolute risk. If both endpoints are available, the equal endpoint prior is a matching lower certificate. For an open range the same lower bound follows by endpoint approximation.*

• *If $\mathscr{V}$ is unbounded, every coefficient-blind randomized decision has infinite worst-case expected absolute risk.*

Proof. A function of the data is constant on fibres; conversely, define the function by its common fibre value. For bounded $\mathscr{V}$,

$$\sup_{v \in \mathscr{V}} |y - v| = \max\{|y - m|, |y - M|\},$$

which gives the formula and the maximum criterion. For a random decision Y,

$$\frac{1}{2}\mathbb{E}|Y - m| + \frac{1}{2}\mathbb{E}|Y - M| \geq \frac{M - m}{2}.$$

Use approximating values if an endpoint is absent. The deterministic midpoint attains the lower bound. Finally, if $\mathbb{E}|Y| = \infty$, the risk is infinite at every finite target. Otherwise $\mathbb{E}|Y - v| \geq |v| - \mathbb{E}|Y|$, which is unbounded over an unbounded target set.

Remark (Randomized attainment). The uniqueness in Lemma concerns deterministic decisions. For a closed interval, any law supported in that interval with mean equal to the midpoint is also minimax for expected absolute loss. For an open target interval, an optimal random law can realize its risk at interior target values even when the deterministic midpoint has no maximizing parameter. We do not transfer deterministic maximum-attainment statements to randomized rules.

Definition (Radiation-type prefix). Fix $c, H_0 > 0$ and $t_i \in (-1/(2H_0), 0)$. Prescribe

$$a_{\mathrm{r}}(t) = \sqrt{1 + 2H_0}\,t, \qquad H_{\mathrm{r}}(t) = \frac{H_0}{1 + 2H_0 t}, \qquad t_i \le t \le 0.$$

A comparison future is a positive smooth scale factor on (t_i, ∞) agreeing with this prefix, with metric

$$g_a = -c^2\,\mathrm{d}t^2 + a(t)^2\,\mathrm{d}\mathbf{x}^2$$

and comoving observer $\gamma(t) = (t, 0)$. Let $\mathscr{F}(K)$, $K > 0$, require $H = \dot{a}/a > 0$ and $H(t) = K$ after some finite time. Set

$$B = \frac{c}{H_0}, \qquad R_0[a] = c\int_0^\infty \frac{\mathrm{d}s}{a(s)}.$$

The endpoint t_i is used through the smooth prefix extension. The excluded zero of a_{r} is not an initial condition.

Lemma (Causality and early horizon radii). *Put $z = a_{\mathrm{r}}(t)$ for $t \le 0$. For a future with finite R_0,*

$$D_{\mathrm{pre}}(t) = c\int_t^0 a_{\mathrm{r}}(s)^{-1}\,\mathrm{d}s = B(1 - z),$$
$$\chi_e(t) = B(1 - z) + R_0,$$
$$R_e(t) = z\,[B(1 - z) + R_0].$$

An early event $p = (t, \mathbf{x})$ reaches the whole observer worldline at finite time exactly when

$$|\mathbf{x}| < \chi_e(t).$$

The equality case is not received at finite time. If the remaining conformal integral is infinite, every finite coordinate radius reaches the observer.

Proof. The primitive of $(1 + 2H_0 t)^{-1/2}$ is $H_0^{-1}\sqrt{1 + 2H_0}\,t$. A future causal curve obeys $|\mathrm{d}\mathbf{x}/\mathrm{d}t| \le c/a(t)$. Integration gives necessity, and an inward radial null curve gives sufficiency. For a finite horizon integral, the finite-time conformal reach increases to, but never attains, its infinite-time limit. Multiplication by $a(t) = z$ gives the proper radius.

par *Scope.* The metric, time orientation, and observer are prescribed. These identities do not supply a radiation state or an Einstein equation. They parallel the early-host construction in Cheung's Big02-A [Big02]. par

Radiation labels, calibration, and general-w extension

Definition (Radiation dictionary and coefficient-free data). Supply $U, \alpha, V_c > 0$ independently of the geometry and keep $\kappa > 0$ external. On the prefix define

$$u(t) = U a_{\mathrm{r}}(t)^{-4}, \qquad V(t) = V_c\, a_{\mathrm{r}}(t)^3,$$

$$T_{\mathrm{r}}^{(\kappa)}(t) = \frac{1}{k_B}\left(\frac{u(t)\kappa^3 c^3}{\alpha}\right)^{1/4}, \qquad S_{\mathrm{r}}^{(\kappa)}(t) = \frac{4u(t)V(t)}{3T_{\mathrm{r}}^{(\kappa)}(t)}.$$

The coefficient-free datum is

$$\mathfrak{E} = (g_a|_{[t_i, 0]}, c, G, k_B, U, \alpha, V_c, \text{coefficient-blind declared data}).$$

When U is varied, it is removed from the fixed datum. Numerical evaluations of the labels are not silently included among coefficient-blind reports.

Theorem (Radiation coefficient and calibration certificate). *At fixed U, write*

$$A = \frac{(c^3/\alpha)^{1/4}}{k_B}, \qquad \widehat{C} = \frac{4}{3} k_B \alpha^{1/4} V_c c^{-3/4}.$$

Then

$$T_{\mathrm{r}} = \frac{A}{a_{\mathrm{r}}} U^{1/4} \kappa^{3/4}, \qquad S_{\mathrm{r}} = \widehat{C}\, U^{3/4} \kappa^{-3/4}.$$

Each label is injective in κ*, with logarithmic derivatives* $3/(4\kappa)$ *and* $-3/(4\kappa)$*. Neither is a function of* $\mathfrak{E}$*; each has infinite absolute minimax risk over* $\kappa > 0$*, also for randomized decisions.*

For two prefix times,

$$\frac{T_{\mathrm{r}}(t)}{T_{\mathrm{r}}(t')} = \frac{a_{\mathrm{r}}(t')}{a_{\mathrm{r}}(t)}, \qquad \frac{S_{\mathrm{r}}(t)}{S_{\mathrm{r}}(t')} = 1.$$

These ratios and the product

$$T_{\mathrm{r}}(t) S_{\mathrm{r}}(t) = \frac{4UV_c}{3a_{\mathrm{r}}(t)}$$

are coefficient-free.

If U is not supplied, temperature alone depends only on $U\kappa^3$*, and entropy alone depends only on* U/κ*. The pair* $(T_{\mathrm{r}}, S_{\mathrm{r}})$ *at one time is a smooth bijection of* $(\kappa, U) \in (0,\infty)^2$ *onto* $(0,\infty)^2$*, with inverse*

$$\kappa = \frac{k_B a_{\mathrm{r}} T_{\mathrm{r}}}{c} \left(\frac{4\alpha k_B V_c}{3 S_{\mathrm{r}}} \right)^{1/3}, \qquad U = \frac{3 a_{\mathrm{r}} T_{\mathrm{r}} S_{\mathrm{r}}}{4 V_c}.$$

Its logarithmic Jacobian determinant is $-3/4$ *in column order* $(\ln U, \ln \kappa)$*, and* $3/4$ *in the reversed order.*

Proof. Substitution in gives . Positive nonzero powers give injectivity and ranges $(0,\infty)$. The fixed data do not depend on κ, so Lemma applies. The ratios and product follow by cancellation. The log-Jacobian in column order $(\ln U, \ln \kappa)$ is

$$\begin{pmatrix} 1/4 & 3/4 \\ 3/4 & -3/4 \end{pmatrix}.$$

Its determinant is $-3/4$. Solving $U\kappa^3$ and U/κ, or using , gives . The inverse is positive and smooth for every positive label pair.

Corollary (Partial calibration). *At fixed time, an exact temperature* T_0 *gives*

$$U\kappa^3 = X, \qquad X = \frac{\alpha (k_B a_{\mathrm{r}} T_0)^4}{c^3}.$$

With $U \in [U_1, U_2]$*, its compatible coefficient interval is*

$$\left[(X/U_2)^{1/3}, (X/U_1)^{1/3} \right].$$

An exact entropy S_0 *instead gives* $U/\kappa = Y$*,* $Y = (S_0/\widehat{C})^{4/3}$*, and compatible interval* $[U_1/Y, U_2/Y]$*. Their absolute minimax risks are their respective half-widths.*

If κ *is supplied but U ranges over* $(0,\infty)$*, both radiation labels still have infinite absolute minimax risk. Supplying* κ *evaluates them as functions of U; it does not fill U.*

Proof. Invert the two power laws. The compatible sets are closed intervals, so Lemma gives their half-widths. At supplied κ, both labels remain positive powers of an unrestricted U.

General-w labels

Proposition (Declared general-*w* dictionary). *Let* $w > 0$ *be declared, put* $n_w = (1+w)/w$*, and supply* $\varepsilon_s > 0$*,*

independently of the geometry and of κ. On any supplied positive scale factor define

$$u_w = Ua^{-3(1+w)}, \qquad V = V_c a^3,$$

$$T_w = \frac{1}{k_B}\left(\frac{u_w \kappa^3 c^3 \varepsilon_s^{n_w - 4}}{\alpha}\right)^{1/n_w}, \qquad S_w = \frac{(1+w)u_w V}{T_w}.$$

With

$$p_w = \frac{3w}{1+w}, \qquad A_w = \frac{1}{k_B}\left(\frac{c^3 \varepsilon_s^{n_w - 4}}{\alpha}\right)^{1/n_w},$$

the labels are

$$T_w = A_w U^{w/(1+w)} \kappa^{p_w} a^{-3w}, \qquad S_w = \frac{(1+w)V_c}{A_w} U^{1/(1+w)} \kappa^{-p_w}.$$

Their logarithmic Jacobian determinant in column order $(\ln U, \ln \kappa)$ *is* $-3w/(1+w)$.

The product and time ratios are coefficient-free:

$$T_w S_w = (1+w)UV_c a^{-3w}, \qquad \frac{T_w(t)}{T_w(t')} = \left(\frac{a(t')}{a(t)}\right)^{3w}, \qquad \frac{S_w(t)}{S_w(t')} = 1.$$

In contrast, $T_w/S_w \propto \kappa^{2p_w}$ *is not coefficient-free. Substitution* $w = 1/3$ *recovers ;* ε_s *then cancels.*

Proof. Take the $1/n_w = w/(1+w)$ power in the definition of T_w and divide the declared product $(1+w)u_w V$ by it. The log-Jacobian is

$$\begin{pmatrix} w/(1+w) & 3w/(1+w) \\ 1/(1+w) & -3w/(1+w) \end{pmatrix},$$

whose determinant is $-3w/(1+w)$. The remaining assertions are direct cancellations.

par *Scope.* The radiation and general-w dictionaries are declarations. The local calculations parallel Cheung's Big02-A [Big02]. The number w is not a derived equation of state. No species, equilibrium state, entropy-conservation law, or physical coefficient value is inferred. par

Exact absolute and relative risks

Theorem (Absolute interval and box risks). *At fixed U and time, put* $A_U = AU^{1/4}/a_{\mathrm{r}}(t)$ *and* $C_U = \widehat{C}U^{3/4}$. *For* $0 < \kappa_- \le \kappa_+ < \infty$, *the temperature endpoint values are*

$$\ell_T = A_U \kappa_-^{3/4}, \qquad u_T = A_U \kappa_+^{3/4},$$

and the entropy endpoint values are

$$\ell_S = C_U \kappa_+^{-3/4}, \qquad u_S = C_U \kappa_-^{-3/4}.$$

For either label L,

$$\sup_{\kappa} |y - L^{(\kappa)}| = \frac{u_L - \ell_L}{2} + \left|y - \frac{u_L + \ell_L}{2}\right|.$$

The half-spread is also the randomized minimax expected absolute risk.

If instead $(\kappa, U) \in [\kappa_-, \kappa_+] \times [U_-, U_+]$, *the exact componentwise minimax risks are*

$$m_T = \frac{A}{2a_{\mathrm{r}}}\left(U_+^{1/4}\kappa_+^{3/4} - U_-^{1/4}\kappa_-^{3/4}\right),$$

$$m_S = \frac{\widehat{C}}{2}\left((U_+/\kappa_-)^{3/4} - (U_-/\kappa_+)^{3/4}\right).$$

These values hold for randomized decisions as well. For the closed boxes, deterministic midpoint worst cases occur at corners.

Proof. Each image is the interval between the stated monotone endpoint or corner values. Apply Lemma . For randomization, use the two parameter points giving the extreme values of the particular target component.

Remark (Correct unbounded-box statement). An unbounded parameter interval does not by itself make *every* target risk infinite. For independent positive parameter intervals with extended endpoints $U_{\pm}, \kappa_{\pm}$, temperature is unbounded above exactly when $U_{+} = \infty$ or $\kappa_{+} = \infty$. Entropy is unbounded above exactly when $U_{+} = \infty$ or $\kappa_{-} = 0$. For example, at fixed U, allowing $\kappa \in [\kappa_0, \infty)$ makes temperature unbounded but leaves entropy bounded. For a general declared parameter set, apply Lemma to its actual target image.

Theorem (Sharp relative risk and its correct endpoint prior). *Suppose a positive label fills* $[\ell, u]$, $0 < \ell < u < \infty$, *on one data fibre. Under relative loss*

$$L_{\mathrm{rel}}(y, v) = |y/v - 1|,$$

the unique deterministic minimax decision and risk are

$$y_{\mathrm{rel}} = \frac{2\ell u}{\ell + u}, \qquad r_{\mathrm{rel}} = \frac{u - \ell}{u + \ell}.$$

The randomized minimax value is the same. A matching least-favourable prior assigns probabilities

$$\pi(\ell) = \frac{\ell}{\ell + u}, \qquad \pi(u) = \frac{u}{\ell + u}.$$

Equal endpoint weights are the absolute-loss certificate, not generally the relative-loss certificate.

For a coefficient calibration

$$\kappa \in [\kappa_0(1 - \delta), \kappa_0(1 + \delta)], \qquad 0 < \delta < 1,$$

the sharp risks for T_w *and* S_w, *at fixed* U, w, *are*

$$r_{T_w} = r_{S_w} = \tanh\left(p_w \operatorname{artanh} \delta\right).$$

In particular,

$$\text{boxed}\, r_{T_{\mathrm{r}}} = r_{S_{\mathrm{r}}} = \tanh\left(\frac{3}{4}\operatorname{artanh} \delta\right) < \delta.$$

The value equals a declared δ *only if* δ *denotes the label contrast* $(u - \ell)/(u + \ell)$, *rather than the relative width of the coefficient interval.*

Proof. Negative decisions are worse than the decision zero. For $y \geq 0$, the maximum relative loss is at a label endpoint. Equalizing $y/\ell - 1 = 1 - y/u$ gives . For every real y,

$$\frac{\ell}{\ell + u}|y/\ell - 1| + \frac{u}{\ell + u}|y/u - 1| = \frac{|y - \ell| + |y - u|}{\ell + u} \geq \frac{u - \ell}{u + \ell}.$$

Taking expectations proves the randomized lower bound.

For a power κ^{p_w}, or its reciprocal, the ratio of the largest and smallest label values is $((1 + \delta)/(1 - \delta))^{p_w}$. Since

$$\frac{q - 1}{q + 1} = \tanh\left(\frac{1}{2}\ln q\right),$$

this gives . For radiation $p_w = 3/4$, and monotonicity of tanh gives the strict inequality in .

Proposition (Coefficient-blind reports and randomized rules). *Adjoining deterministic reports constant on a data fibre does not refine that fibre. If randomized reports have one common joint law throughout the fibre, any coefficient-blind postprocessing also has one common output law. Consequently, the absolute and relative randomized lower bounds above survive such reports.*

For closed coefficient and energy boxes, and for the late-label boxes in Theorem , randomization does not improve the componentwise absolute minimax values.

Proof. The deterministic statement is the definition of a fibre. The randomized statement is composition of a common law with a common decision kernel. The endpoint-prior inequalities then apply to the resulting common decision law. The box claims use the two corners realizing the respective target extrema.

par *Scope.* These are risks for declared parameter uncertainty, not experimental precision bounds or measurements of a physical constant. Cheung's Big03-A gives the corresponding absolute-risk and linear/inverse-label relative-risk discussion [Big03]. The radiation exponent changes its coefficient-window formula; it cannot be replaced by the linear-label value δ. par

Version 6: orbit–split independence, adaptive non-calibration, and the fibre principle

This subsection is inserted in version 6 (plan items 1, 2 and 5). Its results are proved here on the prescribed prefix of Definition with the dictionary of Definition . The words ordinary, dark and vacuum below name three declared slots; no matter model is introduced.

Definition (Split-augmented accounts; inserted in version 6). Declare $\Lambda \in \mathbb{R}$ and a coupling $\chi \neq 0$, written χ and never κ. On the prefix region put $\Theta = G + \Lambda g$ as in Proposition and $T_{\mathrm{tot}} = \chi^{-1}\Theta$; this is a definition, not a field equation. A *split* is a triple $\sigma = (T_{\mathrm{o}}, T_{\mathrm{d}}, T_{\mathrm{v}})$ of symmetric divergence-free tensor fields on the prefix region with $T_{\mathrm{o}} + T_{\mathrm{d}} + T_{\mathrm{v}} = T_{\mathrm{tot}}$. Write $\Sigma(\Lambda, \chi)$ for the set of splits. A *split-augmented account* is $x = (\sigma; U, \kappa, \alpha)$ with $\sigma \in \Sigma(\Lambda, \chi)$ and $U, \kappa, \alpha > 0$; the prefix metric, $c, G, k_B, V_c, \Lambda, \chi$ are fixed. The *split-blind datum* is $D_{\mathrm{tot}} = (g_a|_{[t_i,0]}, c, G, k_B, V_c, \Lambda, \chi, T_{\mathrm{tot}})$. The *coefficient orbit* of x is $\mathcal{O}(x) = \{(\sigma; U, q\kappa, q^3\alpha) : q > 0\}$, and its *split fibre* is $\Sigma(\Lambda, \chi) \times \{(U, \kappa, \alpha)\}$.

Proposition (Orbit–split independence of the radiation labels; inserted in version 6). *On split-augmented accounts, at every prefix time t, put $z = \kappa^3/\alpha$.*

• *The labels are*

$$T_{\mathrm{r}} = \frac{(c^3 U z)^{1/4}}{k_B a_{\mathrm{r}}(t)}, \qquad S_{\mathrm{r}} = \frac{4}{3} k_B V_c c^{-3/4} U^{3/4} z^{-1/4}.$$

They depend on (κ, α) only through z and do not depend on σ. Hence each label is constant on every split fibre and on every coefficient orbit.

• *The fibre of $x \mapsto (D_{\mathrm{tot}}, T_{\mathrm{r}}(t), S_{\mathrm{r}}(t))$ through $x = (\sigma; U, \kappa, \alpha)$ is exactly*

$$\Sigma(\Lambda, \chi) \times \{(U, q\kappa, q^3\alpha) : q > 0\},$$

the product of the split fibre and the coefficient orbit. The pair $(T_{\mathrm{r}}, S_{\mathrm{r}})$ at t determines U and z through

$$U = \frac{3 a_{\mathrm{r}} T_{\mathrm{r}} S_{\mathrm{r}}}{4 V_c}, \qquad z = \frac{(k_B a_{\mathrm{r}} T_{\mathrm{r}})^4}{U c^3}.$$

• *$\Sigma(\Lambda, \chi)$ contains the family $\sigma_s = (sT_{\mathrm{tot}}, (1-s)T_{\mathrm{tot}}, 0)$, $s \in [0, 1]$, with $\sigma_s \neq \sigma_{s'}$ for $s \neq s'$. Consequently a split target τ with $\tau(\sigma_{s_0}) \neq \tau(\sigma_{s_1})$ is not a function of $(D_{\mathrm{tot}}, T_{\mathrm{r}}, S_{\mathrm{r}})$, and every rule has worst-case error at least $\frac{1}{2}|\tau(\sigma_{s_0}) - \tau(\sigma_{s_1})|$. Adjoining the labels identifies no split.*

• *Adjoining a declared split σ to $(D_{\mathrm{tot}}, T_{\mathrm{r}}, S_{\mathrm{r}})$ identifies no point of the orbit: on the resulting fibre κ ranges over $(0, \infty)$. Fixing a point of the orbit by an added supply, of κ or of α, leaves the split factor of the fibre equal to $\Sigma(\Lambda, \chi)$; it selects no split.*

• *At fixed U, T_{r} is strictly increasing and S_{r} strictly decreasing in z. The labels do not vary along an orbit; they vary only across orbits, through z. When α is supplied, as in the admitted datum $\mathfrak{E}$, the orbit through x meets $\{\alpha\text{fixed}\}$ only at x; it is then not a confounding orbit, and the labels are injective in κ (Theorem).*

Proof. (a) Substitute $u = U a_{\mathrm{r}}^{-4}$ and $V = V_c a_{\mathrm{r}}^3$ in : $T_{\mathrm{r}}^4 = c^3 U z/(k_B^4 a_{\mathrm{r}}^4)$ and $S_{\mathrm{r}} = 4UV_c/(3 a_{\mathrm{r}} T_{\mathrm{r}})$, which gives the displayed forms. No slot of σ occurs, and z is unchanged by $(\kappa, \alpha) \mapsto (q\kappa, q^3\alpha)$. (b) By (a), every point of the displayed

product has the same datum and labels. Conversely, if $x' = (\sigma'; U', \kappa', \alpha')$ has the same D_{tot} and labels, the displayed inverse formulas, which follow from and the fourth power of T_{r}, give $U' = U$ and $z' = z$. With $q = \kappa'/\kappa$, $z' = z$ gives $\alpha' = q^3\alpha$, and $\sigma' \in \Sigma(\Lambda, \chi)$ is arbitrary. (c) Multiples of a symmetric divergence-free tensor are symmetric and divergence-free, and the three entries of σ_s sum to T_{tot}. By Proposition , $G_{tt} = 3H_{\mathrm{r}}^2$, and $g_{tt} = -c^2$, so $\Theta_{tt} = 3H_{\mathrm{r}}^2 - \Lambda c^2$. Since H_{r} is strictly decreasing on the prefix, Θ_{tt} vanishes at most at one time, so $T_{\mathrm{tot}} \equiv 0$ and the σ_s are distinct. Two accounts $(\sigma_{s_0}; U, \kappa, \alpha)$ and $(\sigma_{s_1}; U, \kappa, \alpha)$ lie in one fibre by (b). A rule F takes one value y there, and $|y - \tau_0| + |y - \tau_1| \geq |\tau_0 - \tau_1|$ gives the bound; Lemma (a) gives non-factorization. (d) Intersecting the fibre of (b) with $\{\sigma\}$ leaves $\{(\sigma; U, q\kappa, q^3\alpha) : q > 0\}$, on which κ takes every positive value. Supplying κ forces $q = 1$; supplying α forces $q^3 = 1$, so $q = 1$. In both cases the fibre is $\Sigma(\Lambda, \chi) \times \{(U, \kappa, \alpha)\}$. (e) Positive and negative quarter powers are strictly monotone; z is constant on an orbit. If α is fixed, $q^3\alpha = \alpha$ forces $q = 1$.

Status: proved here. Big05-A records the same structure on its enlarged accounts [Big05]; Big04-A-coeff gives the complete κ^3/α action, its invariants U and z, and the scope qualification that the orbit confounds only in a data contract omitting α [Big04Coeff]; Big03-A states the same qualification [Big03v3]. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* Calibrating κ and declaring a split are separate added supplies; neither is a function of D_{tot}. No split is selected, no value of κ is admitted, and no particle species, matter model, quantum coefficient or dark capacity is identified with a label or a slot. The tensor T_{tot} is a definition; no Einstein dynamics is derived. par

Theorem (Adaptive coefficient-blind protocols cannot calibrate the radiation labels; inserted in version 6). *Fix a prefix time t, U, α, V_c and the dictionary constants, and let κ range over a set $\mathcal{K} \subset (0, \infty)$ with at least two points. Consider a finite adaptive protocol of n rounds, with standard Borel outcome spaces, in which*

• the initial datum has a law independent of $\kappa \in \mathcal{K}$;

• the experiment of round $r + 1$, including a decision to stop, is selected from the transcript of rounds $1, \ldots, r$ by a kernel independent of κ;

• conditional on the selected experiment and the past transcript, the report law of round $r + 1$ is independent of κ; for a quantum round this means that the complete accessible process, reference extensions and every register retained across rounds (cross-round memory) included, and the tester are the same for every κ. Then:

• The joint law of the transcript, and the law of every randomized estimate Y computed from it, are the same for all $\kappa \in \mathcal{K}$. If the kernels in (i)–(iii) also do not depend on α, the law is constant on each coefficient orbit of Definition .

• For $\mathcal{K} = (0, \infty)$, every estimate of T_{r} or S_{r} has infinite worst-case expected absolute error. Under relative loss $|y/L - 1|$, every estimate has worst-case expected relative loss at least 1, and the decision 0 attains exactly 1.

• For a window $\mathcal{K} = [\kappa_0(1 - \delta), \kappa_0(1 + \delta)]$, $0 < \delta < 1$, with label endpoint values $\ell < u$, the minimax expected absolute error of the protocol is $(u - \ell)/2$, the value of Theorem , and its minimax expected relative loss is $\tanh(\frac{3}{4}\operatorname{artanh}\delta)$, the value of , for each label. Constant decisions attain these values, so adaptivity does not lower them.

• The hypotheses fail for every protocol one of whose reports has a κ-dependent law, in particular for a report of an exact label value. Such a report is a calibration by added supply (Theorem , Corollary), not an output of a coefficient-blind protocol.

Proof. (a) Let P_r be the law of the first r rounds. P_0 is common by (i). If P_r is common, P_{r+1} is P_r composed with the common kernel of (ii) and the common conditional law of (iii), hence common. For a quantum round, the report law is the tester applied to the output of the complete process; a common process and a common tester give a common law. A randomized estimate is a kernel applied to the transcript and preserves equality of laws. If no kernel depends on κ or α, and U is fixed on the orbit, the same induction gives one law on the orbit. (b) On $(0, \infty)$ both labels have image $(0, \infty)$ (Theorem). Let Y have the common law. If $\mathbb{E}|Y| < \infty$, then $\mathbb{E}|Y - L| \geq L - \mathbb{E}|Y| \to \infty$ as $L \to \infty$; otherwise the error is infinite at every L. The decision 0 has relative loss 1 at every L. If $\mathbb{E}|Y| < \infty$, then

$\mathbb{E}|Y/L - 1| \geq 1 - \mathbb{E}|Y|/L \to 1$ as $L \to \infty$, which occurs as $\kappa \to \infty$ for T_r and as $\kappa \to 0$ for S_r; otherwise the loss is infinite. (c) The endpoint values are attained at the window endpoints, by monotonicity. Under the common law, $\frac{1}{2}\mathbb{E}|Y - \ell| + \frac{1}{2}\mathbb{E}|Y - u| \geq (u - \ell)/2$, so one endpoint has expected error at least $(u - \ell)/2$. For relative loss, the pointwise identity in the proof of Theorem gives $\frac{\ell}{\ell + u}\mathbb{E}|Y/\ell - 1| + \frac{u}{\ell + u}\mathbb{E}|Y/u - 1| \geq \frac{u - \ell}{u + \ell}$. The midpoint and the harmonic mean attain the two values (Theorems and). For either label, $u/\ell = ((1 + \delta)/(1 - \delta))^{3/4}$, and $(q - 1)/(q + 1) = \tanh(\frac{1}{2}\ln q)$ gives $\tanh(\frac{3}{4}\operatorname{artanh}\delta)$. (d) A report equal to $T_r^{(\kappa)}$ has the point law at $T_r^{(\kappa)}$, and these laws differ for distinct κ by injectivity (Theorem).

Remark (Supplied window versus adaptive search). Theorem presupposes a supplied coefficient window, which is itself an added supply, and computes the least risk given that window. Theorem (c) shows that a coefficient-blind adaptive search cannot shrink the window or lower that risk. In both statements the window is an input and no value of κ is selected.

Status: proved here. Parallel statements are Big04-A-coeff [Big04Coeff], Big03-A, which adds stopping and postselection [Big03v3], and En04-B [En04B]. Big03-A also shows that placement blindness or marginal blindness does not imply coefficient blindness [Big03v3]; hypothesis (iii) therefore concerns the complete accessible process. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* A negative certificate for the printed protocol class. No coefficient-blind protocol, geometric observable or recovery error selects a value of κ or a split; an exact label at fixed supplied normalization does calibrate κ (Theorem). No numerical κ and no experimental precision bound is asserted. par

Proposition (The fibre principle for the radiation labels; inserted in version 6). *Let D be a data map on a declared account set $\mathscr{X}$, let τ be a real target on $\mathscr{X}$, for instance T_r, S_r or a horizon radius, and let $\ell(y, v) \geq 0$ be a loss.*

• *A rule recovering τ exactly from D exists iff τ is constant on every fibre of D.*

• *If D is constant on a set $\mathscr{F} \subset \mathscr{X}$, the deterministic minimax risk on $\mathscr{F}$ is $\inf_y \sup_{v \in \tau(\mathscr{F})} \ell(y, v)$, and the randomized minimax risk is $\inf_\mu \sup_{v \in \tau(\mathscr{F})} \int \ell(y, v)\, \mu(\mathrm{d}y)$. Both depend on $\mathscr{F}$ only through the image $\tau(\mathscr{F})$.*

• *On a coefficient fibre $\{\kappa \in \mathscr{K}\}$ at fixed (U, α, t), the label T_r (or S_r) is constant iff $\mathscr{K}$ is a single point; hence no rule recovers it from data constant on such a fibre with two or more points. On a coefficient orbit of Definition both labels are constant; there (a) permits their recovery, for instance from data determining (U, z), and such data do not determine κ.*

• *The value statements of Theorem , Theorem , Corollary , Theorem , Proposition , Theorem , Theorem and Theorem (c) are instances of (b): absolute loss, with functional $(\sup - \inf)/2$ of the image, or relative loss, with functional $(u - \ell)/(u + \ell)$ of a closed image $[\ell, u]$. The theorem-specific content of each is the computation of the image.*

Proof. (a) Lemma (a). (b) A deterministic rule takes one value y on $\mathscr{F}$, with worst-case loss $\sup_{x \in \mathscr{F}} \ell(y, \tau(x)) = \sup_{v \in \tau(\mathscr{F})} \ell(y, v)$. A randomized rule has one law μ on $\mathscr{F}$ because D is constant there; the same substitution applies. (c) Injectivity in κ (Theorem) gives the first claim. Orbit constancy is Proposition (a); the formulas there express both labels through (U, z), and on an orbit $\kappa = q\kappa_0$ takes every positive value. (d) In each cited statement the data are constant on the parameter set considered: the coefficient range at fixed data in Theorem ; the boxes and windows in Theorems , and , Corollary and Proposition ; and the futures with common early data in Theorems and . Each value is the functional of (b) on the image computed there; for absolute loss this functional is Lemma (b),(d),(e), and for relative loss Theorem .

Status: proved here. The same elementary principle is re-proved in Big04-A-coeff [Big04Coeff] and DS-OB-1-fibre [DSOB1fibre]. Big05-A applies it to geometric observables [Big05] and to dominance targets [Big05]. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* The principle proves nothing without a fibre witness, and the image computations remain the substantive content of the cited theorems. No label is recovered from coefficient-blind data, no value of κ is

admitted, and the risks are not experimental precision bounds. par

Version 6: supplied normalizations, late supplies and approximate blindness

This subsection is inserted in version 6 from the author's draft ``Big04-A: Version-6 Additions Only'' (Sections 1 and 2 of that draft); each result is re-proved here, and the draft verdicts F11-*n* are in Appendix . Throughout, $\zeta = \kappa^3/\alpha$ as in Proposition , and $a = a_{\mathrm{r}}(t)$ at a fixed prefix time. With $u = Ua^{-4}$ and $V = V_c a^3$ the dictionary reads

$$T_{\mathrm{r}} = \frac{c^{3/4}}{k_B a}(U\zeta)^{1/4}, \qquad S_{\mathrm{r}} = \frac{4}{3}k_B V_c c^{-3/4} U^{3/4}\zeta^{-1/4},$$

so that the pair determines $U = 3aT_{\mathrm{r}}S_{\mathrm{r}}/(4V_c)$ and $\zeta = (k_B a T_{\mathrm{r}})^4/(Uc^3)$. The split coordinate of Definition ranges over $\Sigma(\Lambda,\chi)$ independently of (U,κ,α): the account class is the Cartesian product $\Sigma(\Lambda,\chi)\times(0,\infty)^3$, and no admissibility condition couples the split to the dictionary coordinates. An added coupling relation is a different account class.

Corollary (Calibration with a supplied normalization window; inserted in version 6). *Suppose an exact radiation pair at one prefix time is supplied, so that U_* and ζ_* are fixed, while α is supplied only through $0 < \alpha_- \le \alpha \le \alpha_+ < \infty$. Then:*

- *the compatible coefficients form exactly $I = [(\zeta_*\alpha_-)^{1/3}, (\zeta_*\alpha_+)^{1/3}] = [\ell, u]$;*
- *the deterministic and randomized absolute minimax risks for κ are $(u-\ell)/2$, attained by the midpoint;*
- *under the relative loss $|Y/\kappa - 1|$ the deterministic and randomized minimax risks are $\tanh(\frac{1}{6}\ln(\alpha_+/\alpha_-))$, attained by the harmonic decision $2\ell u/(\ell+u)$;*
- *if α is restricted only by positivity, every $\kappa > 0$ is compatible, and the pair does not determine κ.*

Proof. (a) $\kappa^3 = \zeta_*\alpha$ is strictly increasing in α. (b) The midpoint has risk $(u-\ell)/2$; for any randomized Y, $\frac{1}{2}\mathbb{E}|Y-\ell| + \frac{1}{2}\mathbb{E}|Y-u| \ge (u-\ell)/2$ by the triangle inequality, so the two-point prior matches. (c) For $Y_h = 2\ell u/(\ell+u)$, $Y_h/\ell - 1 = (u-\ell)/(u+\ell) = 1 - Y_h/u$, and on $[\ell,u]$ the loss $|Y_h/\kappa - 1|$ is maximal at an endpoint. For any randomized Y, with weights $\ell/(\ell+u)$ and $u/(\ell+u)$ at ℓ and u, $\frac{\ell}{\ell+u}|Y/\ell - 1| + \frac{u}{\ell+u}|Y/u - 1| = \frac{|Y-\ell|+|Y-u|}{\ell+u} \ge \frac{u-\ell}{u+\ell}$. Finally $u/\ell = (\alpha_+/\alpha_-)^{1/3}$ and $(u-\ell)/(u+\ell) = \tanh(\frac{1}{2}\ln(u/\ell))$. (d) For every $\kappa > 0$ take $\alpha = \kappa^3/\zeta_*$.

par *Non-claim.* The window on α is supplied; no normalization is derived, and no numerical κ is admitted. par

Corollary (A supplied late label breaks the compensating orbit; inserted in version 6). *Fix an exact radiation pair (so U_*, ζ_* are fixed) and suppose a terminal rate $K > 0$ and one exact late label of are separately supplied. Then*

$$\kappa = \frac{2\pi k_B T_{\mathrm{late}}}{K}, \qquad \kappa = \frac{\pi k_B c^5}{GK^2 S_{\mathrm{late}}}, \qquad \kappa = \frac{\pi c^5}{GK^2 \ln \mathrm{N}_{\mathrm{late}}} \quad (N_{\mathrm{late}} > 1),$$

and $\alpha = \kappa^3/\zeta_*$. Under $(\kappa,\alpha) \mapsto (r\kappa, r^3\alpha)$ the late labels transform as $T_{\mathrm{late}} \mapsto rT_{\mathrm{late}}$, $S_{\mathrm{late}} \mapsto r^{-1}S_{\mathrm{late}}$, $N_{\mathrm{late}}\mathrm{mapsto}\mathrm{N}_{\mathrm{late}}^{1/r}$, so each separates the orbit. None of these supplies selects a split.

Proof. Invert ; $\ln \mathrm{N}_{\mathrm{late}} = S_{\mathrm{late}}/k_B$. Substitute $r\kappa$. No split coordinate occurs in .

par *Non-claim.* The late labels are external dictionaries at a supplied rate (Section); the corollary is calibration by added supply, not a host derivation. par

Proposition (Unrestricted blind families and blind confidence sets; inserted in version 6). *Let a protocol be coefficient-blind in the sense of Theorem on the unrestricted family $\kappa > 0$ at fixed U, α, t, with real-valued (possibly randomized) output Y or interval output I.*

- *If the protocol reports a random interval I whose upper end I^+ is finite almost surely, then $\inf_{\kappa>0} \Pr_\kappa\{\kappa \in I\} = 0$.*
- *If M target values are pairwise separated by more than 2ε, at least one of them has $\Pr\{|Y-L| \le \varepsilon\} \le 1/M$. The unbounded absolute risk and the relative value 1 on this family are Theorem (b).*

Proof. All laws are one common law P (Theorem (a)). (a) $\Pr_\kappa\{\kappa \in I\} \le P\{I^+ \ge \kappa\} \to 0$ as $\kappa \to \infty$, since $I^+ < \infty$ almost surely. (b) Under the common law the M events $\{|Y - L_m| \le \varepsilon\}$ are disjoint, so their probabilities sum to at most one.

par *Non-claim.* Blind acquisition does not manufacture a finite window; a window used in Theorem (c) is supplied. par

Theorem (Two-point bounds from approximate blindness; inserted in version 6). *Let P_0, P_1 be the complete observed laws (transcript and every retained record) at two parameter points, with $\mathrm{TV}(P_0, P_1) \le \eta \in [0,1]$, and let Y be obtained by a common randomized postprocessing.*

• *For targets L_0, L_1:* $\max_i \mathbb{E}_i|Y - L_i| \ge \frac{1}{2}|L_1 - L_0|(1-\eta)$.

• *For targets $0 < \ell < u$:* $\max\{\mathbb{E}_0|Y/\ell - 1|, \mathbb{E}_1|Y/u - 1|\} \ge \frac{u-\ell}{u+\ell}(1-\eta)$.

• *If at each round $j \le n$ the two complete round maps, including returned records, control kernels and persistent memory, differ by at most η_j in half-diamond distance, then $\mathrm{TV}(P_0, P_1) \le \min\{1, \sum_j \eta_j\}$. With $\eta = 0$, (a) and (b) are the exact bounds used in Theorem .*

Proof. A common postprocessing does not increase total variation, so we may take Y a function of the observation and of independent common randomness. Let $\mu = \min(p_0, p_1)\,\mathrm{d}(P_0 + P_1)$ with p_i the densities; $\mu \le P_i$ and $\mu(\Omega) = 1 - \mathrm{TV}(P_0, P_1) \ge 1 - \eta$. (a) $\max_i \mathbb{E}_i|Y - L_i| \ge \frac{1}{2}\int(|Y - L_0| + |Y - L_1|)\,\mathrm{d}\mu \ge \frac{1}{2}|L_1 - L_0|\mu(\Omega)$. (b) The maximum dominates the average with weights $\ell/(\ell+u), u/(\ell+u)$, which is at least $\int \frac{|Y-\ell| + |Y-u|}{\ell+u}\,\mathrm{d}\mu \ge \frac{u-\ell}{u+\ell}\mu(\Omega)$. (c) Let $\rho_j, \widetilde{\rho}_j$ be the complete states after round j (histories included) and $\Phi_j, \widetilde{\Phi}_j$ the round maps. Then $\frac{1}{2}\|\Phi_j(\rho) - \widetilde{\Phi}_j(\widetilde{\rho})\|_1 \le \frac{1}{2}\|\rho - \widetilde{\rho}\|_1 + \frac{1}{2}\|(\Phi_j - \widetilde{\Phi}_j)(\widetilde{\rho})\|_1$, by contractivity of Φ_j; the second term is at most η_j. The initial states agree; induction and the final measurement give the bound, and TV ≤ 1 always.

Remark (Postselection and which orbit is blind). The exact common-law statement survives common positive-probability postselection, since conditioning one common law on one common event of positive probability gives one common law (Theorem (a)); an approximate allowance η does not transfer to normalized selected branches, whose conditional laws need their own bound. Blindness at fixed supplied U, α concerns the fixed-α curve $\kappa \mapsto q\kappa$, along which the labels vary; it gives no common law along $(\kappa, \alpha) \mapsto (r\kappa, r^3\alpha)$, which changes α. Blindness on that orbit alone does not obstruct evaluation of the labels, which are constant there; it obstructs the separation of κ from α.

Documentary parallels: Big03-A version 3 [Big03v3] and Big04-A-coeff [Big04Coeff]. None is a premise above. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* This is not a theorem that every experiment is coefficient-blind; a coefficient-sensitive preparation, apparatus or supplied label changes the hypotheses, and a constant optimized recovery error does not by itself establish process blindness. par

Finite-channel certificates

All Hilbert spaces in this section are finite dimensional. Channels are completely positive and trace preserving. For channels on M_d, define

$$d_\diamond(T,S) = \frac{1}{2}\|T - S\|_\diamond, \qquad \varepsilon(T) = d_\diamond(T, \mathrm{id}), \qquad \varepsilon^*(\mathscr{N}) = \inf_{\mathscr{D}} \varepsilon(\mathscr{D}\mathscr{N}).$$

Unless explicitly restricted, decoders are arbitrary channels on the declared complete receiver interface.

Use output-first Choi operators

$$J(T) = \sum_{ij} T(|i\rangle\langle j|) \otimes |i\rangle\langle j|, \quad \Omega_d = \sum_i |ii\rangle, \quad W_d = |\Omega_d\rangle\langle\Omega_d|, \quad \Phi_d = W_d/d.$$

Thus Ω_d is a vector, W_d an unnormalized operator, and Φ_d a state.

Lemma (Elementary primal and dual bounds). *Put* $C_T = W_d - J(T)$.

• *For every pure referenced input* ψ *and effect* $0 \le P \le I$,

$$\varepsilon(T) \ge \operatorname{Tr} P(\operatorname{proj}\psi - (T \otimes \mathrm{id})(\operatorname{proj}\psi)).$$

In particular, with $P = \operatorname{proj}\psi$, *the value is* $1 - \langle\psi|(T \otimes \mathrm{id})(\operatorname{proj}\psi)|\psi\rangle$.

• *If* $Z_- \ge 0$, $Z_- \ge C_T$, *and* $\operatorname{Tr}_{\mathrm{out}} Z_- \le eI$, *then* $\varepsilon(T) \le e$. *The partner* $Z_+ = Z_- - C_T$ *certifies the opposite Choi sign and has the same partial trace.*

• *If* $J(T) \ge fW_d$, $0 \le f \le 1$, *take*

$$Z_- = (1-f)W_d, \qquad Z_+ = J(T) - fW_d.$$

Then $\varepsilon(T) \le 1 - f$. *If a primal input has overlap f, equality holds. Conjugating by the factor swap converts every witness to input-first order; it is not a partial transpose.*

Proof. The primal bound is the variational formula for the positive part of a traceless Hermitian difference of states. For the dual bound write a unit referenced vector as $(I \otimes A)\Omega_d$, with $\operatorname{Tr} A^\dagger A = 1$. Filtering $C_T \le Z_-$ gives, for every effect P,

$$\operatorname{Tr} P(I \otimes A)C_T(I \otimes A^\dagger) \le \operatorname{Tr}[(\operatorname{Tr}_{\mathrm{out}} Z_-)A^\dagger A] \le e.$$

Pure inputs suffice by purification and convexity. Trace preservation gives $\operatorname{Tr}_{\mathrm{out}} C_T = 0$, proving the partner statement. Part (c) follows by substitution.

This is the elementary weak-duality part of the standard diamond-norm SDP framework [WatrousSDP]. The proofs below do not need an asserted numerical SDP value.

Lemma (Exact replacement spectrum). *For a state* σ *on* $\mathbb{C}^d$, *define*

$$\Im(\sigma) = \sum_i \lambda_i^{-1},$$

with $\Im = \infty$ *if any eigenvalue vanishes and* $1/\infty = 0$. *Then*

$$\varepsilon(\mathscr{R}_\sigma) = 1 - \frac{1}{\Im(\sigma)}, \qquad \Im(\sigma) \ge d^2.$$

Equality $\Im = d^2$ *holds exactly for* $\sigma = I/d$.

For full-rank σ, *put* $f = 1/\Im(\sigma)$. *An exact dual pair is*

$$Z_- = (1-f)W_d, \qquad Z_+ = \sigma \otimes I - fW_d.$$

In an eigenbasis e_i *of* σ, *the primal vector is*

$$\psi = \sum_i \sqrt{\frac{\lambda_i^{-1}}{\Im(\sigma)}}\, e_i \otimes \overline{e_i},$$

with effect $\operatorname{proj}\psi$. *For singular* σ, *a product input in its output kernel gives error one.*

Proof. Here $J(\mathscr{R}_\sigma) = \sigma \otimes I$. Weighted Cauchy–Schwarz gives

$$|\langle\Omega_d, v\rangle|^2 \le \operatorname{Tr}(\sigma^{-1})\langle v, (\sigma \otimes I)v\rangle,$$

so $J(\mathscr{R}_\sigma) \ge fW_d$. The indicated input has replacement overlap $\sum_i \lambda_i(\lambda_i^{-1}/\Im)^2 = f$. Lemma proves equality. If σ is singular, choose a unit vector in its kernel; its output is orthogonal to that input. Finally

$$d^2 \le \Big(\sum_i \lambda_i\Big)\Big(\sum_i \lambda_i^{-1}\Big) = \Im,$$

with equality exactly for equal eigenvalues.

Proposition (Dephasing, classical records, and the reading gap). *Let* $\mathscr{B} = \{b_a\}_{a=1}^d$ *be an orthonormal basis,* $P_a = |b_a\rangle\langle b_a|$, *and* $\Delta_{\mathscr{B}}(X) = \sum_a P_a X P_a$. *Then*

$$\varepsilon(\Delta_{\mathscr{B}}) = \varepsilon^{*}(\Delta_{\mathscr{B}}) = 1 - \frac{1}{d}.$$

Writing $\Pi_{\mathscr{B}} = \sum_a P_a \otimes \overline{P_a}$, an exact dual pair is

$$Z_{-} = (1 - 1/d)W_d, \qquad Z_{+} = \Pi_{\mathscr{B}} - W_d/d.$$

The Bell input is a matching primal.

More generally, every channel whose complete message-bearing output is classical has recovery error at least $1 - 1/d$. The optimum over such interfaces is $1 - 1/d$. For record-ignoring composites the optimum is $1 - 1/d^2$. Their exact difference is

$$\left(1 - \frac{1}{d^2}\right) - \left(1 - \frac{1}{d}\right) = \frac{d-1}{d^2}.$$

Proof. The projector $\Pi_{\mathscr{B}}$ contains the unit Bell vector in its range, so $\Pi_{\mathscr{B}} \geq W_d/d$. Its partial trace is I, giving the stated witnesses. The Bell overlap is $1/d$.

A channel through a classical output is measure-and-prepare. Its referenced Bell output after any decoder is separable. Every normalized separable state has overlap at most $1/d$ with Φ_d, as follows first for product pure states by Cauchy–Schwarz and then by convexity. This proves the classical lower bound, attained by dephasing. Every record-ignoring composite is replacement, so Lemma gives the other optimum and the gap.

The measure-and-prepare interpretation is consistent with [HSR]. Cheung's I-C-CERT interprets as the gain from reading the record on its specified interface [ICcert]. It is not a physical distance between a shredder and a horizon.

Complete causal-access loss and partially dark interfaces

Definition (Located-circuit access contract). A finite acyclic circuit consists of channels with a complete dependency graph, including quantum wires, classical controls, and retained records. Each operation has a position in a separately supplied time-oriented spacetime. Every dependency edge respects its causal relation.

The source inserts an arbitrary payload independently of the circuit's fixed initial resources. An untouched external reference may purify that payload and is not granted to the decoder. The complete receiver interface consists of the declared retained receiver outputs. No message-correlated bypass is omitted from the graph.

Lemma (Joint ancestor lemma). *Under Definition , if no receiver output is a descendant of the source, the joint receiver channel is replacement, $\mathscr{R}_\sigma$, for one state σ. The state may be correlated across receiver factors. If no receiver position is causally reachable from the source, this hypothesis holds.*

Proof. Pull a joint receiver observable backwards through channel adjoints. Gates outside its dependency ancestry act on identity factors and disappear because channel adjoints preserve identity. The pullback is therefore the identity on the payload tensor a fixed observable on the initial circuit resources. Its expectation is independent of the payload. Observables separate finite-dimensional states, so the joint receiver output is constant. Linearity makes the map $X \mapsto \mathrm{Tr}\,(X)\sigma$.

Every descendant path is future-causal by the edge condition and transitivity. Hence causal non-reachability excludes descendants.

Theorem (Exactly dark early sources). *Let the complete receivers lie on γ, and suppose an early source satisfies*

$$|\mathbf{x}| \geq B(1 - a_{\mathrm{r}}(t)) + R_0.$$

Under the located-circuit contract, its joint accessible channel is replacement. For every decoder,

$$\mathscr{D}\mathscr{R}_\sigma = \mathscr{R}_{\mathscr{D}(\sigma)}, \qquad \varepsilon(\mathscr{D}\mathscr{R}_\sigma) \geq a_d, \qquad a_d := 1 - d^{-2}.$$

The optimized error is exactly a_d, attained by a decoder preparing I/d. A decoder attains the optimum exactly when its output on σ is I/d.

This floor depends on the active target dimension and the access contract, not on κ, U, α, V_c or the radiation labels.

Proof. Apply Lemmas and , then Lemma . The Bell input also gives the universal replacement lower bound directly:

$$\langle \Phi_d | \sigma' \otimes I/d | \Phi_d \rangle = d^{-2}.$$

Proposition (Correlated receiver outputs require a joint certificate). *For a replacement on a two-qubit target, take*

$$\sigma = \operatorname{diag}\left(\frac{2}{5}, \frac{1}{10}, \frac{1}{10}, \frac{2}{5}\right).$$

Both qubit marginals are $I_2/2$*, but*

$$\Im(\sigma) = \frac{5}{2} + 10 + 10 + \frac{5}{2} = 25, \qquad \varepsilon(\mathscr{R}_\sigma) = \frac{24}{25}.$$

The product of the marginal replacement parameters would instead predict $1 - 1/16 = 15/16$*, which is not the joint installed error.*

The unrestricted optimized recovery error is nevertheless $15/16$*, because the target has dimension four and its decoder may prepare* $I_4/4$*.*

Proof. The marginal and reciprocal-eigenvalue calculations are exact rational identities. Apply Lemma and Theorem . The marginal-product state is different from σ; its witness is not a joint witness for σ.

Theorem (Robust envelope, continuity, and mixture range). *For finite channels with a common target dimension:*

• *Optimized recovery is* 1*-Lipschitz:*

$$|\varepsilon^*(\mathscr{N}) - \varepsilon^*(\widetilde{\mathscr{N}})| \operatorname{led}_\diamond(\mathscr{N}, \widetilde{\mathscr{N}}).$$

• *If* $d_\diamond(\mathscr{N}, \mathscr{R}_\sigma) \le \eta$*, then*

$$[a_d - \eta]_+ \le \varepsilon^*(\mathscr{N}) \le a_d.$$

The exact infimum over this replacement neighbourhood is $[a_d - \eta]_+$*.*

• *The channel*

$$\mathscr{N}_\lambda = (1-\lambda)\mathscr{R}_{I/d} + \lambda \mathrm{id}$$

satisfies

$$d_\diamond(\mathscr{N}_\lambda, \mathscr{R}_{I/d}) = \lambda a_d, \qquad \varepsilon(\mathscr{N}_\lambda) = \varepsilon^*(\mathscr{N}_\lambda) = (1-\lambda) a_d.$$

It attains the lower envelope for $\lambda = \eta/a_d$ *when* $\eta \le a_d$*; the identity attains it otherwise.*

• *In the declared mixture class*

$$\mathscr{N} = (1-\lambda)\mathscr{R}_\sigma + \lambda \mathscr{M}, \qquad 0 \le \lambda \le 1,$$

with arbitrary visible branch $\mathscr{M}$*, the exact range of optimized errors is*

$$[(1-\lambda)a_d, a_d].$$

The deterministic minimax risk for predicting this optimized error from λ, d *alone is* $\lambda a_d/2$*; randomization does not improve it.*

Proof. For any decoder, contraction and the triangle inequality give

$$\varepsilon(\mathscr{D}\mathscr{N}) \le \varepsilon(\mathscr{D}\widetilde{\mathscr{N}}) + d_\diamond(\mathscr{N}, \widetilde{\mathscr{N}}).$$

Take infima and exchange the channels. This proves (a). Part (b) follows from Theorem ; the universal upper bound uses a constant decoder.

For (c), norm homogeneity gives the distance and installed error. After any decoder, the Bell overlap of the

replacement branch is $1/d^2$ and that of the identity branch is at most one. Thus no decoder improves the stated value. Equivalently, its dual witnesses are

$$Z_{-} = (1-\lambda)a_d W_d, \qquad Z_{+} = (1-\lambda)(I/d - W_d/d^2),$$

where I/d acts on the d^2-dimensional Choi space.

The same branchwise Bell bound proves the lower end in (d). The upper end is attained by a replacement visible branch. For the whole interval take $\sigma = I/d$ and $\mathscr{M} = (1-\mu)\mathscr{R}_{I/d} + \mu\mathrm{id}$, $0 \le \mu \le 1$. The composite is $\mathscr{N}_{\lambda\mu}$. Apply Lemma to its closed error range.

Remark (Mixture weight is not causal transparency). A nonzero declared λ does not prove nonzero physical influence: the visible branch can itself be replacement. The uniform lower floor over the entire mixture class drops below a_d when $\lambda > 0$, but a particular channel may still have error a_d. No geometric derivation of λ is made.

par *Scope.* These results parallel Cheung's cross-epoch certificate [Big02]. They concern a complete declared channel. They do not construct a receiver network, a field-theoretic transport channel, Hawking radiation, or a physical partially transparent horizon. par

A finer declared cut ladder

Proposition (Flagged quantum width and replacement interpolation). *Suppose every message-bearing output consists of a finite classical record and, conditional on that record, a quantum system of dimension at most $m \le d$, without additional shared entanglement. Every decoder satisfies*

$$F_{\mathrm{e}}(\mathscr{D}\mathscr{N}) \le \frac{m}{d}, \qquad \varepsilon(\mathscr{D}\mathscr{N}) \ge 1 - \frac{m}{d}.$$

This floor is attained by the coordinate-subset channel

$$\Lambda_m(X) = \binom{d-1}{m-1}^{-1} \sum_{|S|=m} P_S X P_S.$$

For

$$T_{\lambda,m} = (1-\lambda)\mathscr{R}_{I/d} + \lambda\Lambda_m,$$

with or without delivery of the outer branch flag,

$$\varepsilon(T_{\lambda,m}) = \varepsilon^{*}(T_{\lambda,m}) = (1-\lambda)a_d + \lambda(1 - m/d).$$

Proof. The composite Kraus operators K_j have rank at most m. Therefore

$$F_{\mathrm{e}} = \frac{1}{d^2}\sum_j |\mathrm{Tr}\, K_j|^2 \le \frac{m}{d^2}\sum_j \mathrm{Tr}\, K_j^{\dagger}K_j = \frac{m}{d}.$$

The Bell primal gives the error bound.

Counting subsets gives

$$\Lambda_m = \alpha\mathrm{id} + (1-\alpha)\Delta, \qquad \alpha = \frac{m-1}{d-1}.$$

The instrument recording S transmits its m-dimensional coordinate output, so it obeys the cut contract. Moreover,

$$J(\Lambda_m) = \alpha W_d + (1-\alpha)\Pi \ge (m/d)W_d, \qquad F_{\mathrm{e}}(\Lambda_m) = m/d.$$

Thus Lemma is exact. The mixture has

$$J(T_{\lambda,m}) \ge \big((1-\lambda)/d^2 + \lambda m/d\big)W_d.$$

For any decoder the two branch Bell overlaps are bounded by $1/d^2$ and m/d, respectively. These two matching bounds prove , including the flagged version.

Documentary record (Hybrid cuts and the ceiling/value distinction). For a cut with at most L classical labels and quantum width m, Cheung's Big02-A records

$$F_{\mathrm{e}} \le f_{d,L,m} := \min\{m/d, Lm^2/d^2\}.$$

It also records the exact two-label seeded law

$$d^2 F_{\max} = 2r^2 + 2\sqrt{2}\, rk + k^2, \qquad r = \min(m, d-m), \quad k = \max(0, 2m-d),$$

and consequently

$$e_{\mathrm{seed}}(3,2,2) = \frac{6-2\sqrt{2}}{9}.$$

These are version-qualified source statements [Big02], not proof inputs here. The spectral Ky Fan ceiling is not substituted for an attained channel value.

The general $L \ge 3$ formula and weighted rank-m projector resolutions are not evaluated in this paper. The phrase ``partial'' describes this paper's treatment and the cited version's scope, not a claim that no external existence criterion is available.

Documentary record (Version-6 hybrid-cut closed forms; inserted in version 5). Write $Q_{d,L,m} = d^2 F_{\max}(d,L,m)$ for the seeded value of Documentary record , so that $e_{\mathrm{seed}} = 1 - Q_{d,L,m}/d^2$. Cheung's Big02-A, version 6, records the following closed forms [Big02].

• *Band.* For $L \ge 2$, $(L-1)m \le d \le Lm$ and $k = Lm - d$,

$$Q_{d,L,m} = L\big(m - k + k\sqrt{1 - \tfrac{1}{L}}\big)^2.$$

• *Co-band.* For $L \ge 2$, $m \le d$ and $c = Lm - (L-1)d \ge 0$, with $n = d - m$,

$$Q_{d,L,m} = \big(c + n\sqrt{L(L-1)}\big)^2.$$

• *Three labels.* For $L = 3$ the value is closed for every $1 \le m \le d$. Besides the case $d \ge 3m$ (value $3m^2$), the band and the co-band, the range $\frac{3}{2}m \le d \le 2m$ has $Q_{d,3,m} = 2(q + \sqrt{3}\, p)^2$ with $p = 2m - d$ and $q = 2d - 3m$.

• *Values.* The recorded values include $Q_{7,3,3} = 11 + 4\sqrt{6}$ and $Q_{7,4,5} = 18 + 12\sqrt{2}$. The second rests on the meeting inequality (I3) of prop:v6-valid with $s = 3$. The same record leaves open the middle range $Lm/(L-1) < d < (L-1)m$ for $L \ge 4$ (its open item B2-O27), with the recorded exceptions $(7,4,5)$ and the cases in which I_d is a weighted projector sum. These closed forms are source record; not a premise. They are recorded, not proved, here, and no statement of this paper uses them as a premise.

par *Non-claim.* The middle range for $L \ge 4$ remains open in the sense recorded by the source (B2-O27). No species, interaction, dissipative law or radiation-label value is derived from a cut value. The forms are not imported as premises, and the values of cor:v6-values are documentary only. par

Proposition (The two-label reduction; local re-proof). *Let $L = 2$ and $1 \le m \le d$.*

• The band domain $(L-1)m \le d \le Lm$ and the co-band domain $m \le d$, $(L-1)d \le Lm$, both equal $m \le d \le 2m$. On it, with $r = d - m$ and $k = 2m - d$, the band and co-band expressions of Documentary record coincide:

$$2\big(r + \frac{k}{\sqrt{2}}\big)^2 = (k + \sqrt{2}\, r)^2 = 2r^2 + 2\sqrt{2}\, rk + k^2,$$

and the two recorded deficits agree: $md - Q_{d,2,m} = (3 - 2\sqrt{2})\, rk$.

• For $d \ge 2m$, the orthogonal-block value $Lm^2 = 2m^2$ equals $2r^2 + 2\sqrt{2}\, rk + k^2$ with $r = \min(m, d-m) = m$ and $k = \max(0, 2m-d) = 0$. Hence, at $L = 2$, the version-6 forms reduce exactly to the two-label law recorded in Documentary record .

• At $(d,L,m) = (3,2,2)$, $r = k = 1$, $Q_{3,2,2} = 3 + 2\sqrt{2}$, and

$$e_{\mathrm{seed}}(3,2,2) = 1 - \frac{3+2\sqrt{2}}{9} = \frac{6-2\sqrt{2}}{9},$$

the value already cited in Documentary record .

• *The middle range* $Lm/(L-1) < d < (L-1)m$ *is empty for* $L = 2$ *and is a nonempty real interval for every* $L \geq 3$.

Proof. (a) At $L = 2$ the band conditions read $m \leq d \leq 2m$; the co-band condition $c = 2m - d \geq 0$ with $m \leq d$ is the same set. There $k = c$, $n = d - m = r$ and $m - k = r$. Since $\sqrt{1 - 1/2} = 1/\sqrt{2}$ and $\sqrt{L(L-1)} = \sqrt{2}$, both expressions expand to $2r^2 + 2\sqrt{2}\,rk + k^2$. The band deficit is $L(1 - \sqrt{1 - 1/L})^2 k(m-k) = 2(1 - 1/\sqrt{2})^2 kr = (3 - 2\sqrt{2})kr$, and the co-band deficit is $cn(2L - 1 - 2\sqrt{L(L-1)}) = (3 - 2\sqrt{2})kr$. (b) Substitute $r = m$, $k = 0$. (c) Substitute $d = 3$, $m = 2$. (d) The inequality $L/(L-1) < L - 1$ holds exactly when $L < (L-1)^2$, that is, when $L \geq 3$.

Status: proved here. Only the algebraic reduction is re-proved. The values $Q_{d,L,m}$ themselves, and the reading $e_{\mathrm{seed}} = 1 - F_{\max}$, remain source records of Big02-A.

par *Non-claim.* No closed form for $L \geq 3$ is proved here, and no value in the middle range for $L \geq 4$ is decided. No species or interaction is derived. par

Documentary record (Four-label closed sectors and the open register bracket; inserted in version 5). Cheung's Res06-A, version 4, proves two further closed sectors for $L = 4$ [Res06]: for $\frac{8m}{3} \leq d \leq 3m$,

$$Q_{d,4,m} = 3((3d - 8m) + 2\sqrt{2}\,(3m - d))^2,$$

and for $\frac{3m}{2} \leq d \leq \frac{8m}{5}$, with $a = 8m - 5d$ and $c = 2d - 3m$,

$$Q_{d,4,m} = 2(\sqrt{3}\,a + 2\sqrt{5}\,c)^2.$$

They give $Q_{11,4,4} = 27 + 12\sqrt{2}$ and $Q_{11,4,7} = 46 + 8\sqrt{15}$ [Res06]. Both tuples lie in the middle range of Documentary record . In that record Q is proved locally, while its reading as $d^2 F_{\max}$ is conditional on the Big02-A identification cited there; for $L = 4$ the record keeps B2-O27 open on the central range $\frac{8m}{5} < d < \frac{8m}{3}$. The same record keeps the unassisted-register bracket

$$\frac{3}{5} \leq \varepsilon_1(3,2) \leq \frac{2}{3}$$

open [Res06]. These closed sectors and the bracket are source record; not a premise. The replacement floor $a_d = 1 - d^{-2}$ of Theorem is independent of both: it concerns a complete replacement channel, not a hybrid cut or an unassisted register, and its proof uses neither.

par *Non-claim.* The $(3,2)$ value is not decided. No Page curve is defined. The sector forms are not imported as premises. par

par *Scope.* The cut ladder is an access model, not a radiation species, interaction, or dissipative law. The classical record is part of the complete contract and cannot hide additional message-correlated quantum memory. par

An energy-constrained floor is a separate floor

Documentary record (Energy-constrained replacement; inserted in version 5). Cheung's Big02-A, version 6, considers M bosonic modes with total number operator N, a separately supplied gap $\Delta > 0$ and energy ΔN, and the energy-constrained half-diamond distance ε_E over referenced inputs of mean energy at most E. With $\nu = E/\Delta$ and $F(\nu) = (\nu + \sqrt{4\nu - 3\nu^2})/2$, it records [Big02]:

• for $0 \leq \nu \leq \frac{1}{3}$ and every $M \geq 1$, the optimized replacement floor is exactly $F(\nu)$, attained by the vacuum output; for the cutoff-and-reset channel with $q \leq \frac{1}{3}$ the identity decoder is optimal, so optimized decoding does not improve cutoff-and-reset there;

• for one mode and $\frac{1}{3} \leq \nu \leq \nu_2 := 2 - \sqrt{22}/3$, the floor is $3\nu(1 - \nu)$, with an explicit dual certificate. The same record states that a fixed number of bosonic modes without a cutoff is not a finite dimension [Big02]. Beyond these ranges it gives no elementary closed form; it gives an exact variational formula [Big02], which is neither evaluated nor used here. As elementary algebra, the two expressions agree at $\nu = \frac{1}{3}$ (value $\frac{2}{3}$); ν_2 is the smaller root

of $9v^2 - 36v + 14$; and on $(\frac{1}{3}, 1)$, $F(v) - 3v(1-v) = \frac{1}{2}(\sqrt{4v - 3v^2} - (5v - 6v^2)) > 0$, because $4v - 3v^2 - (5v - 6v^2)^2 = -4v(v-1)(3v-1)^2 > 0$ there.

This floor is conditional on the separately supplied energy promise Δ, together with E. It is a different quantity from the causal replacement floor $a_d = 1 - d^{-2}$ of Theorem , which concerns a complete finite-dimensional replacement channel without an energy constraint, and it involves none of κ, U, α, V_c or the radiation labels. No relation between Δ and κ is assumed. The record is source record; not a premise.

par *Non-claim.* No energy scale is derived from the host. No exact energy-constrained value outside the stated ranges is evaluated here; on this sheet it stays open. The floor is not identified with a physical bath or with Hawking radiation, and it is not converted into a value of a radiation label. par

Thermal operations at one external radiation label

Fix a supplied Hamiltonian H_S on $\mathbb{C}^d$. For $\beta > 0$, set

$$\gamma_\beta = \frac{e^{-\beta \mathrm{H}_S}}{Z(\beta)}, \qquad Z(\beta) = \mathrm{Tr}\, e^{-\beta \mathrm{H}_S}.$$

A finite-bath thermal operation is

$$T(X) = \mathrm{Tr}_B\, U(X \otimes \gamma_{\beta,B})U^\dagger, \qquad [U, \mathrm{H}_S \otimes I + I \otimes \mathrm{H}_B] = 0.$$

This is the thermal-operation convention of Horodecki and Oppenheim [HO]. At a formally fixed radiation-label value, define

$$\beta_\kappa = \frac{1}{k_B T_{\mathrm{r}}^{(\kappa)}(t)}.$$

Using this number in a comparison bath is a declaration.

Theorem (Three thermal interfaces and two different error tasks). *Let $\mathscr{B}$ be a rank-one eigenbasis of H_S. At every $\beta > 0$:*

• id, $\Delta_{\mathscr{B}}$, *and* $\mathscr{R}_{\gamma_\beta}$ *are thermal operations. Every thermal operation fixes γ_β. Complete dephasing in a rank-one basis $\mathscr{B}'$ is thermal exactly when H_S is diagonal in $\mathscr{B}'$. A replacement $\mathscr{R}_\sigma$ is thermal exactly when $\sigma = \gamma_\beta$.*

• *Their installed, or undecoded, errors are*

$$0, \qquad 1 - \frac{1}{d}, \qquad e(\beta) := 1 - \frac{1}{Z(\beta)Z(-\beta)}.$$

The dephasing dual pair is that of Proposition ; the replacement pair is that of Lemma . Both Choi signs and factor orders are thereby covered.

• *With unrestricted decoding their recovery errors are*

$$0, \qquad 1 - \frac{1}{d}, \qquad 1 - \frac{1}{d^2},$$

for every supplied Hamiltonian.

• *If the decoder is itself required to be a thermal operation on the same Hamiltonian at the same β, then the Gibbs-replacement recovery error is $e(\beta)$, not generally $1 - 1/d^2$.*

Proof. Identity uses no interaction. For dephasing use a d-dimensional bath with zero Hamiltonian, state I/d, and

$$U = \sum_a P_a \otimes Z^a,$$

where Z is the clock unitary. The energy commutator vanishes, and the bath trace removes off-diagonal basis blocks. For replacement, swap with a system copy having Hamiltonian H_S in state γ_β.

The joint Gibbs state is proportional to $e^{-\beta \mathrm{H}_{\mathrm{tot}}}$, so every allowed U preserves it; taking the bath trace proves Gibbs

preservation. A rank-one dephasing fixes γ_β exactly when γ_β, and hence its logarithm H_S, is diagonal in that basis. A thermal replacement must output γ_β. The displayed constructions prove sufficiency.

The installed values follow from Proposition and

$$\operatorname{Tr}\gamma_\beta^{-1} = Z(\beta)Z(-\beta).$$

The unrestricted recovery values follow from the same proposition and Theorem . A thermally restricted decoder fixes γ_β, so its composite after Gibbs replacement is still $\mathscr{R}_{\gamma_\beta}$, proving (d).

Proposition (Coefficient dependence of installed Gibbs error). *For eigenvalues E_i of H_S,*

$$\Im(\beta) := Z(\beta)Z(-\beta) = d + 2\sum_{i<j}\cosh\left(\beta(E_i - E_j)\right).$$

If H_S is scalar, $\Im = d^2$ and $e(\beta) = a_d$ for every β. Otherwise $\Im$ is strictly increasing for $\beta > 0$, from d^2 to infinity. At fixed U, t, β_κ is a decreasing bijection of $(0,\infty)$, so the installed error $e(\beta_\kappa)$ ranges over $(a_d, 1)$.

The absolute minimax risk for predicting this error from coefficient-blind data is $1/(2d^2)$ in the nonscalar case, and zero in the scalar case.

Proof. Expand the product of partition functions and pair opposite energy differences. A nonzero gap gives a strictly increasing cosh term and an unbounded limit. The coefficient statement follows from . Apply Lemma to the installed-error image.

Theorem (Restricted O6 minimax, including the scalar case). *At fixed β, let $\mathscr{T}_{3,\beta}$ contain identity, all complete energy-basis dephasings, and Gibbs replacement. For the task of predicting the installed error from β, H_S, d, but without the channel identity:*

• The deterministic and randomized minimax risks are $e(\beta)/2$. The unique deterministic decision is $e(\beta)/2$. The equal prior on identity and Gibbs replacement is a matching lower certificate. The value is unchanged on the convex hull of this channel class.

• On a closed coefficient window, write $\beta_{\mathrm{lo}} = \beta_{\kappa_+}$ and $\beta_{\mathrm{hi}} = \beta_{\kappa_-}$. For Gibbs replacement alone the risk is

$$\frac{1}{2}(e(\beta_{\mathrm{hi}}) - e(\beta_{\mathrm{lo}})).$$

For both the channel and coefficient unknown in the union of the three-channel classes, it is $e(\beta_{\mathrm{hi}})/2$.

• On $\kappa > 0$, a nonscalar Hamiltonian gives risk $1/2$ for the union class. The deterministic worst case is attained at the identity, although the supremum Gibbs error one is not a value. For a scalar Hamiltonian the union-class risk remains $a_d/2$ on every coefficient window, including $(0,\infty)$; the Gibbs-only risk is zero.

Proof. Since $\Im \geq d^2$,

$$0 < 1 - 1/d < a_d \leq e(\beta) < 1.$$

The fixed-label error set has minimum zero and maximum $e(\beta)$. Convexity of the diamond norm bounds every convex-hull error by the same maximum, and both extrema remain available. Lemma gives (a). Monotonicity in Proposition gives (b). For nonscalar Hamiltonians the union error set is $\{0, 1 - 1/d\} \cup (a_d, 1)$; for scalar Hamiltonians it is $\{0, 1 - 1/d, a_d\}$. This proves (c).

par *Scope.* This restricted minimax problem does not identify an unspecified physical noise channel, and it does not close O6. Its task is distinct from optimized quantum recovery. Cheung's Big03-A and Big02-A provide the corresponding source-record separations [Big03,Big02]. The unrestricted-decoder qualification is essential: a constant decoder preparing I/d is not generally a thermal operation for a nonscalar Hamiltonian. par

Late labels and optical-tail non-determination

The named Gibbons–Hawking-form dictionary at a supplied rate K is

$$T_{\text{late}} = \frac{\kappa K}{2\pi k_B}, \qquad S_{\text{late}} = \frac{\pi k_B c^5}{G\kappa K^2}, \qquad N_{\text{late}} = \exp(S_{\text{late}}/k_B).$$

Its form is motivated by Gibbons and Hawking [GH]; here it is an external dictionary. It is not assigned to a quantum state of the comparison host.

Theorem (Supply cases for late labels). *Every $K > 0$ admits a smooth future in $\mathscr{F}(K)$ with the prescribed radiation prefix. At fixed κ, the early geometry and radiation-label history are the same across these futures, but all three late labels have unbounded images. Their absolute minimax risks from early data are infinite.*

Under the geometric condition $H \le 0$, the admissible terminal rates are exactly $0 < K < H_0$. Then, at fixed κ, the temperature image is

$$(0, \kappa H_0/(2\pi k_B)),$$

with deterministic minimax risk $\kappa H_0/(4\pi k_B)$. The deterministic midpoint has no maximizing future. Entropy and exponential-label risks remain infinite.

If (κ, K) is supplied or recoverable from augmented data, all three late labels are determined. Conversely, determining both T_{late} and S_{late} determines the pair:

$$K = \frac{c^5}{2GT_{\text{late}}S_{\text{late}}}, \qquad \kappa = \frac{2\pi k_B T_{\text{late}}}{K}.$$

Proof. Let b be a smooth increasing step, zero up to δ and one after 2δ. For $t \ge 0$ prescribe

$$H(t) = (1 - b(t))\frac{H_0}{1 + 2H_0 t} + b(t)K.$$

It is positive, matches the prefix near zero, and is eventually constant K. The late-label ranges follow by varying $K > 0$.

For nonincreasing H, smooth matching gives $H(0) = H_0$, $\dot{H}(0) = -2H_0^2$, so a terminal K must be strictly smaller than H_0. For $K < H_0$, choose δ sufficiently small that $H_r(2\delta) > K$. Then

$$\dot{H} = (1 - b)\dot{H}_r + \dot{b}(K - H_r) \le 0.$$

This proves sufficiency and the exact rate range. Apply Lemma . Finally, multiplying the first two formulas of gives $T_{\text{late}}S_{\text{late}} = c^5/(2GK)$, proving .

Theorem (Late-label box minimax). *On the independent declared box*

$$\kappa \in [\kappa_-, \kappa_+], \qquad K \in [K_-, K_+], \quad \kappa_-, K_- > 0,$$

the exact componentwise absolute minimax risks are

$$m_T^{\text{late}} = \frac{\kappa_+ K_+ - \kappa_- K_-}{4\pi k_B},$$

$$m_S^{\text{late}} = \frac{\pi k_B c^5}{2G}\left(\frac{1}{\kappa_- K_-^2} - \frac{1}{\kappa_+ K_+^2}\right),$$

$$m_N^{\text{late}} = \frac{1}{2}\left[\exp\left(\frac{\pi c^5}{G\kappa_- K_-^2}\right) - \exp\left(\frac{\pi c^5}{G\kappa_+ K_+^2}\right)\right].$$

They hold for randomized rules as well. Degenerate parameter intervals represent supplied parameters; if both degenerate, all risks are zero.

Proof. Temperature increases in both parameters. Entropy and its exponential decrease in both. The extreme corner values are available by Theorem . Apply Lemma , including its endpoint-prior bound.

Proposition (The prefix does not decide the optical tail). *Continuing $a(t) = \sqrt{1 + 2H_0}t$ for every $t \geq 0$ gives*

$$c\int_0^\infty a(t)^{-1}\,\mathrm{d}t = \infty.$$

Every $\mathscr{F}(K)$ future instead has a finite optical tail. Thus the prefix does not determine whether an event horizon exists.

It does not follow that the status of every early event can change. If $|\mathbf{x}| \leq B(1 - z)$, the event already reaches the observer by the junction and is visible for every future. Status changes require a radius beyond this prefix reach.

Proof. The radiation primitive is unbounded at infinity. An eventually exponential tail is integrable. The last claim follows directly from the finite-prefix causal integral.

Proposition (One coefficient, different label coordinates). *On the independent parameter space $(\kappa, U, K) \in (0,\infty)^3$, the pair (T_r, S_r) determines (κ, U) but leaves K free; $(T_{\text{late}}, S_{\text{late}})$ determines (κ, K) but leaves U free. Consequently neither pair determines the other without the remaining parameter or an additional relation supplying it.*

At fixed U,K, both pairs are coordinates on the same coefficient κ, with logarithmic coefficient slopes $(3/4, -3/4)$ *and* $(1, -1)$*. For instance,*

$$\frac{T_r}{T_{\text{late}}^{3/4}} = \frac{AU^{1/4}}{a_r}(2\pi k_B)^{3/4}K^{-3/4}$$

is coefficient-free but is not independent of U,K.

Proof. Use and , then vary the parameter left free. The slopes and displayed invariant follow from the power laws. par *Scope.* The distinction is between supplied coordinates and functions of the declared data, not between two physically different values of $\hbar$. A coefficient-free expression need not be a function of geometry alone if it still depends on an unsupplied *U* or *K*. These are the radiation/late-coordinate separations documented by Cheung [Big02,Big03]. par

Proposition (Relative risk of the late dimension label; inserted in version 5). *For a supplied terminal rate $K > 0$ put*

$$C_K = \frac{\pi c^5}{GK^2}, \qquad \text{so that} \qquad N_{\text{late}}^{(\kappa)} = \exp\left(\frac{C_K}{\kappa}\right), \qquad \frac{C_K}{\kappa} = \frac{S_{\text{late}}^{(\kappa)}}{k_B}.$$

• *For the calibration $\kappa \in [\kappa_0(1-\delta), \kappa_0(1+\delta)]$, $0 < \delta < 1$, the label N_{late} fills $[\ell_N, u_N]$ with $\ell_N = \exp(C_K/(\kappa_0(1+\delta)))$ and $u_N = \exp(C_K/(\kappa_0(1-\delta)))$. Its sharp relative minimax risk, deterministic and randomized, is*

$$r_N = \tanh\left(\frac{C_K\delta}{\kappa_0(1-\delta^2)}\right) = \tanh\left(\frac{S_{\text{late}}^{(\kappa_0)}}{k_B}\frac{\delta}{1-\delta^2}\right).$$

The unique deterministic decision is $2\ell_N u_N/(\ell_N + u_N)$, and the unequal-weight prior $\pi(\ell_N) = \ell_N/(\ell_N + u_N)$, $\pi(u_N) = u_N/(\ell_N + u_N)$ of Theorem is a matching least-favourable prior. Thus the constant in the form $\tanh(C\delta/(\kappa_0(1-\delta^2)))$ *is exactly $C = C_K$.*

• *On the same window $r_{T_{\text{late}}} = r_{S_{\text{late}}} = \delta$. The value is neither identically δ nor of the power-law form* $\tanh(\text{partanh}\,\delta)$ *of : $r_N > \delta$ whenever $S_{\text{late}}^{(\kappa_0)} \geq k_B$, and $r_N \to 0$ as $C_K/\kappa_0 \to 0$ at fixed δ.*

• *On a declared box $\kappa \in [\kappa_-, \kappa_+]$, $K \in [K_-, K_+]$, $\kappa_-, K_- > 0$, the sharp relative risk of N_{late} is*

$$\tanh\left(\frac{\pi c^5}{2G}\left(\frac{1}{\kappa_- K_-^2} - \frac{1}{\kappa_+ K_+^2}\right)\right).$$

• *If the terminal rate is free, over $K > 0$ or over $0 < K < H_0$ under $H \leq 0$ as in Theorem , then at every fixed κ, and*

a fortiori on every coefficient window, the relative minimax risk of N_{late} *from early data is* 1*, for deterministic and randomized rules. The decision* 0 *attains it.*

Proof. (a) At fixed K, $\kappa \mapsto \exp(C_K/\kappa)$ is continuous and strictly decreasing, so the closed window has image $[\ell_N, u_N]$ with $\ell_N < u_N$. Theorem gives the decision, the prior and the risk $(u_N - \ell_N)/(u_N + \ell_N) = \tanh(\frac{1}{2}\ln(u_N/\ell_N))$, and

$$\ln\frac{u_N}{\ell_N} = \frac{C_K}{\kappa_0}\left(\frac{1}{1-\delta} - \frac{1}{1+\delta}\right) = \frac{2C_K\delta}{\kappa_0(1-\delta^2)}.$$

The second form uses . (b) At fixed K, $T_{\mathrm{late}} \propto \kappa$ and $S_{\mathrm{late}} \propto \kappa^{-1}$, so both endpoint ratios are $(1+\delta)/(1-\delta)$, and $\tanh(\frac{1}{2}\ln\frac{1+\delta}{1-\delta}) = \delta$. Put $s = C_K/\kappa_0 = S_{\mathrm{late}}^{(\kappa_0)}/k_B$. Since $\operatorname{artanh}\delta = \sum_{j\ge 0}\delta^{2j+1}/(2j+1) < \sum_{j\ge 0}\delta^{2j+1} = \delta/(1-\delta^2)$, the condition $s \ge 1$ gives $s\delta/(1-\delta^2) > \operatorname{artanh}\delta$, and monotonicity of tanh gives $r_N > \delta$. The limit is continuity of tanh at zero. (c) The label decreases strictly in each of κ and K, so on the connected box its image is the closed interval between its values at (κ_+, K_+) and (κ_-, K_-). If this interval is a point the risk is zero, as is the displayed value; otherwise apply Theorem as in (a). (d) By Theorem , every admissible K occurs with the same early data, so at fixed κ the image of N_{late} is unbounded above. For $y \ge 0$, $|y/N - 1| \to 1$ as $N \to \infty$, so every such decision has risk at least 1, and $y = 0$ has risk exactly 1; a negative decision is worse than zero. For a coefficient-blind random decision Y with $\mathbb{E}|Y| < \infty$, $\mathbb{E}|Y/N - 1| \ge 1 - \mathbb{E}|Y|/N \to 1$; if $\mathbb{E}|Y| = \infty$ the risk is infinite. Proposition extends this to coefficient-blind reports.

Status: proved here, from Theorems and and . Cheung's Big03-A records the same form on its de Sitter host, with its Hubble rate in the place of K [Big03]; that record is source record; not a premise and is not used in the proof. par

Non-claim. The relative risk does not select a value of κ, and K is an input in (a)–(c). The late labels remain undetermined when the terminal rate is free, as (d) records. No numerical value of κ, K or S_{late} is admitted. par

Version 6: general-w logarithmic certificates and the two-epoch special case

This subsection is inserted in version 6 (plan item 3).

Proposition (Logarithmic certificates of the general-*w* labels; inserted in version 6). *Under the hypotheses of Proposition , at fixed* $(U, \alpha, V_c, \varepsilon_s, w)$ *and on any supplied positive* C^1 *scale factor* a *with* $H = \dot{a}/a$*:*

• $\partial\ln T_w/\partial\ln\kappa = p_w$*, equivalently* $\partial_\kappa \ln T_w = p_w/\kappa = 3w/((1+w)\kappa) > 0$*, and* $\partial\ln S_w/\partial\ln\kappa = -p_w$*.*

• $\partial_t \ln T_w = -3wH$*,* $\partial_t S_w = 0$ *and* $\partial_t \ln(T_w S_w) = -3wH$*. Hence* S_w *is independent of t on every supplied scale factor, and* $T_w(t)/T_w(t') = (a(t')/a(t))^{3w}$*.*

• $p_w = 3w/(1+w)$ *is strictly increasing on* $w > 0$*, with range* $(0, 3)$*,* $p_{1/3} = \frac{3}{4}$*,* $p_{1/2} = 1$ *and* $p_1 = \frac{3}{2}$*. Consequently the window risk* $\tanh(p_w \operatorname{artanh}\delta)$ *of is strictly increasing in w; it is smaller than* δ *iff* $w < \frac{1}{2}$*, equal to* δ *at* $w = \frac{1}{2}$*, and larger for* $w > \frac{1}{2}$*.*

• *The late temperature label of has* $\partial\ln T_{\mathrm{late}}/\partial\ln\kappa = 1 = p_{1/2}$*, yet it is not a general-w label: on a terminal tail* $H = K$ *it is constant in t, whereas every* T_w*,* $w > 0$*, strictly decreases where* $H > 0$*; and* T_{late} *contains K but not U. Equality of logarithmic slopes does not identify two dictionaries.*

Proof. (a) By , $\ln T_w = \ln A_w + \frac{w}{1+w}\ln U + p_w \ln\kappa - 3w\ln a$ and $\ln S_w = \ln\frac{(1+w)V_c}{A_w} + \frac{1}{1+w}\ln U - p_w\ln\kappa$; differentiate. (b) $\frac{\mathrm{d}}{\mathrm{d}t}\ln a = H$, and $\ln S_w$ contains no a. The ratio follows by integration of $\partial_t \ln T_w$. (c) $\frac{\mathrm{d}p_w}{\mathrm{d}w} = 3/(1+w)^2 > 0$, $p_w \to 0$ as $w \to 0$ and $p_w \to 3$ as $w \to \infty$. Since $\delta = \tanh(\operatorname{artanh}\delta)$, $\operatorname{artanh}\delta > 0$ and tanh is strictly increasing, the comparison with δ is the comparison of p_w with 1, and $3w < 1 + w$ iff $w < \frac{1}{2}$. (d) $T_{\mathrm{late}} = \kappa K/(2\pi k_B)$ is linear in κ and constant on the tail, where $\partial_t \ln T_w = -3wK < 0$.

Status: proved here. Big02-A records the scaling $T_w \propto \kappa^{3w/(1+w)} a^{-3w}$, the time independence of S_w and

$\partial_\kappa \ln T_w = 3w/((1+w)\kappa)$ [Big02]. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* The number w is declared, not derived; no equation of state, thermal state or entropy-conservation law is inferred. No general-w label is identified with a quantum coefficient or with dark capacity. par

Corollary (Two-epoch comparison as an equal-exponent certificate; inserted in version 6). *Let two declared label families have the form $L_i = C_i f_i(t)\kappa_i^{p_i}$, $i = 1, 2$, each on its own declared epoch, with positive coefficient-free factors C_i, f_i, nonzero exponents p_i and slots $\kappa_1, \kappa_2 > 0$.*

• The cross ratio is $L_1/L_2 = \frac{C_1 f_1}{C_2 f_2}\kappa_1^{p_1}\kappa_2^{-p_2}$. The common rescaling $(\kappa_1, \kappa_2) \mapsto (q\kappa_1, q\kappa_2)$ multiplies it by $q^{p_1 - p_2}$; it preserves every cross ratio for all $q > 0$ iff $p_1 = p_2$.

• Theorem is the case $p_1 = p_2 = p_{1/3} = \frac{3}{4}$ for the temperatures, , and $p_1 = p_2 = -\frac{3}{4}$ for the entropies, . Its conclusion that an exact cross ratio determines only κ_N/κ_B is (a) with equal exponents.

• If the equation-of-state parameter changes across the epochs by a declared supply, $w_1 \neq w_2$, the general-w temperatures have $p_{w_1} \neq p_{w_2}$ by Proposition (c). For independent slots an exact cross ratio fixes only $\kappa_1^{p_1}\kappa_2^{-p_2}$ and leaves a curve of slot pairs. If the slots are declared equal, $\kappa_1 = \kappa_2 = \kappa$, the cross ratio is strictly monotone in κ, so an exact cross ratio, with every dictionary constant supplied, calibrates κ. This is calibration by the added supplies (the change of w, the dictionary constants and the exact labels), not information produced by the geometry.

Proof. (a) Direct substitution; $q^{p_1 - p_2} = 1$ for every $q > 0$ iff $p_1 = p_2$. (b) In and the factors ρ_T and ρ_S are time and coefficient independent, and the slot powers are $(\kappa_N/\kappa_B)^{3/4}$ and $(\kappa_B/\kappa_N)^{3/4}$. (c) Distinct exponents by strict monotonicity of p_w. For independent slots, each level set $\{\kappa_1^{p_1}\kappa_2^{-p_2} = \text{const}\}$ is the graph $\kappa_2 = (\kappa_1^{p_1}/\text{const})^{1/p_2}$. With equal slots the cross ratio is a constant multiple of $\kappa^{p_1 - p_2}$, $p_1 - p_2 \neq 0$, which is strictly monotone and onto $(0, \infty)$.

Status: proved here. The two-epoch construction parallels Cheung's nuADaM01-A [nuADaM], source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* No change of w is derived or asserted to occur physically, and no slot identification is made. A calibrated slot remains a supplied coordinate; no quantum coefficient is admitted. par

Proposition (General-w quotient coordinate and Jacobian; inserted in version 6). *Under the hypotheses of Proposition put $\eta_w = w/(1+w)$, so $p_w = 3\eta_w$, and $X = Uc^3\zeta/\varepsilon_s^4$ with $\zeta = \kappa^3/\alpha$. Then*

$$T_w = \frac{\varepsilon_s}{k_B} X^{\eta_w} a^{-3w}, \qquad S_w = (1+w)\frac{k_B V_c U}{\varepsilon_s} X^{-\eta_w}.$$

In the coordinates $(\ln U, \ln\kappa, \ln\alpha)$ the map $(\ln T_w, \ln S_w)$ has Jacobian $\begin{pmatrix} \eta_w & 3\eta_w & -\eta_w \\ 1-\eta_w & -3\eta_w & \eta_w \end{pmatrix}$, of rank two with kernel spanned by $(0, 1, 3)$. Hence the compensating orbit $(U, r\kappa, r^3\alpha)$ and the split independence of Proposition hold for every fixed $w > 0$.

Proof. With $n_w = 1/\eta_w$, $u_w \kappa^3 c^3 \varepsilon_s^{n_w - 4}/\alpha = X\varepsilon_s^{n_w} a^{-3(1+w)}$; taking the power η_w gives T_w, since $3(1+w)\eta_w = 3w$. Then $S_w = (1+w)u_w V/T_w$ with $u_w V = UV_c a^{-3w}$ gives S_w; the factor a^{-3w} cancels. Taking logarithms gives the rows. The $(\ln U, \ln\kappa)$ minor is $-3\eta_w^2 - 3\eta_w(1-\eta_w) = -3\eta_w \neq 0$, and both rows annihilate $(0, 1, 3)$. No split coordinate occurs.

par *Non-claim.* The quotient is a reparametrization of a declared dictionary; w is declared, not derived. par

Proposition (Epoch-change ratios and their supply content; inserted in version 6). *At a junction with supplied scale factor $a_j > 0$, declare exponents $w_\pm > 0$, separate constants $A_\pm, V_{c,\pm}, U_\pm > 0$, one common slot $\kappa > 0$, and $T_\pm = A_\pm U_\pm^{w_\pm/(1+w_\pm)}\kappa^{p_\pm} a_j^{-3w_\pm}$, $S_\pm = B_\pm U_\pm^{1/(1+w_\pm)}\kappa^{-p_\pm}$ with $p_\pm = p_{w_\pm}$ and $B_\pm = (1+w_\pm)V_{c,\pm}/A_\pm$. Then*

$$\frac{T_+}{T_-} = C_T \kappa^{p_+ - p_-}, \qquad \frac{S_+}{S_-} = C_S \kappa^{-(p_+ - p_-)}, \qquad \frac{T_+ S_+}{T_- S_-} = \frac{(1+w_+)U_+ V_{c,+}}{(1+w_-)U_- V_{c,-}} a_j^{-3(w_+ - w_-)},$$

with $C_T = \frac{A_+ U_+^{w_+/(1+w_+)}}{A_- U_-^{w_-/(1+w_-)}} a_j^{-3(w_+ - w_-)}$ *and* $C_S = \frac{B_+ U_+^{1/(1+w_+)}}{B_- U_-^{1/(1+w_-)}}$. *If* $w_+ \neq w_-$ *and* C_T *is supplied, an exact ratio calibrates* $\kappa = ((T_+/T_-)/C_T)^{1/(p_+ - p_-)}$. *Neither temperature nor entropy continuity at the junction follows from the dictionaries.*

Proof. Divide the formulas; in the product the powers of κ cancel and $\frac{w}{1+w} + \frac{1}{1+w} = 1$. Since $w \mapsto p_w$ is strictly increasing, $p_+ \neq p_-$ when $w_+ \neq w_-$, which gives the inverse. The constants $A_\pm, V_{c,\pm}, U_\pm$ are free, so arbitrary jumps are allowed; no matching law is imposed.

par *Non-claim.* The ratio is an added coefficient-sensitive datum; no transition law, common coefficient or continuity condition is derived. par

Corollary (A variable exponent breaks entropy constancy; inserted in version 6). *If, as a separate declaration,* $w = w(t) > 0$ *is differentiable and is substituted in the quotient form with* $U, \kappa, \alpha, \varepsilon_s, V_c$ *fixed, then*

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln S_{w(t)} = \dot{w}\left[\frac{1}{1+w} - \frac{\ln X}{(1+w)^2}\right].$$

The fixed-w constancy of Proposition (b) therefore gives neither a junction conservation law nor constancy for a variable exponent.

Proof. $\ln S_w = \ln(1+w) + \ln(k_B V_c U/\varepsilon_s) - \eta_w \ln X$, and $\mathrm{d}\eta_w/\mathrm{d}w = (1+w)^{-2}$.

Remark (Relation to Corollary). Proposition is the explicit form of Corollary (c) with a common slot, and adds the coefficient-free product. The version-5 two-prefix comparison has $w = \frac{1}{3}$ on both sides; with two independent slots, cross ratios do not give the inverse above. The late Gibbons–Hawking-form dictionary is separate: it is not the positive-w dictionary at $w = -1$.

Documentary parallel: Big02-A [Big02]. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* No equation of state, transition dynamics, entropy-transfer law or physical common coefficient is derived; $w(t)$, the junction and the common slot are declarations. par

Rate-bounded futures: exact horizon intervals

Definition (Geometric rate bound). Fix $0 < K < H_0$ and $\Gamma \geq 2$. Let $\mathscr{F}^\Gamma(K) \subset \mathscr{F}(K)$ impose

$$0 \leq \frac{\mathrm{d}}{\mathrm{d}t}(1/H) \leq \Gamma \qquad (t > 0).$$

Let $\mathscr{F}^\Gamma_\tau(K)$ also require $H = K$ by a supplied time $\tau > 0$. The lower restriction $\Gamma \geq 2$ is necessary for smooth matching to the radiation prefix, since $(1/H)'(0) = 2$.

Lemma (Smooth approximation of the extremal profiles). *Write* $u_0 = 1/H_0$, $u_1 = 1/K$, *and* $t_* = (u_1 - u_0)/\Gamma$. *Among smooth functions u that equal* $u_0 + 2t$ *near zero, satisfy* $0 \leq u' \leq \Gamma$, *and are eventually* u_1*:*

• *the upper pointwise envelope is* $u_{\mathrm{fast}}(t) = \min(u_0 + \Gamma t, u_1)$*; it is approached by admissible smooth profiles;*

• *the constant lower envelope* u_0 *is approached on every compact time interval by profiles whose transition is postponed;*

• *with settling by* $\tau > t_*$, *the lower envelope is*

$$u_{\mathrm{slow},\tau}(t) = \max\{u_0, u_1 - \Gamma(\tau - t)\}, \quad 0 \leq t \leq \tau,$$

followed by u_1, *and both envelopes are approached by admissible profiles settling by* τ. *The admissible profile sets are convex.*

Proof. The envelopes follow by integrating the derivative bounds forwards from zero and backwards from the settling time.

For approximation, prescribe $v = u'$. Take $v = 2$ on a sufficiently short interval at zero; smoothly connect it to plateaux with heights zero or Γ; and smoothly decrease it to zero at the end. Choose plateau durations so that $\int_0^\infty v = u_1 - u_0$. Shrinking the smoothing intervals gives the fast envelope. Adding a long zero plateau gives the compact-limit lower envelope. With a deadline, place the maximal-slope plateau as late as permitted, adjusting its duration by the vanishing smoothing mass. The strict slack $\tau > t_*$ permits both the initial matching and terminal smoothing. These operations use smooth transitions staying in $[0, \Gamma]$. They keep $u = u_0 + 2t$ near zero, hence match every prefix derivative there. Convex combinations preserve all constraints; eventual constancy holds after the larger settling time.

Theorem (Exact rate-bounded horizon range). *Put*

$$B = \frac{c}{H_0}, \qquad D = \frac{c}{K}, \qquad \rho = D/B > 1,$$

$$\overline{R}_\Gamma = B\frac{\Gamma\rho^{1-1/\Gamma} - 1}{\Gamma - 1}.$$

Then

•

$$\{R_0[a] : a \in \mathscr{F}^\Gamma(K)\} = (B, \overline{R}_\Gamma), \qquad B < \overline{R}_\Gamma < D.$$

For $\Gamma = 2$, $\overline{R}_2 = 2\sqrt{BD} - B$, and $\overline{R}_\Gamma \to D$ as $\Gamma \to \infty$.

• If $\tau > t_*$, the deadline range is

$$\left(B + (\overline{R}_\Gamma - B)e^{-H_0(\tau - t_*)}, \overline{R}_\Gamma\right).$$

For $\tau \le t_*$, the smooth deadline class is empty.

• At an early time t, the deterministic minimax risks for R_0, the comoving horizon $\chi_e(t)$, and the proper horizon $R_e(t)$, respectively, are

$$\frac{\overline{R}_\Gamma - B}{2}, \qquad \frac{\overline{R}_\Gamma - B}{2}, \qquad \frac{z(\overline{R}_\Gamma - B)}{2}.$$

The midpoint decisions are optimal; their worst-case suprema are not realized by a future.

• Every early source with radius

$$B(1 - z) + B < |\mathbf{x}| < B(1 - z) + \overline{R}_\Gamma$$

is visible for some admissible future and dark for another. Under the located-circuit contract, prediction of its least possible recovery error from the common early data has minimax risk $a_d/2$.

Proof. Write $u = 1/H$ and $b(t) = \ln a(t) = \int_0^t u(s)^{-1}\,\mathrm{d}s$. The horizon functional increases with u:

$$R_0[u] = c\int_0^\infty e^{-b(t)}\,\mathrm{d}t.$$

The fast envelope gives

$$a_{\text{fast}}(t) = (1 + \Gamma H_0 t)^{1/\Gamma}, \quad 0 \le t \le t_*,$$

and an exponential tail thereafter. Its radius is

$$c\int_0^{t_*} (1 + \Gamma H_0 t)^{-1/\Gamma}\,\mathrm{d}t + D\rho^{-1/\Gamma} = \overline{R}_\Gamma.$$

The lower envelope gives B. The lower inequality is strict because $u'(0) = 2$. For $\Gamma > 2$, the fast envelope also violates the initial derivative. For $\Gamma = 2$, its terminal corner is not smooth. Thus the upper inequality is strict as well.

Lemma gives sharpness. All profiles have $H \geq K$, hence $a^{-1} \leq e^{-Kt}$; dominated convergence applies to the approximating profiles and to convex interpolation. The image is therefore the whole open interval. The upper comparison with D also follows because the fast profile has $H > K$ on a nonempty initial interval. The special value and limit in (a) follow from .

For (b), the slow envelope is a de Sitter segment of rate H_0 lasting $\tau - t_*$, followed by the fast transition. Splitting the integral gives its stated radius. Approximation, strictness and convex interpolation give the open range. If $\tau < t_*$, the derivative bound forbids the required total change. Equality would require $u' = \Gamma$ throughout $(0, \tau)$, incompatible with smooth terminal constancy.

Part (c) follows from – and Lemma . Part (d) follows by choosing R_0's on opposite sides of the source threshold. A causal wire gives least error zero when visible, while Theorem gives a_d when dark.

Corollary (The terminal label and early-junction radius are different). *Even at fixed K, Γ, early data do not determine R_0. The settled late proper radius is nevertheless exactly c/K.*

If K is left free with no settling deadline, every $0 < K < H_0$ remains feasible for $\Gamma \geq 2$. At fixed κ, the late-temperature image and its minimax risk remain those of Theorem . The rate bound does not determine the late label.

Proof. The first statement is Theorem . Direct integration on an exponential tail gives c/K. Lemma constructs a transition for every $u_1 > u_0$, proving the last claim.

par *Scope.* The geometric bound is an input, not a derived energy condition or field equation. The limit $\Gamma \to \infty$ recovers the *nonincreasing-Hubble* fixed-terminal interval (B, D), not the unrestricted smooth-future interval $(0, \infty)$. The factor z belongs to the proper radius, not to the comoving conformal distance. These distinctions sharpen the source-record rate theorem [Big02]. No extension obstruction is certified. par

Documentary record (Rate-family energy minimax as a parallel statement; inserted in version 5). On comparison metrics $-c^2\,\mathrm{d}t^2 + e^{2\varphi(t)}\,\mathrm{d}\mathbf{x}^2$ with rate $r = \varphi'$, Red06-G-DSS-CERT proves an exact finite-window minimax for the energy component $\Theta_\Lambda(\mathrm{n}, \mathrm{n}) = 3r^2/c^2 - \Lambda$ at a fixed later time t_1: if the rates of a family agreeing for $t \leq 0$ fill $[a, b]$ at t_1, the least worst-case error of a rule from data in $\{t \leq 0\}$ is $3(b^2 - a^2)/(2c^2)$ [Red06]. Its rate bound enters only through the window $[a, b]$. The same record gives smooth futures with an identical de Sitter prefix, one with a finite and one with an infinite optical tail [Red06]. Both statements are source record; not a premise.

The horizon-radius minimax of Theorem and the energy minimax of prop:RC-rate9 are parallel statements on different targets. The first concerns the conformal integral R_0 of futures with the radiation prefix under the declared bound ; the second concerns a pointwise energy component on a de Sitter prefix under a declared rate window. Neither determines the other, and neither is derived from the other here. Likewise, Proposition and prop:RC-tails9 are parallel prefix non-determination statements on different prefixes.

par *Non-claim.* No dynamics is selected, and the rate bound remains an input. Inextendibility stays open. The energy minimax is not converted into a value of a radiation label or into a dynamical consequence. par

Documentary record (Late-slice expansion and FRW null invariance; inserted in version 5).

Hawk07-C-CERT considers deformations g_{w} of a zero-shift metric whose spatial factor is rescaled by $e^{2\mathrm{wb}(t)}$, with b = 1 on $\{t \geq 1\}$. It proves that the late-slice expansion separates every pair of distinct deformation parameters, $\theta_{\mathrm{w}_1} - \theta_{\mathrm{w}_2} = 3(\mathrm{w}_1 - \mathrm{w}_2)/N$ [HawkC]; on a flat FLRW base these deformations shift the late rate.

Hawk07-C-Dyn-CERT proves that, for spatially flat FLRW metrics $a(\eta)^2(-c^2\,\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2)$, whole-fibre null invariance holds exactly where $Q = a^2a'''' - 8aa'a''' - aa''\text{^}2 + 14a'\text{^}2a''$ vanishes, and that linear conformal factors,

the radiation metric among them, solve $Q = 0$ [HawkDyn]. Both statements are source record; not a premise. On the prescribed prefix, $d\eta = dt/a_r$ gives $\eta = a_r/H_0$ up to a constant, so the conformal factor $a_r = H_0\eta$ is linear and the cited null invariance holds there. On a terminal tail with $H = K$, the expansion of $c^{-1}\partial_t$ is $3K/c$. Neither statement supplies κ: both concern the geometry, and the expansion is not among the admitted data of this paper. If a late expansion were separately supplied, it would supply K only, and $T_{late} = \kappa K/(2\pi k_B)$ would still require κ. When K is free, the late temperature label is not fixed, as in Theorem .

par *Non-claim.* No dynamics is selected, and no Einstein equation, derived metric or field equation is claimed. par

Version 6: comparison-future formulae, apparent radius and a sharp modulus

This subsection is inserted in version 6 (plan item 4).

Theorem (Event and apparent radii on comparison futures; inserted in version 6). *Use Definition and put* $z = a_r(t)$, $t_i \le t \le 0$.

• *For every comparison future with finite* $R_0[a]$, $\chi_e(t) = B(1 - z) + R_0$ *and* $R_e(t) = z[B(1 - z) + R_0]$, *and the apparent (Hubble) radius on the prefix is* $R_A(t) = c/H_r(t) = Bz^2$. $R_A(t)$ *is a function of the prefix alone;* $R_e(t)$ *is not. Moreover* $R_e(t) - R_A(t) = z[B(1 - 2z) + R_0]$.

• *On* $\mathscr{F}^{\Gamma}(K)$, $0 < K < H_0$, $\Gamma \ge 2$: $R_e(t) - R_A(t) > 2Bz(1 - z) \ge 0$ *at every prefix time, so the event radius strictly exceeds the apparent radius, and* $\dot{R}_e > 0$ *on* $[t_i, 0]$. *The set of values of* $R_e(t)$ *is the open interval* $(Bz(2 - z), z[B(1 - z) + \overline{R}_\Gamma])$, *and that of* $R_e(t) - R_A(t)$ *is* $(2Bz(1 - z), z[B(1 - 2z) + \overline{R}_\Gamma])$. *The deterministic minimax risks from early data are* 0 *for* $R_A(t)$ *and* $z(\overline{R}_\Gamma - B)/2$ *for both* $R_e(t)$ *and* $R_e(t) - R_A(t)$; *the midpoints are optimal and their worst-case suprema are not realized.*

• *(Sharp modulus.) Let* $a, \widetilde{a}$ *be positive* C^1 *functions on* $[0, \infty)$ *with* $a(0) = \widetilde{a}(0) = 1$ *whose Hubble profiles satisfy* $H, \widetilde{H} \ge h > 0$ *and* $\sup|H - \widetilde{H}| \le \delta$, $\delta \ge 0$. *Then*

$$|R_0[a] - R_0[\widetilde{a}]| \le \frac{c\delta}{h(h + \delta)},$$

with equality for the constant profiles h *and* $h + \delta$. *For two comparison futures with the common prefix,* $|R_e(t) - \widetilde{R}_e(t)| \le zc\delta/(h(h + \delta))$.

• *On* $\mathscr{F}^{\Gamma}(K)$ *every profile satisfies* $K \le H \le H_0$ *on* $[0, \infty)$. *Taking* $h = K$ *and* $\delta = H_0 - K$ *in (c) gives* $|R_0[a] - R_0[\widetilde{a}]| \le D - B$, *consistent with, and weaker than, the exact width* $\overline{R}_\Gamma - B < D - B$ *of Theorem (a). The equality profiles of (c) do not match the prefix and are not in* $\mathscr{F}(K)$.

Proof. (a) Lemma gives χ_e and R_e. From , $H_r = H_0/(1 + 2H_0 t) = H_0/z^2$, so $c/H_r = Bz^2$. Subtraction gives the difference. R_A involves only the prefix; R_e varies with R_0 at a fixed prefix by Theorem (a). (b) By Theorem (a), R_0 fills $(B, \overline{R}_\Gamma)$. Both R_e and $R_e - R_A$ are affine in R_0 with slope $z > 0$, so their images are the stated open intervals; at $R_0 = B$ the difference is $z(2B - 2Bz) = 2Bz(1 - z) \ge 0$, and $R_0 > B$ gives strictness. From $R_e(t) = a_r(t)\chi_e(t)$ and $\dot{\chi}_e = -c/a_r$, $\dot{R}_e = H_r R_e - c = (c/z)(1 - 2z + R_0/B) > (c/z)\,2(1 - z) \ge 0$. Since R_A is constant on the early-data fibre its risk is zero; Lemma gives the other two risks and the attainment statement. (c) Write $b = \ln a = \int_0^t H$ and $\widetilde{b} = \ln \widetilde{a}$. Then $\min(b, \widetilde{b}) \ge ht$ and $|b - \widetilde{b}| \le \delta t$, so

$$|e^{-b} - e^{-\widetilde{b}}| = e^{-\min(b, \widetilde{b})}(1 - e^{-|b - \widetilde{b}|}) \le e^{-ht}(1 - e^{-\delta t}).$$

Integrating over $[0, \infty)$ gives $1/h - 1/(h + \delta) = \delta/(h(h + \delta))$; multiply by c. Constant profiles give $R_0 = c/h$ and $c/(h + \delta)$. For the proper radius, a common prefix gives a common z and $B(1 - z)$. (d) On $\mathscr{F}^{\Gamma}(K)$, $(1/H)' \ge 0$, so H is

nonincreasing from $H(0) = H_0$ to its eventual value K. Hence $|H - \widetilde{H}| \le H_0 - K$, and $c(H_0 - K)/(KH_0) = c/K - c/H_0 = D - B$. Theorem (a) gives $\overline{R}_\Gamma < D$. A constant profile h has $H(0) = h$; it is prefix-matched only if $h = H_0$, and then it is not eventually $K < H_0$.

Status: proved here, from Lemma , Lemma and Theorem .

par *Non-claim.* Printed radii, intervals and risks are calculations on a prescribed geometry, not measurements. No future is selected, no energy condition or Einstein dynamics is derived, the rate bound remains an input, and no entropy is assigned to a horizon. par

Documentary record (A prescribed past does not determine the future: parallel records; inserted in version 6). Big02-A gives part (a) of Theorem on its radiation host, the uncertainty propagation $z(v - u)/2$, and the modulus of part (c) [Big02]. Big05-A proves, for a de Sitter past and smooth futures with $H \le 0$, that the event-horizon radius at $t = 0$ ranges exactly over $[c/H_E, \infty]$, that every rule of the past metric has infinite worst-case error for it, and that the existence predicate has worst-case error $\frac{1}{2}$ under randomized decisions [Big05]. Its prefix is de Sitter, not radiation-type, and its class $H \le 0$ differs from the bound . On the present prefix the non-determination statements are local: Theorem , Proposition , Theorem and Theorem (b). Neither record is used in a proof here. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* Past-only prediction of a horizon is refused, not performed; no continuation law is imported, and no printed radius is a measurement. par

Proposition (Finite-settling functional and the apparent radius; inserted in version 6). *Let $a(0) = 1$, $H = \dot{a}/a > 0$ continuous on $[0, \infty)$, with $H = K > 0$ for $t \ge T$, and $b(t) = \int_0^t H$. Then*

$$R_0[a] = c\int_0^T e^{-b(s)}\,\mathrm{d}s + \frac{c}{K}e^{-b(T)}, \qquad R_e(t) = ce^{b(t)}[\int_t^T e^{-b(s)}\,\mathrm{d}s + \frac{e^{-b(T)}}{K}] \ (0 \le t \le T),$$

$R_e(t) = c/K$ for $t \ge T$, and $\dot{R}_e = HR_e - c$. On the spatially flat metric the apparent (Hubble) radius is $R_A = c/H$, the positive zero of $g^{ab}\partial_a R\,\partial_b R = 1 - H^2R^2/c^2$ for the areal radius $R = ar$; on the prefix it is Bz^2 (Theorem (a)). If $H = h$ on $[s, s + \ell]$, then $R_e(s) = \frac{c}{h}(1 - e^{-h\ell}) + e^{-h\ell}R_e(s + \ell)$.

Proof. Split $c\int_t^\infty \mathrm{d}s/a$ at T; for $s \ge T$, $a(s) = e^{b(T)}e^{K(s-T)}$, whose reciprocal integrates to $e^{-b(T)}/K$. Multiply by $a(t) = e^{b(t)}$ for R_e; for $t \ge T$ the same computation gives c/K. Differentiating $R_e = a\int_t^\infty c\ \mathrm{d}s/a$ gives $\dot{R}_e = HR_e - c$. With $R = ar$ and $\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + a^2\,\mathrm{d}r^2 + \ldots$, $\partial_t R = HR$, $\partial_r R = a$, so $g^{ab}\partial_a R\partial_b R = -H^2R^2/c^2 + 1$. On a segment with $H = h$, $a(s + v) = a(s)e^{hv}$; split the integral at $s + \ell$.

par *Non-claim.* These are integrals over prescribed profiles; no future is selected and no radius is measured. par

Theorem (A fixed terminal rate alone leaves every positive radius; inserted in version 6). *For every $K > 0$, $\{R_0[a] : a \in \mathscr{F}(K)\} = (0, \infty)$, where $\mathscr{F}(K)$ is the class of Definition without a monotonicity or rate bound. Consequently, with K supplied: the early apparent radius is fixed while the early comoving and proper event radii $B(1 - z) + R_0$ and $z[B(1 - z) + R_0]$ are not; their absolute minimax risks from early data are infinite, also for randomized rules; and c/K is the settled proper radius, not R_0.*

Proof. Every member has $0 < R_0 < \infty$ by Proposition . *Small values.* For $n \ge 2$ put $\delta_n = 1/(n^2H_0)$; keep $H = H_r$ on $[0, \delta_n]$, pass smoothly and positively to nH_0 by $2\delta_n$, keep nH_0 for time $1/H_0$, pass smoothly to K within time $1/H_0$ and keep K. Since $H > 0$, $a \ge 1$, and after the plateau $a \ge e^n$; hence $R_0 \le c[2\delta_n + 1/(nH_0) + e^{-n}(1/H_0 + 1/K)] \to 0$. *Large values.* Fix $t_0 > 0$; for $0 < \varepsilon < H_0$ pass smoothly from the prefix to $H = \varepsilon$ by t_0 with $0 < H \le H_0$, keep ε on $[t_0, t_0 + 1/\varepsilon]$, then pass to K. Since $a(t_0) \le e^{H_0t_0}$, $R_0 \ge c\,e^{-H_0t_0}\int_0^{1/\varepsilon} e^{-\varepsilon v}\,\mathrm{d}v = c\,e^{-H_0t_0}(1 - e^{-1})/\varepsilon \to \infty$. *Connectedness.* For two members, $H_\theta = (1 - \theta)H_1 + \theta H_2$ is smooth, positive, prefix-matched and equal to K after a common time T; by

the settling formula $\theta \mapsto R_0$ is continuous. So the image is an interval in $(0,\infty)$ with infimum 0 and no upper bound. The radii are affine in R_0 with positive slope on one early-data fibre, and the two-point inequality gives infinite risk.

Remark (Overlaps). With the rate bound, Theorem (b) is the stronger form of the event–apparent separation (it also gives the intervals and the risks), and Theorem gives the finite-time reception envelopes. The fact that a source within $B(1-z)$ is visible for every continuation is Proposition . The limit $\Gamma \to \infty$ in Theorem gives (B, D), not the interval $(0,\infty)$ of Theorem . Causal reachability permits a channel placement; it does not make the accessible channel an identity channel.

Documentary parallel: Big02-A [Big02]. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* All radii are calculations on prescribed metrics, not measurements. No causal order, horizon, field equation or transport channel is inferred from a label value. par

Two-prefix comparison and coefficient separation

Cheung's nuADaM01-A uses a separately prescribed two-epoch comparison and a radiation example [nuADaM]. The proposition below concerns only the explicit early profiles and label formulas stated here.

Theorem (Early-profile isometry and label factors). *Let*

$$a_{\mathrm{N}}(\tau) = (\tau/\tau_i)^{1/2}, \qquad h(\tau) = \frac{1}{2\tau}, \qquad 0 < \tau_i < \tau \le \tau_*.$$

Set

$$H_0 = \frac{1}{2\tau_*}, \quad a_* = \sqrt{\tau_*/\tau_i}, \quad t = \tau - \tau_*, \quad t_i = \tau_i - \tau_*.$$

Then

$$a_{\mathrm{N}} = a_* a_{\mathrm{r}}, \qquad h = H_{\mathrm{r}},$$

and $(\tau, \mathbf{x}) \mapsto (t, a_*\mathbf{x})$ *is an observer-preserving isometry of the two early metric regions.*

Define a second label family by

$$T_{\mathrm{N}} = \frac{\kappa_{\mathrm{N}}^{3/4}}{k_B}\left(\frac{45c^5h^2}{4\pi^3 G g_*}\right)^{1/4},$$

$$S_{\mathrm{N}} = \frac{8\pi^3}{135} g_{*s} k_B \left(\frac{k_B T_{\mathrm{N}}}{\kappa_{\mathrm{N}} c}\right)^3 a_{\mathrm{N}}^3 r^3,$$

where all displayed dictionary constants are supplied and $r > 0$ *is a declared comoving radius. For the first family write its slot* κ_{B}*. Then*

$$\frac{T_{\mathrm{N}}}{T_{\mathrm{r}}} = \rho_T (\kappa_{\mathrm{N}}/\kappa_{\mathrm{B}})^{3/4}, \; \rho_T = \left(\frac{45c^2H_0^2\alpha}{4\pi^3 G g_* U}\right)^{1/4},$$

$$\frac{S_{\mathrm{N}}}{S_{\mathrm{r}}} = \rho_S (\kappa_{\mathrm{B}}/\kappa_{\mathrm{N}})^{3/4},$$

where the time- and coefficient-independent factor is

$$\rho_S = \frac{2\pi^3}{45} g_{*s} c^{-9/4} \left(\frac{45c^5}{4\pi^3 G g_*}\right)^{3/4} H_0^{3/2} a_*^3 r^3 \alpha^{-1/4} U^{-3/4} V_c^{-1}.$$

The two temperature ratio histories agree and both entropy ratio histories are constant. An exact cross ratio determines only $\kappa_{\mathrm{N}}/\kappa_{\mathrm{B}}$*. Common rescaling* $(\kappa_{\mathrm{B}}, \kappa_{\mathrm{N}}) \mapsto (q\kappa_{\mathrm{B}}, q\kappa_{\mathrm{N}})$ *preserves every time ratio and cross ratio, so these augmented data do not determine either absolute slot.*

If the slots are declared equal, an exact absolute label at fixed dictionary constants does calibrate that shared slot. No assertion of cross-host non-identifiability overrides that calibration fact.

Proof. Since $1 + 2H_0 t = \tau/\tau_*$, the profile identities and metric pullback are immediate. Also $h = H_0/a_r^2$. Substituting this in T_N and dividing by gives . In S_N, the product $h^{3/2}a_N^3$ equals $H_0^{3/2}a_*^3$; division by S_r gives . The ratio and common-rescaling statements follow from these powers. Exact absolute calibration follows from Theorem .

par *Scope.* The isometry identifies the early metric regions only. It does not identify their later junction constructions, the static de Sitter patch, Host A, or Host B. The dictionary constants are not derived Friedmann data. No physical atomic dark-sector model is reconstructed from these formulas. par

Conservation and spectator declarations

Proposition (Inverse-source vacuity). *For a smooth prescribed a, a constant Λ, and every separately supplied $\chi \neq 0$,*

$$G_{tt} = 3H^2, \qquad G_{ij} = -\frac{a^2}{c^2}(2\dot{H} + 3H^2)\delta_{ij}.$$

The tensor $\Theta = G + \Lambda g$ is conserved, and $T = \chi^{-1}\Theta$ satisfies $G + \Lambda g = \chi T$ by definition.

If auxiliary homogeneous coefficients are defined by

$$\rho_{\mathrm{cmp}} = \frac{3H^2 - \Lambda c^2}{\chi}, \qquad p_{\mathrm{cmp}} = \frac{-2\dot{H} - 3H^2 + \Lambda c^2}{\chi},$$

then

$$\dot{\rho}_{\mathrm{cmp}} + 3H(\rho_{\mathrm{cmp}} + p_{\mathrm{cmp}}) = 0.$$

Neither κ nor the radiation dictionary occurs. For $k = \partial_t + (c/a)\partial_{x_1}$,

$$\Theta(k,k) = G(k,k) = -2\dot{H}.$$

Proof. The nonzero connection coefficients needed for the calculation are

$$\Gamma^t_{ij} = \frac{a\dot{a}}{c^2}\delta_{ij}, \qquad \Gamma^i_{tj} = H\delta^i_j.$$

They give $R_{tt} = -3(\dot{H} + H^2)$, $R_{ij} = a^2(\dot{H} + 3H^2)\delta_{ij}/c^2$, and $R = 6(\dot{H} + 2H^2)/c^2$, hence the displayed Einstein tensor. The contracted Bianchi identity and metric compatibility give conservation. The auxiliary continuity identity follows from $\dot{\rho}_{\mathrm{cmp}} = 6H\dot{H}/\chi$ and $\dot{\rho}_{\mathrm{cmp}} + p_{\mathrm{cmp}} = -2\dot{H}/\chi$. The null contraction is direct.

Remark (No entropy bridge). The preceding conservation identity holds for every prescribed scale factor. It selects no matter model and imposes no equality between S_r, a horizon-area proxy, and S_{late}. At fixed geometry $S_r/S_{\mathrm{late}} \propto \kappa^{1/4}$, so imposing their numerical equality is an additional condition, not a consequence of conservation. The vacuity statement parallels the Hawk07 dynamics certificates [HawkDyn].

Proposition (Spectator action and the metadata distinction). *Let a monoid be generated by:*

- *appending a fixed, payload-independent spectator state τ_n, disconnected from every message-bearing operation;*
- *adjoining metadata that are coefficient-blind;*
- *changing only the symbolic presentation of a label formula, without supplying its numerical value. The active geometry, source placement, complete receiver access contract, target, and label dictionary constants remain fixed.*

Then

$$\varepsilon^*(\mathcal{M} \otimes \tau_n) = \varepsilon^*(\mathcal{M}).$$

Every word in the generators preserves the corresponding active recovery values and preserves the original coefficient-identifiability problem.

Adjoining an exact numerical radiation label is different: it can refine the data fibre and, under the calibration hypotheses, supply κ. It still does not change a channel when used only as unused metadata, but it does change the decision problem.

Proof. A decoder for $\mathscr{N}$ can discard the spectator. Conversely, a decoder for $\mathscr{N} \otimes \tau_n$ can prepare τ_n internally. Thus decoder composites transfer in both directions. Blind metadata and symbolic formulas do not refine the fibre. Induction proves the word statement. The final distinction follows from or single-label calibration at fixed *U*.

Proposition (Capacity promises are not automatically preserved). *Suppose a separately supplied promise counts all declared q-dimensional sites:*

$$n \ln q \le \frac{\mathrm{etaS}_{\mathrm{r}}^{(\kappa)}}{k_B}, \qquad q \ge 2.$$

Appending n' idle sites preserves every active channel value and every label, but changes the left side to $(n + n')\ln q$. The promise fails as soon as

$$n' > \frac{\mathrm{etaS}_{\mathrm{r}}^{(\kappa)}}{k_B \ln q} - n.$$

Thus the spectator action is an action on the promised class only while the count remains admissible.

An arbitrary host-preserving declaration can instead replace an accessible replacement channel by an identity bypass, lowering recovery error. There is no invariance theorem for all such declarations.

Proof. Count the appended sites and use Proposition . For the bypass, compare $\varepsilon^*(\mathscr{R}_{I/d}) = a_d$ and $\varepsilon^*(\mathrm{id}) = 0$.

par *Scope.* These are bookkeeping statements about declared comparison accounts, parallel to Cheung's monoid discussion [Big01] and its spectator qualification [Big02,Big03]. They neither derive a physical resource price nor close a capacity promise. A physically coupled early phase is not constructed by adding an epoch label. par

Documentary record (The radiation labels do not fix the vacuum term; inserted in version 5). Let $\mathscr{F}_{\mathrm{vac}} = \{T + c_0 g + c_G G + c_1 E^{(1)} + c_2 E^{(2)} : (c_0, c_G, c_1, c_2) \in \mathbb{R}^4\}$ be the conserved local fibre of DS-OB-1-CERT [DSOB1]. That record proves that every member is symmetric and divergence-free; that on every four-dimensional open set with $\mathrm{Ric} = \frac{R}{4} g$ every member equals $T + (c_0 - \frac{R}{4} c_G) g$; and that the constant $\lambda = c_0 - \frac{R}{4} c_G$ is not a function of the metric there. Cheung's Big03-A adds that κ does not occur in the fibre and that a supplied coefficient selects no c_0 and no λ [Big03]. Both statements are source record; not a premise.

On the present host, the radiation labels are functions of (κ, U, α, V_c), of the dictionary constants and of the prescribed prefix only. None of c_0, c_G, c_1, c_2 or Λ occurs in them. Hence the supply of an exact radiation label, and with it the supply of κ under the calibration hypotheses of Theorem , selects neither the fibre coordinate c_0 nor Λ.

A local qualification applies. By the curvature formulas in the proof of Proposition , the prefix has $\dot{H}_{\mathrm{r}} = -2H_{\mathrm{r}}^2$, hence $R = 0$ and $R_{tt} = 3H_{\mathrm{r}}^2 \neq 0$. The radiation prefix is therefore not an Einstein region, and the reduction of the fibre to multiples of *g* does not apply on it. On a terminal de Sitter tail of a future in $\mathscr{F}(K)$, $\dot{H} = 0$ gives $\mathrm{Ric} = \frac{R}{4} g$ with $R = 12K^2/c^2$, so the cited reduction applies there.

par *Non-claim.* No vacuum energy is computed, and the absolute vacuum energy remains open. The separation is not a premise for any dynamical claim, and no Einstein equation is derived. par

Proposition (A constant volume term and the coefficient orbit are independent; inserted in version 6). *Adjoin $\lambda \in \mathbb{R}$ to the split-augmented accounts of Definition : the account with shift λ carries the source $T_{\mathrm{tot}} + \lambda g$ and the cosmological constant $\Lambda + \chi\lambda$ (plan item 7).*

• $T_{\mathrm{tot}} + \lambda g$ *is symmetric and divergence-free, and* $\chi(T_{\mathrm{tot}} + \lambda g) = G + (\Lambda + \chi\lambda)g$.

• *Every radiation label, its ratio histories, the product , the inverse and the orbit invariant* $z = \kappa^3/\alpha$ *are unchanged by the shift; none contains* λ, Λ *or* χ.

• *On the data (prefix metric,* T_{r}, S_{r}*), and also with* κ *supplied,* λ *ranges over* $\mathbb{R}$ *in one fibre, and every rule for* λ *has infinite worst-case error. A supplied point of the coefficient orbit selects no* λ *and no* Λ.

• *Conversely, supplying* λ *or* Λ *changes no label, so the coefficient fibre and the risks of Section are unchanged. The vacuum slot of a split absorbs the shift: if* $(T_{\mathrm{o}}, T_{\mathrm{d}}, T_{\mathrm{v}}) \in \Sigma(\Lambda, \chi)$*, then* $(T_{\mathrm{o}}, T_{\mathrm{d}}, T_{\mathrm{v}} + \lambda g) \in \Sigma(\Lambda + \chi\lambda, \chi)$*, with the same labels on both accounts.*

Proof. (a) $\nabla g = 0$ and g is symmetric; $\mathrm{chiT}_{\mathrm{tot}} = G + \Lambda g$ by Definition . (b) The dictionary contains only (κ, U, α, V_c) , c, k_B and a_{r}, and the prefix metric is unchanged by the shift. (c) By (b), every $\lambda \in \mathbb{R}$ gives the same data, also at a fixed κ; the target image is $\mathbb{R}$, and Lemma (e) applies. (d) (b), and $T_{\mathrm{o}} + T_{\mathrm{d}} + T_{\mathrm{v}} + \lambda g = T_{\mathrm{tot}} + \lambda g = \chi^{-1}(G + (\Lambda + \chi\lambda)g)$.

Status: proved here. DS-OB-1-fibre gives effective rank two on the radiation prefix, where a supplied tensor $X - T$ determines (c_0, c_G) while no label does [DSOB1fibre]; rank one on de Sitter regions with label blindness [DSOB1fibre]; and that effective rank does not imply selection [DSOB1fibre]. Big03-A gives the de Sitter box risk and the compensated ambiguity $T \mapsto T + \lambda g$, $\Lambda \mapsto \Lambda + \chi\lambda$ [Big03v3], and the orbit qualification [Big03v3]. Source record; not a premise. Citations identify source records only; no proof transfer.

Remark (In what sense the κ**-slot is not selected).** The radiation labels are not functions of the admitted datum $\mathfrak{E}$ (Theorem). An exact label value at fixed supplied normalization (U, α) is itself an added supply that calibrates κ ; it is not information produced by the host. Neither a vacuum shift (Proposition) nor a split (Proposition) selects κ. Calibration remains by an added supply only.

par *Non-claim.* No vacuum energy is computed, and absolute vacuum energy stays open. The relation $\mathrm{chiT}_{\mathrm{tot}} = G + \Lambda g$ is a definition, not an imposed Einstein equation, and no value of Λ, λ or κ is admitted. par

Corollary (Action form of the shift and interval risk; inserted in version 6). *Where an action variation is declared, with* $T_{\mu\nu} = -2|g|^{-1/2}\delta S/\delta g^{\mu\nu}$*, the change* $S_v = S_0 - v\int\sqrt{|g|}\ \mathrm{d}^4x$ *gives* $T^{(v)}_{\mu\nu} - T^{(0)}_{\mu\nu} = -v\,g_{\mu\nu}$*, the shift of Proposition with* $\lambda = -v$*. At fixed* U, α, V_c, κ*, every rule for* v *from the labels has infinite worst-case absolute risk on* $\mathbb{R}$ *and, on a supplied closed interval* $[v_-, v_+]$*, deterministic and randomized minimax risk* $(v_+ - v_-)/2$.

Proof. $\delta\sqrt{|g|} = -\frac{1}{2}\sqrt{|g|}\,g_{\mu\nu}\delta g^{\mu\nu}$, so the added term contributes $\frac{v}{2}\sqrt{|g|}g_{\mu\nu}$ to $\delta S/\delta g^{\mu\nu}$ and $-vg_{\mu\nu}$ to $T_{\mu\nu}$. No label contains v, so the fibre is the whole range of v; the two-point inequality and the midpoint give the risks. The proof does not use a rank reduction, and U is not identified with an action-derived energy density: a model in which U changes with v is a different fibre.

par *Non-claim.* No absolute vacuum energy, cosmological constant, matter split or physical normalization is selected; a calibrated dictionary coordinate is not a vacuum-energy calculation. par

A separate approximate-Singleton resource statement

The robust replacement envelope concerns near-constant *accessible channels*. Approximate Singleton bounds concern near-perfect recovery of specified erasures on a declared factorization. These are different hypotheses.

For $0 \le x \le 1$, write

$$h(x) = -x\ln x - (1-x)\ln(1-x), \qquad g(x) = (1+x)h\left(\frac{x}{1+x}\right),$$

with the continuous endpoint conventions.

Theorem (Approximate Singleton with two erasure allowances). *Let* $V : \mathbb{C}^D \to A \otimes B \otimes C$ *be an isometry. Suppose erasures of A and B have half-diamond recovery errors at most* $\varepsilon_A, \varepsilon_B$*. Then*

$$(1-\varepsilon_A-\varepsilon_B)\ln D \le \ln\dim C + \frac{1}{2}(g(\varepsilon_A)+g(\varepsilon_B)).$$

Equivalently, the entropy argument allows a defect of size

$$(\varepsilon_A+\varepsilon_B)\ln D + \frac{1}{2}(g(\varepsilon_A)+g(\varepsilon_B)).$$

It is not a dimension-free $O(\varepsilon\ln(1/\varepsilon))$ allowance.

Proof. Let ψ_{RABC} be the encoded normalized Bell state. A decoder for erasure of A produces ω_{RL} within trace distance ε_A of Φ_D, with $\omega_R = I/D$.

We use the Alicki–Fannes–Winter conditional-entropy bound [Winter2016]:

$$|S(R|L)_\omega - S(R|L)_{\Phi_D}| \le 2\varepsilon_A \ln D + g(\varepsilon_A).$$

Together with data processing of mutual information [WatrousBook], this gives

$$I(R:BC)_\psi \ge 2\ln D - 2\varepsilon_A\ln D - g(\varepsilon_A).$$

Purity implies $I(R:A) + I(R:BC) = 2\ln D$, hence

$$I(R:A) \le 2\varepsilon_A\ln D + g(\varepsilon_A).$$

The same argument applies to B. Purity and subadditivity also give

$$2\ln D \le 2S(C) + I(R:A) + I(R:B).$$

Substitute both information bounds and $S(C) \le \ln\dim C$, then rearrange.

Corollary (Conditional joint ledger). *Suppose a declared n-site q-ary memory protects a d-dimensional target against every r_{er}-site erasure with half-diamond error at most $\varepsilon < 1/2$, and suppose $n \ge 2r_{\mathrm{er}}$. Then*

$$\ln d \le \frac{(n-2r_{\mathrm{er}})\ln q + g(\varepsilon)}{1-2\varepsilon}.$$

If complete communication also passes through an (L,m) cut with permitted recovery error $e < 1$, then

$$d \le \min\left\{\frac{m}{1-e}, m\sqrt{\frac{L}{1-e}}\right\}.$$

With the additional supplied capacity promise in Proposition , the first bound permits the substitution $n\ln q \le \mathrm{eta}\mathrm{S}_{\mathrm{r}}^{(\kappa)}/k_B$. This is a numerical resource bound only when its dictionary constants, including U, and its coefficient input are specified.

Proof. Use two disjoint erased sets in Theorem . For the communication bound, write branch effects E_x and encoder/decoder Kraus operators A_{xa}, B_{xb}. The branch contribution to d^2F_{e} obeys

$$\sum_{a,b}|\mathrm{Tr}\,(B_{xb}A_{xa})|^2 \le m\,\mathrm{Tr}\,E_x, \qquad \sum_{a,b}|\mathrm{Tr}\,(A_{xa}B_{xb})|^2 \le m^2.$$

The first inequality uses rank at most m; the second uses $E_x \le I$ and decoder trace preservation. Summing gives $F_{\mathrm{e}} \le \min\{m/d, Lm^2/d^2\}$. Since $1-e\mathrm{leF}_{\mathrm{e}}$, rearrangement gives the bound. The promise substitution is algebraic.

par *Scope.* The continuity inequality is an explicitly cited external mathematical input, not a companion proof transfer. The factorization, erasure guarantees, and access cut are supplied comparison hypotheses. No physical threshold or overhead law is derived. In particular, neither a replacement-neighbourhood allowance nor a weight-t approximate noise statement automatically supplies the erasure hypotheses of . The corresponding companion formula is [Big02]. par

Interpretation, additional source separations, and limitations

The four-part reading remains an analogy

Cheung's access-relative classification has four categories [Big02]:

• coding, syndrome, or gauge allocations outside the target;

• information excluded from the complete accessible channel;

• unused sectors or residual factors;

• information excluded by a specified causal cut. These are not disjoint physical substances or a canonical Hilbert-space decomposition.

The dark-radiation reading in this paper is primarily a (D4) interpretation when an early source is outside the observer's causal past. Its quantitative statement is the complete-access replacement theorem, not a species claim. A (D3) unused orthogonal sector alone forces no error: an isometric encoding into a larger register has a channel left inverse on its code. Consequently, junk is not assigned the replacement floor. An epoch tag does not become a fifth independent category.

Casimir and coupling coordinates

Documentary record (A third named coefficient coordinate). The supplied Hawk07-F-CERT passages describe a coefficient q_{C} occupying the slot of $\varepsilon\hbar$ in the fixed-inverse-length stress family. Its fixed-mass variant instead lets q_{C} occupy the $\hbar$-slot while changing $\mu = mc/q_{\mathrm{C}}$. Hawk07-H-CERT states an analogous fixed-contract coefficient separation [HawkFcert,HawkHcert].

These are distinct declared comparison coordinates from κ in the radiation dictionary and from the gravitational coupling χ. No equality between them is stipulated. An exact supply in the radiation dictionary is not thereby an evaluation of a Casimir stress in a different account. The anisotropic null-combination test of those records is not a cosmological-constant calculation.

par *Scope.* No Casimir series is evaluated here. No end stress, field state, coupling normalization, or cross-model coefficient identification is supplied. par

Additional Cheung catalogue-companion edges

Documentary record (Different target objects). The supplied excerpts support the following limited documentary comparisons.

• Oai211-A studies labelled path sums on a conditioned tree [Oai211]. Such a path-sum coordinate is not T_{r}, S_{r}, or the coefficient κ.

• Oai212-A states a conditional transfer theorem for random passage fronts [Oai212]. An asymptotic front or its deterministic limiting norm is not an event horizon of the prescribed metric.

• Oai280-C distinguishes whole-code recovery from subsystem and algebra recovery [Oai280C]. Our replacement floor fixes the complete d-dimensional logical target. It is not a theorem about every smaller observable or subsystem. The earlier stream-slot and vacuum-Ward distinctions in Big04-A version 3 remain documentary: a stream slot is not an $\hbar$-slot, and a Ward identity is not an absolute vacuum-energy calculation [Big04v3]. No OpenAI family proof is verified or imported.

Remark (Complete access versus an outgoing factor). A supplied complete isometry has a channel left inverse on its *whole* output:

$$\mathscr{L}(Y) = V^{\dagger}YV + \operatorname{Tr}\left[(I - VV^{\dagger})Y\right]\tau.$$

This does not produce a decoder on an arbitrarily restricted output factor. Cheung's En07-B makes this interface distinction explicit [En07B]. It is consistent with the exact error-correction structure of Knill and Laflamme [KL].

Documentary record (Two direct catalogue edges; inserted in version 5).

• Oai280-B, Theorem thm:oneslot, is a sharp one-slot recovery error $\frac{1}{2}$ in a streaming interface [Oai280B]. A stream slot is not the $\hbar$-slot, and the radiation labels are not read as stream-slot data.

• Oai282-B, equation eq:vacward, is a Ward identity for declared fields in a vacuum vector [Oai282B]. A vacuum Ward identity is not a vacuum energy, and the radiation labels are not read as one. These one-line edges cite the two catalogue entries directly; the version-3 record cited above contains the same distinctions.

par *Non-claim.* No family proof is verified or imported. par

Version 6: access and recovery separations

Lemma (Two different finite floors; inserted in version 6). *For every integer $d \geq 2$, $a_d = 1 - d^{-2}$ and $s_d = 1 - 1/d$ satisfy*

$$a_d - s_d = \frac{d-1}{d^2} > 0, \qquad \frac{a_d}{s_d} = \frac{d+1}{d}.$$

In particular $a_d \neq s_d$ for every $d \geq 2$, and $a_d - s_d \to 0$ as $d \to \infty$.

Proof. $1 - d^{-2} - 1 + d^{-1} = (d-1)/d^2$, and $\frac{(d-1)(d+1)}{d^2} \cdot \frac{d}{d-1} = \frac{d+1}{d}$.

Status: proved here. Both numbers are half-diamond distances to the identity on M_d. The value a_d of Theorem is attained by preparing I/d after a replacement channel, whose output carries no record of the input; the value s_d below concerns channels that deliver a classical record. The inequality $a_d > s_d$ makes the two statements consistent.

par *Non-claim.* A comparison of two numbers. It does not identify the two interfaces or derive either from the other. par

Documentary record (Access and recovery separations; inserted in version 6).

• I-C proves that for every finite classical-output channel and deterministic decoder the reference-preserving half-diamond error is at least $1 - 1/d$, sharply, and that the Haar-average infidelity bound is $(d-1)/(d+1)$ [ICv2]. The located-circuit floor $1 - d^{-2}$ of Theorem concerns a source with no causal path to the complete receiver interface; it is not the Shredder floor (Lemma).

• I-C-CERT gives the exact error $1 - 1/S$, $S = \sum_i \lambda_i^{-1} \geq d^2$, of every record-ignoring decoder through a replacement channel with output spectrum (λ_i) [ICCERT]; its optimum $1 - 1/d^2$ is a_d. Equality of this scalar optimum across a parameter family does not imply equality of the installed output states or of the complete accessible processes.

• En04-B proves that on its account class with arbitrary declared finite carriers complete residual leakage is not a host functional (values 0 and $\frac{1}{2}$ on one host) [En04B]. The floor a_d is not a host functional either: it is fixed by the located-circuit contract, the source placement and d (Definition), not by the host alone.

• En04-B gives an exact clock-feedback criterion: the O-witness of a destructive clock readout followed by record-controlled feedback vanishes on all code pairs iff $\sum_k m_k F_k$ is scalar [En04B]. The radiation labels are evaluations of at supplied (κ, U, α, V_c); no clock POVM, destructive readout or record-controlled feedback occurs in Definition . The labels are not obtained by destructive Enter on a clock register.

• En04-A proves that on fibres of a declared magic label (Clifford flag, T-count or stabilizer entropy) carriers with leakage 0 and $\frac{1}{2}$ coexist [En04A], and states that residual response is not yet geometric response. The floor a_d is not a magic-fibre response, and no magic label enters the radiation dictionary.

• Hawk05-B proves the exact finite Haar mean of a subsystem entropy, the Page–Sen average, in a declared Haar model [Hawk05B]. It is a mean entropy, not a recovery error. The floor a_d is not a point of a Page curve, and S_{r} is not an entanglement entropy of a state.

• The Big05-A, Big04-A-coeff, Big03-A and DS-OB-1-fibre records cited with Propositions , and and Theorems and are parallel statements on their own hosts.

• The OpenAI catalogue [OpenAIcat], as served, contains no occurrence of the strings ``radiation" or ``dark"; it contains no radiation-label or dark-matter statement and remains documentary only. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* No Page curve, island, Shredder identification, magic-resource identification or geometric backreaction is claimed. The floor $1 - d^{-2}$ remains a located-circuit certificate under the declared contract, and d is a declared payload dimension. par

Version 6: a clock-feedback factorization test

Proposition (Exact clock-feedback factorization criterion; inserted in version 6). *Let C have orthonormal basis $\{|j\rangle\}$, let A contain every returned quantum register, and let $\Lambda_j : A \to B$ be declared channels. Destructive readout of C followed by outcome-controlled feedback has, after the record is discarded, the channel $\mathscr{F}(X) = \sum_j \Lambda_j(X_{jj})$ with $X_{jj} = (\langle j| \otimes I)X(|j\rangle \otimes I)$. For a channel $\Lambda : A \to B$,*

$$\mathscr{F} = \Lambda \circ \mathrm{Tr}_C \Leftrightarrow \Lambda_j = \Lambda \text{ for every } j.$$

For an output observable O, $\mathrm{Tr}(O\mathscr{F}(X))$ factors through Tr_C iff all $\Lambda_j^*(O)$ equal one operator. Both statements persist with an untouched reference.

Proof. If $\Lambda_j = \Lambda$ for all j, then $\mathscr{F}(X) = \Lambda(\sum_j X_{jj}) = \Lambda(\mathrm{Tr}_C X)$. Conversely, apply the identity to $X = \mathrm{proj}\, j \otimes \rho$: $\Lambda_j(\rho) = \Lambda(\rho)$ for every state, hence by linearity for every operator. For the observable, $\mathscr{F}^*(O) = \sum_j \mathrm{proj}\, j \otimes \Lambda_j^*(O)$, which equals $I_C \otimes B_O$ iff every block equals B_O. Tensoring with the identity on a reference preserves these identities.

Remark (What a clock record does not establish). A coefficient-blind record marginal does not establish blindness of the retained residual channel: the initial state, the feedback maps and the complete correlations must also satisfy hypothesis (iii) of Theorem . With diagonal POVM $F_k = \mathrm{proj}\, k$ and scalar values $m_k = \mathrm{Tr}(O\Lambda_k(\sigma))$, the criterion reduces to m_k independent of k, which is the scalar condition $\sum_k m_k F_k \propto I$ of En04-B [En04B]; that record treats general POVMs and a fixed probe state. No instrument here is supplied with a coefficient-sensitive encoding of the labels, so destructive readout provides no calibration mechanism; this is not an impossibility theorem for a separately supplied coefficient-sensitive apparatus. Source record; not a premise. Citations identify source records only; no proof transfer.

par *Non-claim.* No external clock parameter is removed, no relational-time construction is completed, and no radiation label is derived from clock consumption. par

Limitations

• No finite Hilbert space of a cosmological host is derived. All target and register dimensions are declared. O.1 is not closed.

• The causal-to-channel step uses the located-circuit contract. It is not a construction of a continuum quantum field channel.

• A thermal-operation comparison at a label temperature does not identify the physical noise channel. O6 remains open.

• No w, Γ, λ, U, ε_s, χ, or κ is inferred as physical data.

• The two-prefix isometry supplies no later-junction identification or entropy-transfer law.

• General hybrid-cut elementary formulas, and their weighted-projector existence classification, are not proved in this paper.

• No inextendibility, bounce, physical de Sitter-state construction, Page curve, island, or downstream characteristic quantum admission is established.

• (Version 5.) The hybrid-cut middle range for $L \geq 4$ stays open in the sense of the cited records. The version-6 closed forms, the four-label sectors and the bracket for $\varepsilon_1(3, 2)$ are source records, not results of this paper (Documentary records and).

• (Version 5.) No energy-constrained value outside the ranges of Documentary record is evaluated, and the vacuum term is not fixed (Documentary record). The late dimension-label risk of Proposition requires a supplied terminal rate.

• (Version 6.) The split of Definition is a declaration on the prefix region; no constitutive law or split-coupled observation is supplied, so no split target is identified (Proposition). The orbit invariant $z = \kappa^3/\alpha$ is not a selected coefficient.

• (Version 6.) Theorem concerns protocols whose complete accessible process is coefficient-blind; it says nothing about coefficient-sensitive experiments, which are added supplies.

• (Version 6.) The prediction layer of Section rests on supplied closures outside the ledger; it makes no observational claim and uses no measured number.

• (Version 6.) The orbit–split product of Proposition holds on the uncoupled product account class; an added relation coupling the split to (U, κ, α) defines a different class. The approximate-blindness bounds of Theorem need a supplied total-variation or round-wise half-diamond bound on the complete processes, cross-round memory included.

Conditional prediction layer: six supplied-closure theorems

This section is inserted in version 6 from the user's prediction layer (PRED_t345u). Each theorem states its supplied hypotheses explicitly. *Supplied closures are optional hypotheses outside the ledger*: they are not admitted into the standing ledger, they are not consequences of the label construction, and nothing earlier in this paper depends on them. The theorems are relations among declared functions on prescribed geometries; they contain no observational claim and no measured number. Verdicts X6-1–X6-6 are in Appendix .

Lemma (Kinematic drift identity on a prescribed flat FLRW metric). *Let $a > 0$ be C^1 on an interval containing $t_e < t_o$, and let a comoving source at fixed coordinate radius send radial null signals to the comoving observer. With $1 + z = a(t_o)/a(t_e)$, the emission time is a C^1 function of the reception time and*

$$\frac{\mathrm{d}z}{\mathrm{d}t_o} = (1 + z)H(t_o) - H(t_e).$$

Proof. The comoving distance $c\int_{t_e}^{t_o} \mathrm{d}s/a(s)$ is the fixed source radius. Differentiating in t_o gives $\mathrm{d}t_e/\mathrm{d}t_o = a(t_e)/a(t_o)$.

Then $\frac{\mathrm{d}}{\mathrm{d}t_o}\frac{a(t_o)}{a(t_e)} = \frac{\dot{a}(t_o)}{a(t_e)} - \frac{a(t_o)\dot{a}(t_e)}{a(t_e)^2}\frac{a(t_e)}{a(t_o)} = (1 + z)H(t_o) - H(t_e).$

par *Non-claim.* A kinematic identity on a prescribed metric; no field equation is used and no drift is measured. par

Theorem (Radiation consistency on the prefix interval; inserted in version 6). *Supplied hypotheses. (H1) The observer is comoving at the junction time $t_o = 0$, where $a_r(0) = 1$; (H2) sources are comoving and emit at $t_e \in [t_i, 0]$; (H3) U, α, κ, V_c are fixed. Optionally, (C1) a declared bridge identifies T_r with a separately declared temperature reading; (C1) is a supplied closure and is not used in the proof. Put $z_i = 1/a_r(t_i) - 1$. Then for $z \in [0, z_i]$:*

• *$T_r(z) = T_r(0)(1 + z)$ and $S_r(z) = S_r(0)$;*

• *$H_r(z) = H_0(1 + z)^2$;*

• *$T_r(z)^2/H_r(z) = T_r(0)^2/H_0$, a constant in z whose value is proportional to $\kappa^{3/2}$ at fixed U, α;*

• *for every comparison future, $\mathrm{d}z/\mathrm{d}t_o|_{t_o = 0} = -H_0 z(1 + z)$. For $z > z_i$, or for an observer at $t_o > 0$, nothing is asserted: the latter drift involves $H(t_o)$, which the prefix does not fix.*

Proof. (a) Theorem : $T_r \propto a_r^{-1}$ and S_r is constant; $a_r(t_e) = (1 + z)^{-1}$ by (H1). (b) $H_r = H_0 a_r^{-2}$ from . (c) Divide the square of (a) by (b); $T_r(0)^2 \propto \kappa^{3/2}$ by . (d) Every comparison future is smooth and agrees with the prefix on $[t_i, 0]$, so $H(0) = H_0$. Lemma gives $(1 + z)H_0 - H_0(1 + z)^2 = -H_0 z(1 + z)$.

par *Non-claim.* This is a set of relations among declared functions on the prefix interval. It does not assert that any epoch of the universe follows the radiation prefix, it supplies no temperature measurement, and the bridge (C1) is a supply. The constant in (c) depends on κ and does not calibrate it from the geometry. No observational claim is made. par

Theorem (General-*w* cross-consistency under a supplied closure; inserted in version 6). *Supplied closure (C2): on an interval J of a supplied positive C^2 scale factor with $H > 0$, the cosmological constant is declared*

zero, the coupling $\chi > 0$*, and the auxiliary coefficients of Proposition satisfy* $p_{\mathrm{cmp}} = w\rho_{\mathrm{cmp}}$ *with the constant* $w > 0$ *of the dictionary of Proposition . Put* $\ln(1+z) = \ln a(t_o) - \ln a(t)$ *for a fixed observer time* t_o*, and* $s_T = \mathrm{dln}\, T_w / \mathrm{dln}(1+z)$, $s_H = \mathrm{dln}\, H / \mathrm{dln}(1+z)$.

• *Without (C2),* $s_T = 3w$ *on every supplied scale factor, and* $s_H = -\dot{H}/H^2 = \frac{\mathrm{d}}{\mathrm{d}t}(1/H)$.

• *Under (C2),* $s_H = \frac{3}{2}(1+w)$ *and* $2s_H - s_T = 3$ *on J; equivalently* $H^2/(T_w(1+z)^3)$ *is constant on J, and* $\rho_{\mathrm{cmp}} \propto a^{-3(1+w)}$.

• *Conversely, with* $\Lambda = 0$ *declared,* $2s_H - s_T = 3$ *on J implies (C2) on J. Thus the two inferred parameters* $w_T = s_T/3$ *and* $w_H = \frac{2}{3}s_H - 1$ *agree exactly when (C2) holds.*

• *On the radiation prefix with* $\Lambda = 0$, $p_{\mathrm{cmp}} = \frac{1}{3}\rho_{\mathrm{cmp}}$ *identically; (C2) holds there iff* $w = \frac{1}{3}$*, and then* $s_H = 2$, $s_T = 1$*, in agreement with Theorem .*

Proof. (a) $\mathrm{dln}(1+z) = -H\,\mathrm{d}t$ along the observer's past cone, and Proposition (b) gives $\mathrm{dln}\, T_w = -3wH\,\mathrm{d}t$; also $\mathrm{dln}\, H = (\dot{H}/H)\,\mathrm{d}t$. (b) With $\Lambda = 0$, $\rho_{\mathrm{cmp}} = 3H^2/\chi > 0$ and $p_{\mathrm{cmp}} = -(2\dot{H} + 3H^2)/\chi$. Then $p_{\mathrm{cmp}} = w\rho_{\mathrm{cmp}}$ iff $-2\dot{H} = 3(1+w)H^2$, that is $s_H = \frac{3}{2}(1+w)$. Hence $2s_H - s_T = 3(1+w) - 3w = 3$, and integrating $\mathrm{dln}\, H = \frac{3}{2}(1+w)\,\mathrm{dln}(1+z)$ gives $H^2 \propto (1+z)^{3(1+w)} \propto T_w(1+z)^3$ and $\rho_{\mathrm{cmp}} \propto H^2 \propto a^{-3(1+w)}$. (c) By (a), $2s_H - s_T = 3$ gives $s_H = \frac{3}{2}(1+w)$, which is (C2) by the equivalence in (b). (d) On the prefix $\dot{H}_{\mathrm{r}} = -2H_{\mathrm{r}}^2$, so $p_{\mathrm{cmp}} = H_{\mathrm{r}}^2/\chi = \frac{1}{3}\rho_{\mathrm{cmp}}$ and $s_H = 2$.

par *Non-claim.* The closure (C2) is a supplied restriction on the prescribed geometry, written through the auxiliary coefficients of Proposition , which are definitions; no Einstein dynamics, fluid, species or vacuum value is derived. That a general-*w* label is a physical temperature is not established; it would need an equilibrium, kinetic or detector model. No observational claim is made. par

Theorem (Finite-time reception envelopes; inserted in version 6). *Supplied hypotheses. A future* $a \in \mathscr{F}^{\Gamma}(K)$, $0 < K < H_0$, $\Gamma \geq 2$*, with the conventions of Theorem :* $a(0) = 1$, $B = c/H_0$, $D = c/K$, $\rho = D/B = H_0/K$, $t_* = (1/K - 1/H_0)/\Gamma$*, and* $\overline{R}_{\Gamma}$ *of . For* $T > 0$ *put* $C(T) = c\int_0^T \mathrm{d}t/a(t)$,

$$C_-(T) = B(1 - e^{-H_0 T}), \qquad C_+(T) = \begin{cases} \frac{B}{\Gamma - 1}[(1 + \Gamma H_0 T)^{1 - 1/\Gamma} - 1], & T \leq t_*, \\ \overline{R}_{\Gamma} - D\rho^{-1/\Gamma} e^{-K(T - t_*)}, & T \geq t_*. \end{cases}$$

• *The two expressions for* C_+ *agree at* t_**, and* $C_+(T)$uparrow$\overline{R}_{\Gamma}$, $C_-(T)$uparrow$B$ *as* $T \to \infty$.

• *Every admissible future satisfies* $C_-(T) < C(T) \leq C_+(T)$*. The lower bound is an infimum that no admissible future attains. The upper bound is a supremum that is not attained when* $\Gamma > 2$ *or* $T \geq t_*$*, and is attained by smooth members of* $\mathscr{F}^{\Gamma}(K)$ *when* $\Gamma = 2$ *and* $0 < T < t_*$.

• *The set* $\{C(T) : a \in \mathscr{F}^{\Gamma}(K)\}$ *is the interval with endpoints* $C_-(T)$ *and* $C_+(T)$*, open at* $C_-(T)$*, and open or closed at* $C_+(T)$ *as in (b).*

• *An inward radial signal emitted at* $t = 0$ *from coordinate radius r reaches the observer by time T iff* $r \leq C(T)$*. Hence it does so for every admissible future if* $r \leq C_-(T)$*, and for none if* $r > C_+(T)$*, or* $r \geq C_+(T)$ *when* $C_+(T)$ *is not attained. For emission at a prefix time t, replace r by* $r - B(1 - a_{\mathrm{r}}(t))$.

Proof. Write $u = 1/H$, $u_0 = 1/H_0$, $u_1 = 1/K$, so $a(t) = \exp\int_0^t \mathrm{d}s/u$. Admissibility gives $u(0) = u_0$, $\dot{u}(0) = 2$, $0 \leq \dot{u} \leq \Gamma$ and $u = u_1$ eventually, so $u_0 \leq u \leq u_{\mathrm{fast}} = \min(u_0 + \Gamma t, u_1)$, using $\dot{u} \geq 0$ for $u \leq u_1$. Larger u gives smaller $\ln a$ and larger $C(T)$. (a) For $u = u_0$, $a = e^{H_0 t}$ and $C = B(1 - e^{-H_0 T})$. For $u = u_{\mathrm{fast}}$ and $t \leq t_*$, $\int_0^t \mathrm{d}s/(u_0 + \Gamma s) = \Gamma^{-1}\ln(1 + \Gamma H_0 t)$,

so $a = (1 + \Gamma H_0 t)^{1/\Gamma}$, and integrating a^{-1} gives the first expression. Since $1 + \Gamma H_0 t_* = \rho$, $a(t_*) = \rho^{1/\Gamma}$, and for $t \ge t_*$, $a = \rho^{1/\Gamma} e^{K(t - t_*)}$. Adding $D\rho^{-1/\Gamma}(1 - e^{-K(T - t_*)})$ to $C_+(t_*) = B(\rho^{1 - 1/\Gamma} - 1)/(\Gamma - 1)$ and using $D = B\rho$ gives $\overline{R}_\Gamma - D\rho^{-1/\Gamma} e^{-K(T - t_*)}$, as in the proof of Theorem . (b) Since $u'(0) = 2 > 0$, $u > u_0$ on some $(0, \varepsilon)$, so $C(T) > C_-(T)$. Lemma (b) gives profiles equal to $u_0 + o(1)$ uniformly on $[0, T]$, so $C_-(T)$ is the infimum. If $\Gamma > 2$, $u'(0) = 2 < \Gamma$ gives $u < u_0 + \Gamma t$ on some $(0, \varepsilon)$, so $C(T) < C_+(T)$. If $\Gamma = 2$ and $T \ge t_*$, equality would force $u = u_{\text{fast}}$ on $[0, T]$ by continuity; then $u' = 2$ on $(0, t_*)$, so $u'(t_*) = 2$ and $u > u_1$ just after t_*, which is impossible. If $\Gamma = 2$ and $T < t_*$, choose $T < T' < t_*$, put $u = u_0 + 2t$ on $[0, T']$, and continue with $u' = 2\psi((t - T')/L)$, where ψ is smooth, nonincreasing, equal to 1 near 0 and to 0 on $[1, \infty)$, with $\int \psi = c_\psi \in (0, 1)$, and $L = (u_1 - u_0 - 2T')/(2c_\psi)$. This u is smooth, satisfies $0 \le u' \le 2$, equals u_1 after $T' + L$, and agrees with $1/H_r$ near 0, so a extends the prefix smoothly; it lies in $\mathscr{F}^2(K)$ and gives $C(T) = C_+(T)$. In the remaining cases Lemma (a) and dominated convergence on $[0, T]$ show that $C_+(T)$ is the supremum. (c) The admissible profiles form a convex set (Lemma), and $C(T)$ is continuous along segments by dominated convergence on $[0, T]$, so the image is connected; combine with (b). (d) A future causal curve obeys $|\mathrm{d}\mathbf{x}/\mathrm{d}t| \le c/a$, so reaching the centre by T requires $r \le C(T)$; conversely the inward radial null ray arrives at the time $T' \le T$ with $C(T') = r$, and the signal then stays on the worldline. For emission at prefix time t, the prefix contributes $B(1 - a_r(t))$ by .

par *Non-claim.* The envelopes are calculations for the declared class; the rate bound is an input. No arrival is observed, no future is selected, and causal arrival does not by itself give successful communication. par

Theorem (Redshift-drift envelope from the geometric rate bound; inserted in version 6). *Supplied hypotheses. A positive C^2 scale factor with $H > 0$ on $[t_e, t_o]$, a comoving source emitting at t_e and received by the comoving observer at t_o, and the rate bound $0 \le \nu(t) = \frac{\mathrm{d}}{\mathrm{d}t}(1/H) \le \Gamma$ on $[t_e, t_o]$. Put $H_o = H(t_o)$ and $1 + z = a(t_o)/a(t_e)$.*

- $\mathrm{d}\ln H/\mathrm{d}\ln(1 + z) = \nu(t_e)$ *along the past cone, and* $1 \le H(t_e)/H_o \le (1 + z)^\Gamma$.
- $H_o[(1 + z) - (1 + z)^\Gamma] \le \mathrm{d}z/\mathrm{d}t_o \le H_o z$.
- *The lower ratio bound is an equality iff $\nu = 0$ on $[t_e, t_o]$, and the upper iff $\nu = \Gamma$ there.*
- *For $a \in \mathscr{F}^\Gamma(K)$, the rate bound holds on the whole interval (t_i, ∞), because $(1/H_r)' = 2 \in [0, \Gamma]$ on the prefix; the envelope therefore applies to every emission time $t_e \in (t_i, t_o]$, including emission before the junction. Emission before t_i is not covered.*

Proof. (a) Parametrize the past cone by $t = t_e$: $\mathrm{d}\ln(1 + z) = -H\,\mathrm{d}t$ and $\mathrm{d}\ln H = (\dot{H}/H)\,\mathrm{d}t$, so the ratio is $-\dot{H}/H^2 = \nu$. Integrating $0 \le \nu \le \Gamma$ from $z = 0$ gives $0 \le \ln(H/H_o) \le \Gamma\ln(1 + z)$. (b) Lemma : $\mathrm{d}z/\mathrm{d}t_o = (1 + z)H_o - H(t_e)$; substitute (a). (c) Equality in an integrated inequality of continuous integrands holds iff the integrands agree throughout. (d) $1/H_r = 1/H_0 + 2t$, and the class bound holds for $t > 0$; smooth matching gives $\nu(0) = 2$.

par *Non-claim.* The band is a calculation for the declared rate class. No drift reconstruction or measurement is reported, and the rate bound remains an input; no temperature dictionary is used. par

Theorem (Eternal acceleration does not decide horizon existence; inserted in version 6). *For $A, t_s > 0$ and $p > 0$ let $a_p(s) = A\frac{s}{t_s}[\ln(s/t_s)]^p$, $s > t_s$, and $x = \ln(s/t_s)$.*

- $\dot{a}_p = \frac{A}{t_s}x^{p-1}(x + p)$, $\ddot{a}_p = \frac{Ap}{t_s s}x^{p-2}(x + p - 1)$, $H = \frac{x + p}{sx} \to 0$, *and* $q = -\ddot{a}_p a_p/\dot{a}_p^2 = -p(x + p - 1)/(x + p)^2 \to 0^-$. *The tail accelerates for $x > \max(0, 1 - p)$, and $\int^\infty \mathrm{d}s/a_p < \infty$ iff $p > 1$.*
- *For every $p > 0$ there is a comparison future a in the sense of Definition , with the prescribed radiation prefix and $H > 0$, and constants $A' > 0$, $s_0 > t_s$, $t_1 > 0$, such that $a(t) = A'\frac{t + s_0}{t_s}[\ln((t + s_0)/t_s)]^p$ for $t \ge t_1$.*

• *Such a future accelerates for all sufficiently late times, has* $q \to 0^-$*, and has finite* $R_0[a]$*, hence an event horizon for the comoving observer (Lemma), iff* $p > 1$*. It is in no class* $\mathscr{F}(K)$*, since* $H \to 0$*.*

Proof. (a) $\mathrm{d}x/\mathrm{d}s = 1/s$. Differentiating $\dot{a}_p = \frac{A}{t_s}(x^p + px^{p-1})$ gives $\frac{A}{t_s s}[(p-1)x^{p-2}(x+p) + x^{p-1}] = \frac{Ap}{t_s s}x^{p-2}(x+p-1)$. The forms of H and q follow by division. With $\mathrm{d}s/s = \mathrm{d}x$, $\int \mathrm{d}s/a_p = \frac{t_s}{A}\int x^{-p}\,\mathrm{d}x$, finite at infinity iff $p > 1$. (b) Put $s_0 = et_s$, $H_L(t) = \frac{x+p}{(t+s_0)x}$ with $x = \ln((t+s_0)/t_s) \geq 1$ for $t \geq 0$, and let β be a smooth nondecreasing function with $\beta = 0$ on $[0, 1/H_0]$ and $\beta = 1$ on $[2/H_0, \infty)$. For $t \geq 0$ set $H = (1-\beta)H_{\mathrm{r}} + \beta H_L > 0$ and $a = \exp\int_0^t H$; for $t \leq 0$ set $a = a_{\mathrm{r}}$. On $[0, 1/H_0]$, $a = a_{\mathrm{r}}$, so a is smooth and agrees with the prefix. For $t \geq t_1 = 2/H_0$, $a(t) = a(t_1)\,a_p(t+s_0)/a_p(t_1+s_0)$, which is the stated form with $A' = a(t_1)A/a_p(t_1+s_0)$. (c) Derivatives in t and in $s = t + s_0$ coincide, so (a) applies; the optical tail $\int_0^\infty \mathrm{d}t/a$ is finite iff $\int^\infty \mathrm{d}s/a_p$ is.

Status: proved here. Proposition already shows that the prefix does not decide the optical tail; the theorem adds that late-time acceleration with $q \to 0^-$ does not decide it either. In a different setting, an open universe with a scalar potential, Andriot, Tsimpis and Wrase report asymptotically accelerating solutions without a cosmological event horizon [ATW2023]; Source record; not a premise.

par *Non-claim.* No energy condition, field equation or matter law is claimed for these tails, and none is an observational model. A horizon prediction needs quantitative asymptotic control of the optical tail, which is an input here. par

Theorem (Conditional information isolation; inserted in version 6). *Supplied hypotheses. A future in* $\mathscr{F}^{\Gamma}(K)$*; the located-circuit contract of Definition with receivers on γ and no message-correlated bypass; a payload of declared dimension* $d \geq 2$ *inserted at prefix time t and coordinate radius r;* $z = a_{\mathrm{r}}(t)$*.*

• *If* $r \geq B(1-z) + \overline{R}_\Gamma$*, then for every admissible future the joint accessible channel to the complete receiver interface is a replacement channel and* $\varepsilon^* = a_d = 1 - d^{-2}$*. At the junction (*$z = 1$*) the condition is* $r \geq \overline{R}_\Gamma$*.*

• *For* $T > 0$*, the receiver interface retained by T is the set of declared retained outputs of receiver operations located at* $\gamma(s)$*,* $0 \leq s \leq T$*. If* $r - B(1-z) > C_+(T)$*, or* $r - B(1-z) \geq C_+(T)$ *when* $C_+(T)$ *is not attained (Theorem (b)), then for every admissible future the joint channel to that interface is a replacement channel with optimized error* a_d*.*

Proof. (a) By Theorem (a), $R_0 < \overline{R}_\Gamma$ for every admissible future, the supremum being unattained. Hence $r \geq B(1-z) + \overline{R}_\Gamma > B(1-z) + R_0$, and Theorem applies. (b) By Theorem (d), $\gamma(s)$ is causally reachable from the source iff $r - B(1-z) \leq C(s)$, and C is increasing; the hypothesis and Theorem (b) give $r - B(1-z) > C(T) \geq C(s)$ for all $s \leq T$. No retained receiver position is reachable, so Lemma gives a joint replacement channel and Lemma , as in Theorem , gives a_d.

par *Non-claim.* Causal accessibility does not guarantee successful communication, and non-reception by T does not place a source outside an event horizon. The dimension d is declared, not a derived dimension of cosmological space. The statement is conditional on the declared future class and access contract. par

Remark (Priority note and programme of the prediction layer). The prediction layer recommends developing Theorems , and together, and a programme: specify the epoch and model class; calibrate on a subset of data; predict a withheld temperature, expansion, drift or reception quantity; propagate uncertainty through the theorem; reject the model if the predicted relation fails. This is recorded as the user's proposal, not as a theorem or a result of this paper. The temperature–redshift law is an established test in cosmology [AvgoustidisJCAP2012], and redshift drift is studied as a future probe [AlvesMNRAS2019]; both records are source record; not a premise, and no number from them is used. Citations identify source records only; no proof transfer.

par *Non-claim.* No observational claim, measured number, primordial spectrum, radiation species or unique cosmic future is asserted. par

Conclusion

The paper separates three kinds of input that should not be identified merely because their formulas share the words ``radiation", ``temperature", or ``horizon". The label dictionary provides calibration maps and exact risks. The causal contract provides a complete-access replacement statement. The finite comparison system provides installed and optimized channel errors.

The version-4 refinements make these separations quantitative: a corrected radiation relative-risk law and prior; an exact rate-bounded early-horizon interval; a joint correlated-output certificate; and explicit qualifications on decoder classes and coefficient-carrying metadata. They sharpen negative identification statements without converting them into a physical derivation.

Version 5 adds, by insertion only, the two-label reduction of Proposition and the late dimension-label risk of Proposition , whose constant is $C_K = \pi c^5/(GK^2)$ at a supplied terminal rate. Its other additions are documentary separations. None converts a negative certificate into a positive derivation, and none alters the causal or channel theorems above.

Version 6 adds, by insertion only, orbit–split independence of the labels, an adaptive non-calibration theorem, the fibre principle that unifies the absolute and relative risks, general-w logarithmic certificates with the two-epoch comparison as their equal-exponent case, the event/apparent radius separation with a sharp modulus, the vacuum-shift/orbit independence, and the comparison of the floors $1 - d^{-2}$ and $1 - 1/d$. Its prediction layer states six conditional theorems under supplied closures. The re-proved draft results add the quotient and epoch ratios, approximate-blindness bounds, the settling functional, the unrestricted radius range at a fixed terminal rate and a clock-feedback test. None of these selects a split, a future or a vacuum term; κ is calibrated only by added supply, and none alters a version-5 theorem.

appendix

Corrections and editorial disposition

The supplied version 3 remains an unchanged source record. The following corrections are integrated here rather than reproducing false statements in the submission text.

1.12 toprule Issue Correction and location

midrule endfirsthead toprule Issue Correction and location

midrule endhead bottomrule endfoot

Relative coefficient window For a radiation power 3/4, the risk is $\tanh(\frac{3}{4}\operatorname{artanh}\delta)$, not δ. The latter formula belongs to a linear or inverse-linear coefficient dependence, or to a window specified directly by label contrast. Theorem .

addlinespace

Relative-loss prior Equal endpoint probabilities certify expected absolute loss. Relative loss instead uses $\ell/(\ell + u)$ and $u/(\ell + u)$. Theorem .

addlinespace

Exact label metadata An exact absolute label can encode κ. Appending its numerical value does not preserve the coefficient-blind data fibre. Symbolic formulas and coefficient-blind metadata are distinguished in Proposition .

addlinespace

Unbounded energy/coefficient boxes Not every unbounded parameter interval makes both radiation labels unbounded. The correct targetwise endpoint criteria follow Theorem .

addlinespace

Scalar Hamiltonian in unbounded-window O6 The union-class risk is $a_d/2$, not 1/2, when the Hamiltonian is scalar. The Gibbs-only risk is then zero. Theorem .

addlinespace

Unrestricted versus thermal decoding The replacement optimum a_{d} permits arbitrary decoders. A decoder constrained to preserve the same Gibbs state cannot replace it by I/d in general. Theorem (d).

addlinespace

Correlated joint state The installed error 24/25 is not the optimized four-dimensional value 15/16. Marginal-product certificates describe a different joint state. Proposition .

addlinespace

Mixture weight and darkness A positive nominal visible weight does not imply nonconstant influence. A replacement visible branch is a counterexample. Theorem and its following remark.

addlinespace

Conformal versus proper radius The factor $z = a_{\mathrm{r}}(t)$ occurs in the proper-radius risk, not in the comoving conformal-distance risk. Equation .

addlinespace

Rate-bound limit The limit $\Gamma \to \infty$ recovers the monotone fixed-terminal interval (B, D), not the unrestricted smooth-future interval. Theorem .

addlinespace

Deadline endpoint Smooth settling by $\tau = t_{*}$ is impossible. The deadline class is empty for $\tau \le t_{*}$. Theorem (b).

addlinespace

Every-event status claim Events reaching the observer by the junction stay visible for every continuation. Only a specified radial shell has future-dependent status. Proposition and Theorem (d).

addlinespace

Attainment terminology An optimal decision can exist without a maximizing parameter. Randomization can change the realization of the worst-case supremum even when the value is unchanged. Lemma and its remark.

addlinespace

Unverified computational claims The historical claims of executed scripts, source-byte identity, manifests, and clean pdfLaTeX builds are not asserted as results of this revision. Appendix .

Plan disposition

toprule Proposed item Disposition in version 4

midrule General-w labels Locally proved in Proposition .

Rate-bounded futures Locally proved with matching-germ and deadline qualifications in Theorem .

Correlated replacement Locally proved exact rational example, Proposition .

Relative and randomized risk Locally proved with two mathematical corrections, Theorem .

Finer cut ladder Local flagged-width interpolation plus a version-qualified hybrid-cut documentary record.

Approximate Singleton Locally derived using explicitly cited external entropy continuity and data processing.

Casimir, I-C, catalogue edges Documentary only, with separate comparison contracts.

Insertion-only sequence Version 3 is not edited. The present document is a corrected submission redaction, not a bytewise insertion-only assembly.

Re-run Big04-01–Big04-39 Not asserted: the referenced programs and build environment were not part of the available executable materials.

bottomrule

Audit row. Dark-radiation reading, version 4—version 3 remains unedited; general-w and rate bounds are declared; correlated joint and relative risk are proved locally; general hybrid formulas are not completed here; O.1 and O6 remain open; no Host C.

Version 5: plan corrections, errata and audit row

Version 5 follows the insertion sequence of its plan (PLAN_t305u, section 9). Each cited statement was compared with the source records read for this version (Appendix), and each version-4 label cited by the plan was compared with version 4. The plan corrections below record the mismatches; the insertions state the corrected forms.

toprule Item Plan correction

midrule endfirsthead toprule Item Plan correction

midrule endhead bottomrule endfoot PC5-1 Item 2 cites Big02-A thm:t5-energy for the floors $(v + \sqrt{4v - 3v^2})/2$ and $3v(1 - v)$. In Big02-A version 6 these floors, and the optimality of the identity decoder for cutoff-and-reset at $q \le \frac{1}{3}$, are thm:v4-rep; thm:t5-energy records cutoffs, admissible-test bounds, and (b), that a fixed number of modes without a cutoff is not finite dimensional. Documentary record cites both.

addlinespace PC5-2 Item 2 says the exact value beyond the stated ranges stays partial. Big02-A version 6 retains thm:v5-allenergy, an exact variational formula for the replacement value at every energy, and lists the replacement part of B2-O10 as closed. What is not available is an elementary closed form beyond the ranges. On this sheet no such value is evaluated, and the plan-audit row's phrase ``stay open'' is read in that sense.

addlinespace PC5-3 Item 2 refers to ``the energy-constrained section of version 4''. Version 4 has no energy-constrained section. The edge is new and is placed after the cut ladder in Section .

addlinespace PC5-4 Item 1 says that the middle range for $L \ge 4$ stays open. Big02-A version 6 records B2-O27 as open with the exceptions $(7, 4, 5)$ and the weighted projector sums. Res06-A version 4 closes, under its cited identification, three $L = 4$ sectors inside that range and keeps B2-O27 open on $\frac{8m}{5} < d < \frac{8m}{3}$. The non-claims state openness in the sense of those records.

addlinespace PC5-5 Item 1 states the co-band as $(L - 1)d \le Lm$. The source theorem also requires $m \le d$ and $L \ge 2$; the band requires $L \ge 2$. At $L = 2$ both domains are $m \le d \le 2m$ (Proposition).

addlinespace PC5-6 Item 3 states the reduction to a constant multiple of g on Einstein regions. The radiation prefix is not one ($R = 0$, $R_{tt} = 3H_r^2$), so the reduction holds on terminal de Sitter tails, not on the prefix. The statement that κ does not occur is Big03-A prop:R-vac-sep(d); DS-OB-1-CERT thm:v5-vac does not mention κ.

addlinespace PC5-7 Item 6 gives $r_N = \tanh\left(C\delta/(\kappa_0(1 - \delta^2))\right)$ without identifying C. Re-derived: the form holds exactly at a supplied terminal rate K, with $C = C_K = \pi c^5/(GK^2)$, equivalently $C_K/\kappa_0 = S_{\text{late}}^{(\kappa_0)}/k_B$. On a (κ, K) box the argument changes, and with K free the relative risk is 1 (Proposition). The plan's ``not δ'' holds as a difference of forms, not as an inequality at every parameter: $r_N > \delta$ when $S_{\text{late}}^{(\kappa_0)} \ge k_B$, while $r_N \to 0$ as $C_K/\kappa_0 \to 0$.

addlinespace PC5-8 Item 7 says that late-slice expansion separates every pair of distinct rates. Hawk07-C-CERT prop:v5-expansion separates distinct deformation parameters w of its family g_w; the rate formulation is Red06-G-DSS-CERT prop:RC-rate9. Hawk07-C-Dyn-CERT thm:v5-frw gives null invariance for an FRW metric exactly when $Q = 0$, not for every FRW metric; the radiation prefix satisfies $Q = 0$.

addlinespace PC5-9 Item 5: prop:RC-rate9 and prop:RC-tails9 use a de Sitter prefix, and the former uses a declared rate window at one later time, not a bound on $\frac{\mathrm{d}}{\mathrm{d}t}(1/H)$. The edge records the statements as parallel, on different prefixes and targets.

addlinespace PC5-10 Item 4: the values $Q_{11,4,4}$ and $Q_{11,4,7}$ are proved locally in Res06-A; their reading as $d^2F_{\max}$ is conditional on the Big02-A identification cited there.

addlinespace PC5-11 Section 9, step 7: version 4 already uses Big04-40–Big04-47 for proposed, unexecuted checks. The version-5 checks are therefore numbered Big04-48–Big04-60.

Errata to version 4. The version-4 sentences below are superseded in version 5. They are retained unedited.

toprule Erratum Superseded sentence and its version-5 reading

midrule endfirsthead toprule Erratum Superseded sentence and its version-5 reading

midrule endhead bottomrule endfoot ER5-1 Title page: ``Big04-A, Version 4" and the author line ``Ngo Cheung". For version 5 the title reads ``Big04-A, Version 5", and the author line is ``Ngo Cheung (authorship requires confirmation)", by inserted redefinitions after the retained version-4 lines.

addlinespace ER5-2 Manuscript status: the present text ``integrates the additions and corrections rather than claiming bytewise insertion-only preservation". This describes version 4 relative to version 3. Version 5 is insertion-only over version 4.

addlinespace ER5-3 Appendix : ``Attempts to retrieve the Big-series manuscript endpoints did not yield accessible full texts". For version 5, read-only requests returned full texts (Appendix).

addlinespace ER5-4 Appendix : the bibliography ``omits inherited claims about … retrieved-file hashes". The version-4 bibliography entries are unchanged; Appendix records the sha256 digests of the texts read for version 5.

addlinespace ER5-5 Scope after Proposition : Big03-A is described as giving the ``linear/inverse-label relative-risk discussion". Big03-A prop:R-relative also states the exponential-label risk r_{N}; Proposition re-derives it locally for N_{late}.

addlinespace ER5-6 Appendix : the build commands name Big04-A-v4.tex. The version-5 source is Big04-A-v5.tex.

Disposition of the version-5 plan. Item 1: Documentary record and Proposition . Item 2: Documentary record . Item 3: Documentary record . Item 4: Documentary record . Item 5: Documentary record . Item 6: Proposition . Item 7: Documentary records and ; Oai280-B and Oai282-B were not cited directly in version 4 and are now cited. Item 8 is observed: no numerical coefficient is admitted outside Appendix , and no parent is revised.

Audit row, version 5. Dark-radiation reading, version 5 — insertion-only over version 4; hybrid-cut middle range and energy-constrained value outside the stated ranges stay open; vacuum term not fixed; O1 and O6 remain open; no Host C.

Version 6: verdicts, plan corrections, crosswalk and site-version findings

Version 6 follows PLAN_t344u (seven items, insertion points, a what-not-to-import list and an appendix note) and the prediction layer PRED_t345u (six candidate theorems). Plan corrections are numbered PC3-*n*, as requested for this revision; the draft's own corrections PC6-1–PC6-8 are referred to as draft PC6-*n* and are folded into PC3-*n* below. Every label cited by the plan was compared with the record served by a read-only GET request to https://enter.grok.me/api/manuscript/<ID> on 25 October 2026 (Appendix); every cited label exists there. Mismatches of statement or scope are recorded as plan corrections PC3-*n*; the insertions state the proved forms.

Verdicts on the plan items. toprule Plan item Verdict Location in version 6 and reason

midrule endfirsthead toprule Plan item Verdict Location in version 6 and reason

midrule endhead bottomrule endfoot 1. Orbit–split independence corrected Definition , Proposition . The labels are constant along orbits, not varying along them (PC3-1); the split is declared locally (PC3-2); with α supplied the orbit is not confounding (PC3-3).

2. Adaptive non-calibration held Theorem , Remark . All three cited labels state the described results; the hypothesis is the complete process with cross-round memory (PC3-16).

3. General-w certificates corrected Proposition , Corollary . The derivative is taken in $\ln\kappa$ (PC3-4); the version-5 two-epoch theorem has no change of w (PC3-5).

4. Horizon interval corrected Theorem , Documentary record . Big05-A concerns a de Sitter past (PC3-6); the modulus is not sharp in $\mathscr{F}^{\Gamma}(K)$ (PC3-7); without a rate bound R_{0} fills $(0,\infty)$ (PC3-17).

5. Fibre principle corrected Proposition . Geometric observables are thm:Big05-flagship (PC3-8); labels are recoverable on orbits (PC3-9).

6. Access separations corrected Lemma , Documentary record (PC3-10, PC3-11).

7. Vacuum non-selection corrected Proposition , Remark (PC3-12, PC3-13).

Insertion points held After Section 's results; after the late-label material; after the rate-bounded section; a new separation subsection in Section . The general-w re-proof is placed after the late-label material, as the plan's

insertion list says.

Appendix note corrected Big03-A version 3 is served; Big06-A is served as version 2, not as version 1 (PC3-14).

What not to import held Observed; checked by Big04-68 and Big04-72.

Plan corrections. toprule Item Plan correction

midrule endfirsthead toprule Item Plan correction

midrule endhead bottomrule endfoot PC3-1 Item 1 says each label ``varies only along the orbit''. The labels are constant on every coefficient orbit $(U, q\kappa, q^3\alpha)$; at fixed U they vary only across orbits, through $z = \kappa^3/\alpha$ (Proposition (a),(e)).

addlinespace PC3-2 Item 1 refers to ``the split fibre of a fixed host''. Big04-A version 5 has no split coordinate. Version 6 declares splits on the prefix region (Definition) following def:Big05-split of Big05-A; split independence of the labels is then by construction, and the content is the product fibre.

addlinespace PC3-3 Items 1 and 7: the admitted datum $\mathfrak{E}$ supplies α, so on Big04-A's own data the orbit is not a confounding orbit and the labels are injective in κ; this is Big04-A-coeff prop:C-orbit(d) and Big03-A prop:R-v3-orbit.

addlinespace PC3-4 Item 3 writes $\partial_\kappa \ln T_w = 3w/(1+w)$. The correct forms are $\partial \ln T_w / \partial \ln \kappa = 3w/(1+w)$ and $\partial_\kappa \ln T_w = 3w/((1+w)\kappa)$, as in Big02-A thm:v4-wlabels(a).

addlinespace PC3-5 Item 3 calls the version-5 two-epoch comparison a special case ``when the equation-of-state parameter changes''. Theorem compares two $w = \frac{1}{3}$ families; it is the equal-exponent case (Corollary (b)). A declared change of w breaks the common-rescaling invariance and, with equal slots and supplied constants, calibrates κ by added supply (part (c)).

addlinespace PC3-6 Item 4: Big05-A thm:Big05-futures concerns a de Sitter past and futures with $H \le 0$, not the radiation prefix and not the rate bound ; Big02-A thm:v3-horizon gives the radius formulas but not the non-determination statement. The non-determination on this prefix is local (Theorems , , (b)).

addlinespace PC3-7 Item 4: the modulus of Big02-A is attained by constant profiles, which are not prefix-matched; on $\mathscr{F}^\Gamma(K)$ it gives the bound $D - B$, weaker than the exact width $\overline{R}_\Gamma - B$ (Theorem (d)).

addlinespace PC3-8 Item 5: Big05-A thm:Big05-fibre applies the fibre principle to dominance targets; the application to geometric observables is thm:Big05-flagship.

addlinespace PC3-9 Item 5: on a coefficient orbit the labels are constant, so the fibre principle does not forbid their recovery from orbit-constant data; it forbids recovery from data constant on a coefficient fibre at fixed (U, α, t) (Proposition (c)).

addlinespace PC3-10 Item 6: En04-A contains no ``host update'' wording; thm:magic-fibre is a magic-label fibre statement for complete residual leakage, and the record states that residual response is not yet geometric response.

addlinespace PC3-11 Item 6: I-C thm:A is stated for classical-output channels and deterministic decoders (reference-preserving half-diamond error); En04-B thm:fibre is stated on the account class permitting arbitrary declared finite carriers.

addlinespace PC3-12 Item 7: DS-OB-1-fibre thm:F-rank and Big03-A prop:R-v3-vacuum-box concern de Sitter regions. On the radiation prefix the effective rank is two, thm:F-flrw(c) of DS-OB-1-fibre, which is cited in addition.

addlinespace PC3-13 Item 7: ``the κ-slot is not selected by the radiation labels'' holds in the sense of Remark : an exact label value is itself a supply that calibrates κ (Theorem).

addlinespace PC3-14 Appendix note: on 25 October 2026 the id Big06-A first returned HTTP 404 and later returned Big06-A version 2, which contains version 1 verbatim; version 1 itself is a local version (Appendix). Big03-A version 3 is served.

addlinespace PC3-15 Version 5 uses Big04-48–Big04-60; the version-6 checks are numbered Big04-61–Big04-78.

addlinespace PC3-16 Item 2 (from draft PC6-5): non-calibration requires one complete accessible process for all

parameter values, including every register kept across rounds; marginal blindness, placement blindness or a constant recovery optimum is weaker. Hypothesis (iii) of Theorem states this.

addlinespace PC3-17 Item 4 (from draft PC6-6): a terminal rate fixes the settled proper radius c/K, not R_0; without a monotonicity or rate bound the image of R_0 on $\mathscr{F}(K)$ is $(0,\infty)$ (Theorem).

Verdicts on the prediction layer (X6). toprule Item Proposal Verdict Disposition

midrule endfirsthead toprule Item Proposal Verdict Disposition

midrule endhead bottomrule endfoot X6-1 Radiation consistency held Theorem . Algebra and drift $-H_0 z(1+z)$ confirmed for the junction observer, on $z \in [0, z_i]$ only; the temperature bridge is a supplied closure; ``observable histories'' is restated as relations among declared functions.

X6-2 General-w consistency held (conditional) Theorem . $2s_H - s_T = 3$ confirmed under the supplied closure, restated through the auxiliary coefficients of Proposition ; the relation is equivalent to the closure, and on the radiation prefix it forces $w = \frac{1}{3}$. No Einstein dynamics is derived.

X6-3 Finite-time reception corrected Theorem . Formulas confirmed and conventions match Theorem ; the lower envelope is an unattained infimum, the upper envelope is attained only for $\Gamma = 2$, $T < t_*$; reception by T is $r \le C(T)$.

X6-4 Drift envelope held Theorem . For $\mathscr{F}^{\Gamma}(K)$ the rate bound also holds on the prefix ($v = 2$), so emission before the junction is covered down to t_i; earlier emission is not.

X6-5 Logarithmic tails corrected Theorem . Algebra confirmed; the tails are not in any $\mathscr{F}(K)$; an explicit gluing construction (with a time shift) replaces ``can be attached smoothly''; the cited PRD record concerns an open universe and is documentary.

X6-6 Information isolation held Theorem . $r \ge \overline{R}_{\Gamma}$ suffices because $\overline{R}_{\Gamma}$ is an unattained supremum; the retained interface is defined; for finite T the non-strict form needs $C_+(T)$ unattained.

Priority note programme not a theorem Remark .

Errata to version 5. The version-5 sentences below are superseded in version 6. They are retained unedited. toprule Erratum Superseded sentence and its version-6 reading

midrule endfirsthead toprule Erratum Superseded sentence and its version-6 reading

midrule endhead bottomrule endfoot ER6-1 Title page ``Big04-A, Version 5''. For version 6 the title reads ``Big04-A, Version 6'' by inserted redefinitions after the retained lines; the author line is unchanged.

addlinespace ER6-2 Status paragraph of version 5: ``This version is not registered''. On 25 October 2026 the site serves, under the id Big04-A, the version-5 text with 47 added lines (22 browse-URL lines, five bibliography entries and blank lines) and no removed line. Version 6 is not registered.

Crosswalk of cited labels. toprule Record Labels (all present in the served text) Used at

midrule endfirsthead toprule Record Labels (all present in the served text) Used at

midrule endhead bottomrule endfoot Big05-A prop:Big05-orbit-split, eq:radiationorbit, thm:Big05-futures, thm:Big05-fibre, thm:Big05-flagship, def:Big05-split Propositions , ; Documentary record

Big04-A-coeff prop:C-orbit, thm:C-adaptive, lem:C-fibre Proposition , Theorem , Proposition

Big03-A thm:R-v3-adaptive, prop:R-v3-blind-boundary, prop:R-v3-vacuum-box, prop:R-v3-orbit Theorem , Propositions and

Big02-A thm:v4-wlabels, thm:v3-horizon Proposition , Documentary record

En04-B prop:coefficient, thm:fibre, prop:clock-confound Theorem , Documentary record

En04-A thm:magic-fibre Documentary record

I-C thm:A Lemma , Documentary record

Hawk05-B thm:page Documentary record

DS-OB-1-fibre lem:F-fibre, thm:F-rank, thm:F-flrw, cor:F-ranknotsel Propositions and

OpenAI catalogue none (string search) Documentary record

What is not imported. No Einstein dynamics, metric or particle species is derived; the labels are not identified with a quantum coefficient or with Big02-A dark capacity; printed radii and risks are calculations, not measurements; no OpenAI catalogue item is a premise; XV-B is not opened, XVI-N is not discharged and no quantum coefficient is admitted; no protocol, geometric observable or recovery error selects κ or a split.

Audit row, version 6. Dark-radiation reading, version 6 — insertion-only over version 5; orbit and split independent; adaptive blind protocols do not calibrate; prediction layer conditional on supplied closures outside the ledger; absolute vacuum energy open; O1 and O6 remain open; no Host C.

Version 6: the author's draft of additions (t347u)

The author's draft ``Big04-A: Version-6 Additions Only'' (findings_t347u.tex, sha256 eb7a8ac12292629f9f61d7938659cff9908593df47c2fc7e85a80fcb3180a6a7) supplies the backbone of the plan sections. Each of its results is re-proved in this paper; overlapping results are merged and the stronger form is kept. Its verdicts are F11-*n* and its corrections DC3-*n*. The prediction layer keeps its own Section .

Backbone map. toprule Draft section Version-6 location

midrule endfirsthead toprule Draft section Version-6 location

midrule endhead bottomrule endfoot S 1 Two orbits and a split coordinate Section (Definition , Proposition); Section (product class, Corollaries and)

S 2 Adaptive blindness and approximate blindness Theorem ; Proposition , Theorem , Remark

S 3 General-w quotients and epoch changes Proposition , Corollary ; Propositions and , Corollary

S 4 Future functionals and event–apparent separation Theorem ; Proposition , Theorem , Remark ; overlaps with Theorems , and

S 5 Clock-feedback factorization Section

S 6 Vacuum shifts Proposition , Corollary , Remark

S 7 Documentary separations Documentary records and

toprule Item Draft result Verdict Disposition midrule endfirsthead toprule Item Draft result Verdict Disposition midrule endhead bottomrule endfoot F11-1 def:v6-product held Product class stated in Section; it is the account set of Definition. F11-2 thm:v6-orbit held Merged with Proposition, which also gives the split fibre of the split-blind datum; the inverse formulas are in Section. F11-3 remark on ``varies only along the orbit'' held PC3-1. F11-4 cor:v6-alpha-calibration held Corollary. F11-5 cor:v6-late-break held Corollary; the late formulas agree with. F11-6 def:v6-adaptive held Hypothesis (iii) of Theorem, now naming cross-round memory. F11-7 thm:v6-adaptive held Merged with Theorem; postselection in Remark. F11-8 cor:v6-confidence corrected Proposition; the upper end is written I^+, not U_I (DC3-8). F11-9 remark on which orbit has a common law held Remark. F11-10 thm:v6-approxblind held Theorem; the round maps include the control kernels. F11-11 remark on postselection held Remark. F11-12 prop:v6-w held Propositions and. F11-13 prop:v6-epochs held Proposition; stronger than Corollary (c), which is kept. F11-14 remark on the two-prefix comparison held Remark after Corollary; PC3-5. F11-15 cor:v6-variablew held Corollary. F11-16 prop:v6-horizonformula held Proposition and Theorem (a). F11-17 cor:v6-plateau held Last statement of Proposition. F11-18 thm:v6-unrestricted held Theorem; PC3-17. F11-19 prop:v6-rategap held Merged into Theorem (b), which also gives the interval of R_e; $R_e > 0$ added there. F11-20 remark on non-determination held Its first statement is Proposition of version 5; Remark. F11-21 prop:v6-clockfeedback held Proposition. F11-22 remark on clock records corrected The served En04-B prop:clock-confound is read and related in Remark (DC3-9). F11-23 prop:v6-vacuum held Proposition and Corollary. F11-24 remark on fixed U held Proof of Corollary. F11-25 doc:v6-access corrected Merged into Documentary record; the I-C-CERT statement is the served $1 - 1/S$ formula (DC3-4). F11-26 doc:v6-geometric corrected The plan-reported locators are verified in the served texts (DC3-3), and the OpenAI catalogue is searched in full (DC3-10). F11-27 unchanged-ledger paragraph corrected The

verbatim ledger is used (DC3-6). F11-28 source-handling and registration paragraphs rejected Superseded by the read-only retrievals (DC3-1, DC3-2). F11-29 assembly-and-verification paragraph rejected Version 6 is an assembled insertion-only revision with a checking program (DC3-7). F11-30 audit row held Audit row, version 6.

The draft's own plan corrections. toprule Draft item Verdict Folded into

midrule endfirsthead toprule Draft item Verdict Folded into

midrule endhead bottomrule endfoot draft PC6-1 held PC3-1 and PC3-3; checked against Big05-A eq:radiationorbit and Big04-A-coeff prop:C-orbit(a),(b), whose pair is preserved iff $s = 1$, $q = r^{3}$.

draft PC6-2 held PC3-2; the product class is stated in Section .

draft PC6-3 held PC3-4.

draft PC6-4 held PC3-5.

draft PC6-5 held PC3-16.

draft PC6-6 held PC3-17.

draft PC6-7 held PC3-13; the version-6 wording is audited so that no inserted sentence denies calibration by an exact label at fixed supplied normalization.

draft PC6-8 overturned PC3-14; the site status was read (DC3-2).

Draft corrections (DC3). toprule Item Correction to the draft

midrule endfirsthead toprule Item Correction to the draft

midrule endhead bottomrule endfoot DC3-1 The draft says retrieval failed. Read-only GET requests on 25 October 2026 returned every cited record; versions and digests are in Appendix .

addlinespace DC3-2 Registration: the site serves Big03-A version 3 (its header states insertion-only assembly over version 2), Big01-A version 7, and, after an earlier HTTP 404 the same day, Big06-A version 2, byte-identical with the local file /workspace/NewSheets341/Big06-A/deliver/Big06-A-v2.tex (sha256 fb7927fcc52c470d7b1e17d78b3c2d7a4c7d3bdffe9356a135bb9dee293753c4). No Big06-A result is used.

addlinespace DC3-3 Locators the draft calls plan-reported are present in the served texts: thm:Big05-fibre, thm:Big05-futures, lem:C-fibre, thm:C-adaptive, prop:coefficient, prop:clock-confound, prop:R-v3-vacuum-box, prop:R-v3-orbit, lem:F-fibre, cor:F-ranknotsel, thm:v3-horizon. So are the draft's thm:C-equiv, cor:R-v3-supply, thm:v4-rate, thm:F-einstein and I-C-CERT thm:replacement.

addlinespace DC3-4 Bibliography versions: I-C-CERT is served as version 6, Big02-A as version 6, DS-OB-1-fibre as version 5, Big05-A as version 3, En04-B as version 4, I-C as version 2 and Big04-A-coeff as version 5, as the draft says; the served Hawk05-B carries no version marker (the draft says version 1), and the served En04-A has header version 2 and title version 3 (the draft says version 3).

addlinespace DC3-5 The draft has an empty author line; the required line is ``Ngo Cheung (authorship requires confirmation)".

addlinespace DC3-6 The draft paraphrases the ledger; version 6 repeats the version-5 ledger verbatim in the abstract and at the end.

addlinespace DC3-7 The draft is an additions sheet without assembly or checks. Version 6 is assembled by insertion only over version 5, and a checking program accompanies it; this text does not report the program's outcome.

addlinespace DC3-8 In the draft's confidence corollary the upper end U_{I} clashes with the energy constant U; it is written I^{+}.

addlinespace DC3-9 The draft says the hypotheses of En04-B prop:clock-confound were not available; the served statement (general POVM, fixed probe state, scalar $\sum_{k} m_{k} F_{k}$) is read and its relation to Proposition is stated.

addlinespace DC3-10 The draft restricts its OpenAI statement to excerpts; the full served catalogue contains neither ``radiation" nor ``dark".

Evaluation-only declaration

Definition (Evaluation-only supply). Only in the table below, use the dimensionless sample declaration

$$c = k_B = G = \alpha = V_c = H_0 = U = 1, \qquad t = -3/8,$$

and evaluate the coefficient at $\kappa \in \{1, 16, 81\}$. For interval examples use the separately declared boxes $\kappa \in [1, 16]$, $U \in [1, 16]$, and, where indicated, $\kappa = 1$, $K \in [1/4, 1/2]$.

These are sample evaluation inputs, not admitted physical coefficient values and not SI units. No equation identifies a sample coefficient with $\hbar$.

toprule Evaluation-only quantity Exact value

midrule $a_{\mathrm{r}}(t), H_{\mathrm{r}}(t), B(1 - a_{\mathrm{r}}(t))$ 1/2, 4, 1/2

$(T_{\mathrm{r}}, S_{\mathrm{r}})$ at $\kappa = 1, 16, 81$, $U = 1$ $(2, 4/3)$, $(16, 1/6)$, $(54, 4/81)$

Temperature absolute risk on $\kappa \in [1, 16]$, $U = 1$ 7, with deterministic midpoint 9

Radiation box risks on $\kappa, U \in [1, 16]$ $m_T = 15, \quad m_S = 21/4$

Temperature relative risk on $\kappa \in [1, 16]$, $U = 1$ 7/9, with relative decision 32/9

Late-label risks at $\kappa = 1$, $K \in [1/4, 1/2]$ $m_T^{\mathrm{late}} = 1/(16\pi), \quad m_S^{\mathrm{late}} = 6\pi$

bottomrule

The relative decision in the table is the harmonic mean of the temperature endpoints 2 and 16, not their arithmetic midpoint. Every displayed value follows by substitution in the printed formulas; no numerical execution is claimed.

Verification scope and proposed exact checks

The supplied version 3 reports Big04-01–Big04-39 as historical executed checks. The scripts, JSON outputs, original source bytes, and build environment needed to reproduce those claims were not available as executable inputs for this revision. Accordingly:

• no prior check is relabelled as a new pass;

• no source digest is fabricated from a displayed transcript;

• no successful pdfLaTeX run or zero-warning log is asserted;

• no external manuscript version is called the latest on the basis of its inherited bibliography.

The source is designed for two ordinary pdfLaTeX runs:

pdflatex -interaction=nonstopmode -halt-on-error Big04-A-v4.tex?pdflatex -interaction=nonstopmode -halt-on-error Big04-A-v4.tex

There is no external bibliography, figure, or data dependency.

toprule Proposed check Exact target; status: not reported as executed

midrule Big04-40 General-w powers, product, and both Jacobian column orders.

Big04-41 Relative-risk harmonic decision, unequal endpoint prior, and the radiation coefficient-window correction.

Big04-42 Correlated joint state: marginals $I_2/2$, $\Im = 25$, installed error 24/25.

Big04-43 Rate-envelope integration, $\Gamma = 2$ reduction, and the $\Gamma \to \infty$ limit.

Big04-44 Distinct comoving and proper minimax factors, and deadline expression.

Big04-45 Scalar and nonscalar restricted-O6 cases.

Big04-46 Replacement/dephasing witnesses in both signs and factor orders, including singular replacement outputs.

Big04-47 Coefficient-blind versus exact-label metadata, and capacity-count change under padding.

bottomrule

The following optional listing checks selected algebraic identities. It is printed for reproducibility and is not a report of execution.

Optional checks_Big04A_v4_selected.py?# Requires SymPy. Not reported as executed.??import sympy as s??# Big04-40: logarithmic Jacobian, column order (ln U, ln kappa).?w = s.symbols("w", positive=True)?J = s.Matrix([[w/(1+w), 3*w/(1+w)],? [1/(1+w), -3*w/(1+w)]])?assert s.simplify(J.det() + 3*w/(1+w)) == 0?assert J.det().subs(w, s.Rational(1,3)) == -s.Rational(3,4)??# Big04-41: relative-risk decision and prior.?ell, upper = s.symbols("ell upper", positive=True)?y = 2*ell*upper/(ell+upper)?r = (upper-ell)/(upper+ell)?assert s.simplify(y/ell-1-r) == 0?assert s.simplify(1-y/upper-r) == 0?prior_lo = ell/(ell+upper)?prior_hi = upper/(ell+upper)?assert s.simplify(? prior_lo*(y/ell-1)+prior_hi*(1-y/upper)-r?) == 0?# A coefficient ratio 16 gives a radiation-label ratio 8.?assert s.Integer(16)**s.Rational(3,4) == 8?assert (s.Integer(8)-1)/(s.Integer(8)+1) == s.Rational(7,9)??# Big04-42: correlated two-qubit replacement.?lam = [s.Rational(2,5), s.Rational(1,10),? s.Rational(1,10), s.Rational(2,5)]?assert lam[0]+lam[1] == s.Rational(1,2)?assert lam[2]+lam[3] == s.Rational(1,2)?S = sum(1/x for x in lam)?assert S == 25?assert 1-1/S == s.Rational(24,25)?assert 1-1/S != s.Rational(15,16)??# Big04-43: rate-bound closed form and limit.?Gamma, rho, B = s.symbols("Gamma rho B", positive=True)?Rbar = B*(Gamma*rho**(1-1/Gamma)-1)/(Gamma-1)?assert s.simplify(? Rbar.subs(Gamma,2) - B*(2*s.sqrt(rho)-1)?) == 0?assert s.limit(Rbar, Gamma, s.oo) == B*rho??# Big04-44: deadline affine transfer.?H0, tau, tstar = s.symbols("H0 tau tstar", positive=True)?lower = B+(Rbar-B)*s.exp(-H0*(tau-tstar))?assert s.simplify(? lower - (? B*(1-s.exp(-H0*(tau-tstar)))+? s.exp(-H0*(tau-tstar))*Rbar?)?) == 0??# Evaluation-only table: no physical coefficient admission.?Tlo, Thi = s.Integer(2), s.Integer(16)?assert (Thi-Tlo)/2 == 7?assert (Thi-Tlo)/(Thi+Tlo) == s.Rational(7,9)?assert 2*Tlo*Thi/(Tlo+Thi) == s.Rational(32,9)

Version-5 exact checks

The checks below concern only the version-5 insertions. They are numbered from Big04-48 because version 4 already uses Big04-40–Big04-47 for proposed checks. The historical checks Big04-01–Big04-39 and the proposed checks Big04-40–Big04-47 are not reasserted as executed results of this revision, and none is relabelled as a new pass. The version-5 checks are implemented by the accompanying program checks_Big04A_v5.py; this text does not report their outcome.

toprule Check Exact target (version-5 insertions only)

midrule Big04-48 The version-4 input hash, its identity with the attached original, and the plan hash.

Big04-49 Insertion-only structure: balanced, uniquely named blocks; deleting them returns version 4 byte for byte.

Big04-50 Proposition and the values of Documentary record : band and co-band at $L = 2$, deficits, $e_{\mathrm{seed}}(3, 2, 2)$, $Q_{7,3,3}$, $Q_{7,4,5}$, middle-range membership; cited labels present in the source text.

Big04-51 Documentary record : continuity at $\frac{1}{3}$, $v_2 = 2 - \sqrt{22}/3$ as a root, the polynomial identity, and the source locations of PC5-1 and PC5-2.

Big04-52 Documentary record : curvature of the prefix ($R = 0$, $R_{tt} \neq 0$) and of a de Sitter tail; absence of κ and c_0 from the radiation labels.

Big04-53 Proposition : endpoint ratio, the tanh form with $C = C_K$, decision and prior, the linear-label value δ, the comparison with δ, the box form, and the free-rate value 1.

Big04-54 Documentary record : the sector formulas, $Q_{11,4,4}$, $Q_{11,4,7}$, sector and middle-range membership, and the bracket in the source text.

Big04-55 Documentary record : the minimax value $3(b^2 - a^2)/(2c^2)$ and the finite and infinite optical tails.

Big04-56 Documentary records and : the cited labels, $Q = 0$ for the radiation conformal factor, and the tail expansion $3K/c$.

Big04-57 Every inserted result is followed by a non-claim, and each non-claim of the plan is present.

Big04-58 The plan-audit row verbatim, the plan corrections and errata, the required version-4 lines, and the

restrictions of the plan's section 8 on the inserted text.

Big04-59 Every reference and citation in the insertions resolves, and every cited source label exists in the text read.

Big04-60 Two pinned builds: byte-identical output, no errors, no undefined references and no duplicate labels; frozen records unchanged.

bottomrule

Version-6 exact checks

The checks below concern only the version-6 insertions and are implemented by the accompanying program checks_Big04A_v6.py; mutation tests are in mutations_Big04A_v6.py. Checks Big04-01–Big04-60 are not reasserted as results of this revision, and this text does not report the outcome of the version-6 checks.

toprule Check Exact target (version-6 insertions only)

midrule Big04-61 Frozen inputs: version-5 manifest, version-5 source hash, plan and prediction-layer hashes.

Big04-62 Insertion-only structure: deleting the version-6 blocks returns version 5 byte for byte; every version-5 label keeps its number.

Big04-63 Proposition : orbit invariance, kernel $(0, 1, 3)$, product fibre, inverse formulas; negative control (q, q^2).

Big04-64 Theorem : exact laws of finite adaptive protocols, blind and coefficient-sensitive; window values.

Big04-65 Proposition and Corollary : logarithmic derivatives, time independence, p_w values and comparison with δ, cross-ratio rescaling.

Big04-66 Theorem : $R_A = Bz^2$, the difference formula, $R_e > R_A$, the modulus with equality, and the $D - B$ consistency.

Big04-67 Proposition : image-functional values of the version-5 risks recomputed from images.

Big04-68 Lemma , Documentary record and Proposition : floor comparison, the OpenAI string search, the vacuum shift.

Big04-69 Theorems and : drift identity by quadrature, the prefix relations, the closure equivalence.

Big04-70 Theorem : envelopes against quadrature, continuity at t_*, limits, admissible profiles strictly inside, the attained case $\Gamma = 2$, $T < t_*$.

Big04-71 Theorems –: drift band, logarithmic tails, gluing, isolation thresholds.

Big04-72 Every inserted result is followed by a non-claim; documentary edges carry the source-record tag and the citation note.

Big04-73 Standing ledger verbatim in the abstract and at the end; the required lines; author line; no ratings or scores; no numerical κ.

Big04-74 References, citations and cited source labels resolve in the served texts; external references verified, without tracking parameters.

Big04-75 Site-version findings: served Big03-A, Big04-A and the Big06-A request.

Big04-77 Draft results re-proved: the normalization-window risks, the late-label inverses and orbit actions, the general-w quotient and Jacobian, the epoch ratios and product, the variable-w derivative, the settling functional and its differential equation, the unrestricted radius range, the clock-feedback criterion and the action shift.

Big04-78 Draft integration: backbone map, verdicts F11, corrections DC3, the folding of the draft's plan corrections, the PC3 numbering, and the calibration wording.

Big04-76 Two pinned builds: byte-identical output, no errors, no undefined references and no duplicate labels; frozen trees unchanged.

bottomrule

Source handling

The supplied manuscript and excerpts are the basis for the Cheung attributions below. Attempts to retrieve the Big-series manuscript endpoints did not yield accessible full texts through the available browsing interface; this is not a claim that the manuscripts are absent. No byte-identity or hash verification is inferred from those attempts.

Published references were used for their stated standard frameworks and bibliographic information. An unpublished source-record attribution is not upgraded into a proof verification by adding a bibliography entry. The bibliography intentionally omits inherited claims about manifests, execution logs, and retrieved-file hashes.

Source records read for version 5

For version 5, read-only GET requests to https://enter.grok.me/api/manuscript/<ID> on 24 October 2026 returned the full texts listed below. Each was byte-identical to the latest version of the same manuscript in the local companion copies. The sha256 digests identify the bytes read. They identify source records only; no proof is transferred, and citations stay source records, not premises.

toprule Record Version sha256 of the text read

midrule Big02-A version 6 98dd1fb1796a8e838796b21f9ad88242497638f86a2c9dae1c14b7efcc8dd804

Big03-A version 2 c71007c5e5455227925f625ff9fc488e369cbd66a64b95799f8f34e68519bd6a

DS-OB-1-CERT version 5 a0805f8eb081a250f78e8074b9de5986ccd23106749df9e0fe5fac2f71e157f9

Res06-A version 4 d2229f7a039d9837de1cb1d4741999d92341b848dbdd9763742d9a731e07e555

Red06-G-DSS-CERT version 10 43324387beffae4384e142e1c348b4cebdb6269cadeb640fd1e386f4d2eee247

Hawk07-C-CERT version 6 fcb6b12ae27cbe3f6fca50a1bbe4e009f85e472bf3908451b2ea9b76a9840250

Hawk07-C-Dyn-CERT version 6 97397c8357a69bdd7939b7415d64364678d8582dd9e8738df600ee18f2e2afea

Oai280-B version 1 5aebd1cbaa4b6364a09f706befd9b2121a26e80b41b6872bca52d897099d6a5c

Oai282-B dated 12 October 2026 d42f3cdeed913ef22ba08d5d088a760cd03b77a0a28d333195404c66b99a9063

Big04-A version 4 (user) input 7053d2962dc8f035bbda55c9628480c9a970f04c99262c28b7b3813e11274010

PLAN_t305u input 7d13903ad9228b1f2e4590f1ff9fc3ec5b6d02686f159360e852f5b1f3412fb0

bottomrule

Source records read for version 6

For version 6, read-only GET requests to https://enter.grok.me/api/manuscript/<ID> on 25 October 2026 returned the texts below. The sha256 digests identify the bytes read. Citations identify source records only; no proof transfer. Each record is source record; not a premise.

toprule Record Version served sha256 of the text read

midrule Big05-A version 3 1788cf6f5abe4bf139f54aa18167a61c1130cdb6a0aacff84f2f45521d5946a3

Big04-A-coeff version 5 9276db3c2eca392e3af5af1e960230367160d061874b332b4dc7342d5859ca70

Big03-A version 3 3f30924250f6b0e24d74c090653b6a91b110f9a374119ac33ff494a665fa0fd3

Big02-A version 6 1d8fc18c752c82edb1235282cbd56ccd8540e41ae675589337bbd6622ffa8f7a

En04-A header v2, title v3 b4cb98c038318b366c79b8fe932250e3e626e6e2e75edcb0d61e70bfc118568c

En04-B version 4 60a0c284e6415e9cf3b64a7362af8ad06a4a0ec2f25bda26b1467d7966c4045a

I-C version 2 d6535e00b845bfcf005d90efa39cb4092f0118a4177c0613809efa83ceb2bf8b

Hawk05-B no version marker e8ae780e771c04d876d91505de79fb11e47c89925596cb67f50db641945470fd

DS-OB-1-fibre version 5 4626dbb721ab0c0cb1f374699897dde85d5cc20f599f1a3470a620d286f425f8

OpenAI catalogue 9a20a47b380d2de3d68e80a47877c563460260d7acf8aafdcab6aa97285ca5a8

Big04-A version 5 plus 47 lines 0bb4d9b2dd7dcfbef9da194f6be160ae1c8b4a756c0d5550b817df31226a352d

I-C-CERT version 6 7c06063bf80e13fc914edbba1497b8c2ee9378d97601b9dbeb4e6df7e8ae6336

Big06-A (second read) version 2 fb7927fcc52c470d7b1e17d78b3c2d7a4c7d3bdffe9356a135bb9dee293753c4

Big01-A (not cited) version 7 7d3989b447c4a0dd282b1ffeb97eb38d1b303986b82be3130d8db7a5e952169a bottomrule

Site-version findings. (i) Big03-A: the served text is the local version 3 (/workspace/NewSheets336/Big03-A/deliver/Big03-A-v3.tex, sha256 f00b8dbfd827150b3fa8510fc4e896b21368f3ad5cc04a0120ef4ed5b4f89eb4) with one blank line and five bibliography entries added and no line removed; version 3 is cited as served. (ii) Big06-A: the first requests for the ids Big06-A and Big06-A-v1, and with version queries, returned HTTP 404; a second read of Big06-A the same day returned version 2, byte-identical with /workspace/NewSheets341/Big06-A/deliver/Big06-A-v2.tex, which contains version 1 verbatim. Version 1 itself is a local version (/workspace/NewSheets338/Big06-A/deliver/Big06-A-v1.tex, sha256 eca341929d98422c3c6475d8175d1ec48d71b87e4fcc1057842000d658145e79); it is recorded here [Big06v1] and not used elsewhere. (iii) Big04-A: see erratum ER6-2. (iv) A second read of Big03-A, Big02-A and DS-OB-1-fibre returned the same bytes; Big01-A is served as version 7. (v) The author's draft of version-6 additions: eb7a8ac12292629f9f61d7938659cff9908593df47c2fc7e85a80fcb3180a6a7. Version 5 input: sha256 396f8a6849f4b4d4fafc2107b156e791c8e21e04d407ac4151e12678d6b32987; PLAN_t344u: 56afb12c1dc6c52cf2863a41f70086e7cc9e6ceeb17950254d26006452d922f1; PRED_t345u: 341ea3bbaa3f5e950fc1d6676791909b692821495b2341b833920550a683be5a.

External references. The three references of the prediction layer were verified against Crossref and arXiv on 25 October 2026; the tracking parameters of the supplied links are removed, and each is source record; not a premise?[AlvesMNRAS2019,AvgoustidisJCAP2012,ATW2023].

Standing ledger, version 6

Host A and Host B stay separate. Host C is not created. O.1 stays NOT_CLOSED. XV-B is not opened. XVI-N is not discharged. N-Ins.4 stays unresolved. U6-sel is not moved. Gate 0 stays NOT_SUPPLIED. ST4b stays NOT_SUPPLIED. No quantum coefficient is admitted. No quantum coefficient is derived. No Einstein dynamics, derived metric, spacetime emergence, Page curve, island, vacuum-energy computation, or quantized relational time without an external clock is claimed. A source record is not a premise. The OpenAI catalogue is not a premise.

The ledger above is the verbatim block of Section . In addition: H2, I3, J2 and T3 stay unresolved; absolute vacuum energy stays open; no numerical κ is admitted; supplied closures are optional hypotheses outside the ledger. Citations identify source records only; no proof transfer. Author line: Ngo Cheung (authorship requires confirmation).

References

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A. Winter, ``Tight uniform continuity bounds for quantum entropies: conditional entropy, relative entropy distance and energy constraints," *Communications in Mathematical Physics* **347** (2016), 291–313. doi:10.1007/s00220-016-2609-8. arXiv:1507.07775.

N. Cheung, *Big01-A: Horizon-Bounded Capacity on a Prescribed de Sitter Host*, version 5. Enter framework companion manuscript. Source identity and statements supplied with the present materials; authorship requires confirmation. source record; not a premise.

N. Cheung, *Big02-A: Dark-Capacity Certificates on the Prescribed Static Patch: Redundancy, Complete Causal-Access Loss, Junk Sectors, and Future-Dependent Horizons*. Version 4 source supplied earlier; version 6 excerpts and locators supplied with the present materials. References to thm:v4-wlabels, thm:v4-rate, thm:v4-kyfan, thm:v4-var, thm:v4-L2, and eq:approxsingleton refer to the specified version-4 statements. Enter framework companion manuscript; authorship requires confirmation. source record; not a premise.

N. Cheung, *Big03-A: A Supplied-Coefficient Class beside the Negative Coefficient Certificate*, version 2. Enter framework companion manuscript. Supplied excerpts include thm:R-unsupplied and prop:R-relative. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Big04-A: Dark Radiation as a Declared Reading on the Prescribed Comparison Host: Radiation Labels, Recovery Floors, Thermal Errors and Late Labels*, consolidated edition, version 3, 24 October 2026. User-supplied source Big04-A-v3.tex. Authorship requires confirmation. source record; not a premise.

N. Cheung, *nuADaM01-A: A Tightly Coupled and a Decoupled Dark Sector on a Prescribed Two-Epoch Host: Coefficient, Access, Replacement, Residual and Junction Certificates*, version 5. Enter framework companion manuscript. Early-profile and label formulas are supplied through the Big04-A source and accompanying excerpts. Authorship requires confirmation. source record; not a premise.

N. Cheung, *I-C-CERT: Exact Dual Certificates for the Shredder Recovery Obstruction*, version 6. Supplied excerpts include thm:capacity; the accompanying plan identifies prop:v3-gap. Enter framework companion manuscript; authorship requires confirmation. source record; not a premise.

N. Cheung, *Hawk07-C-Dyn-CERT: Dynamics Certificate for the Dynamical Conical-Formation Spacetime*, version 6. The supplied record includes the prop:s2-vac conservation-vacuity statement. Enter framework companion manuscript; authorship requires confirmation. source record; not a premise.

N. Cheung, *Hawk07-F-CERT: Casimir Coefficient Certificate and Dynamics Certificate for the Semiclassical Toroidal Throat*, version 6. Supplied excerpts: thm:s2-cas and prop:s2-cas-mass. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Hawk07-H-CERT: Casimir Coefficient Certificate and Dynamics Certificate for the Toroidal-Throat Response*, version 6. Supplied excerpt: thm:s2-cas. Authorship requires confirmation. source record; not a premise.

N. Cheung, *En07-B*, version 1. Enter framework companion manuscript. Supplied excerpts: thm:global and thm:exact, concerning complete-output restoration and the exact complete-complement criterion. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Oai211-A*, version 1. Enter framework companion manuscript. Supplied excerpt: thm:path, uniform offspring-bounded path sums. Authorship requires confirmation. Documentary only; not a premise.

N. Cheung, *Oai212-A*, version 1. Enter framework companion manuscript. Supplied excerpt: thm:transfer, conditional transfer of an asymptotically identical passage front. Authorship requires confirmation. Documentary only; not a premise.

N. Cheung, *Oai280-C*, version 1. Enter framework companion manuscript. Supplied excerpt: thm:complement, with its whole-code/subsystem qualification. Authorship requires confirmation. Documentary only; not a premise.

N. Cheung, *Landmark XIV-I*, version 1. Supplied manuscript excerpts: thm:I2, thm:I3, and thm:release. The internal-amplitude construction does not supply the joint characteristic map. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Landmark XIV-J*, version 1. Supplied manuscript excerpts: thm:J2, thm:release, and cor:noadmission.

The full characteristic map and quantum admission remain unconstructed in that record. Authorship requires confirmation. source record; not a premise.

N. Cheung, *DS-OB-1-CERT: Coefficient certificate for the named Gibbons–Hawking labels on the prescribed de Sitter background*, version 5. Enter framework companion manuscript; text read for version 5 (Appendix): thm:v5-vac. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Res06-A: A Packaging Certificate for Five Named Residuals*, version 4. Enter framework companion manuscript; text read for version 5: prop:R6v4-sector2, thm:R6v4-1144, thm:R6v4-1147, rem:R6v4-32. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Red06-G-DSS-CERT: Coefficient, Dynamics and Exact Dual Certificates on the Prescribed de Sitter Host of Red06-G-DSS*, version 10. Enter framework companion manuscript; text read for version 5: prop:RC-rate9, prop:RC-tails9. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Hawk07-C-CERT: Dynamics certificate and exact dual recovery certificate for the prescribed conical host*, version 6. Enter framework companion manuscript; text read for version 5: prop:v5-expansion. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Oai280-B: Finite-Confidence Certification of Boundary Reconstruction in Five-Qubit Perfect-Tensor Codes*, version 1, 12 October 2026. Enter framework companion manuscript; text read for version 5: thm:oneslot. Authorship requires confirmation. Documentary only; not a premise.

Oai282-B: From Finite Ward Certificates to Local Conformal Symmetry: Physical Improvement, Explicit Contacts, and Complete Coverage for the Four-Dimensional Free Massless Scalar, dated 12 October 2026; the record states no author and no version number. Enter framework companion manuscript; text read for version 5: eq:vacward. Documentary only; not a premise.

N. Cheung, *Big05-A*, version 3. Enter framework companion manuscript; text read for version 6: prop:Big05-orbit-split, eq:radiationorbit, thm:Big05-futures, thm:Big05-fibre, thm:Big05-flagship. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Big04-A-coeff*, version 5. Enter framework companion manuscript; text read for version 6: prop:C-orbit, thm:C-adaptive, lem:C-fibre. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Big03-A*, version 3, as served on 25 October 2026. Enter framework companion manuscript; text read for version 6: thm:R-v3-adaptive, prop:R-v3-blind-boundary, prop:R-v3-vacuum-box, prop:R-v3-orbit. Authorship requires confirmation. source record; not a premise.

N. Cheung, *En04-A* (served record: header version 2, title version 3). Enter framework companion manuscript; text read for version 6: thm:magic-fibre. Authorship requires confirmation. source record; not a premise.

N. Cheung, *En04-B: Finite Carriers on Prescribed Enter Spacetimes*, version 4. Enter framework companion manuscript; text read for version 6: prop:coefficient, thm:fibre, prop:clock-confound. Authorship requires confirmation. source record; not a premise.

N. Cheung, *I-C-v2: The Shredder Interface*, version 2. Enter framework companion manuscript; text read for version 6: thm:A. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Hawk05-B: A Finite Page-Curve Derivation under the Enter Discipline*. Enter framework companion manuscript; text read for version 6: thm:page. Authorship requires confirmation. source record; not a premise.

N. Cheung, *DS-OB-1-fibre*, version 5. Enter framework companion manuscript; text read for version 6: lem:F-fibre, thm:F-rank, thm:F-flrw, cor:F-ranknotsel. Authorship requires confirmation. source record; not a premise.

OpenAI, *OpenAI Research Catalogue*, catalogue sheet of the Enter companion (public release 6 October 2026). The companion does not verify those proofs. Documentary only; not a premise.

N. Cheung, *I-C-CERT-v6: Exact Dual Certificates for the Shredder Recovery Obstruction*, version 6. Enter framework companion manuscript; text read for version 6: thm:replacement. Authorship requires confirmation. source record; not a premise.

N. Cheung, *Big06-A*, version 1, contained verbatim in the served version 2. Local version

(/workspace/NewSheets338/Big06-A/deliver/Big06-A-v1.tex, sha256 eca341929d98422c3c6475d8175d1ec48d71b87e4fcc1057842000d658145e79); served on 25 October 2026 only inside version 2. Authorship requires confirmation. source record; not a premise.

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D. Andriot, D. Tsimpis and T. Wrase, ``Accelerated expansion of an open universe and string theory realizations,'' *Physical Review D* **108** (2023), 123515. doi:10.1103/PhysRevD.108.123515. arXiv:2309.03938. source record; not a premise.