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Big02-A: Dark-Capacity Certificates on the Prescribed Static Patch: Redundancy, Complete Causal-Access Loss, Junk Sectors, and Future-Dependent Horizons

A companion separation manuscript to Big01-A in the Enter framework

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Abstract

Big01-A turned the fixed-size quantum-computer analogy for the de Sitter static patch into theorems about a *prescribed classical* host DSS_H. In that analogy one may call ``dark" any part of a finite, horizon-bounded register that constrains the usable computation while lying outside the controllable, readable logical subspace. The analogy names four such parts: error-correction redundancy, horizon leakage, junk sectors, and information that has left the patch. This sheet asks which sentences about these four parts are theorems, and proves the ones that are. Each statement here is either a local theorem proved from hypotheses stated in the sheet, or a documentary separation that depends on a source record and is tagged ``source record; not a premise". Temperature, entropy and dimension enter only as named labels with an external coefficient slot κ in the place of $\hbar$.

Four main theorems. *(I) Redundancy.* Suppose a code on n declared q-dimensional sites corrects the erasure of any $\delta - 1$ sites (a Knill–Laflamme condition). Then the reference decouples from every set of $\delta - 1$ sites, and $\ln K \le (n - 2(\delta - 1))\ln q$. The same bound holds for the logical factor of a subsystem code. Under a supplied capacity promise P_η this becomes $\ln K \le \mathrm{eta}\mathrm{S}_{\mathrm{GH}}/k_B - 2(\delta - 1)\ln q$. Without the promise, K is unbounded at fixed distance. No code improves on a replacement channel. *(II) Complete causal-access loss.* A source with $u = X_4 - X_0 \le 0$ reaches a comoving observer only through a replacement channel. The least error over located circuits is exactly $(1 - 1/d^2)\,1[p \notin J^-(\gamma)]$. This error is independent of κ, it is not a function of placement-blind data, and the same temperature label is compatible with errors 0, $1 - 1/d$ and $1 - 1/d^2$. *(III) Junk.* An orthogonal complement forces no error at all. Flagged loss costs exactly $p(1 - 1/k^2)$. A flagged m-dimensional quantum cut costs exactly $1 - m/d$. Scrambling the complement costs exactly $1 - 1/(1 + (d - k)^2)$, which is strictly below the horizon floor. The cut floor lies below the horizon floor by $(md - 1)/d^2$. *(IV) Information that has left the patch.* On smooth futures that share the entire past, the junction radius R_0 takes every value in $(0, \infty)$. A fixed past event can be dark for one future and visible for another, so any rule of past data has worst-case error at least $\frac{1}{2}(1 - 1/d^2)$ on the least error. The area proxies, the late label $N^{(\kappa)}(K)$ and $\Theta_{tt} = 3(K^2 - H_1^2)$ all differ across these futures.

Tools. A primal–dual sandwich lemma: if $J(T) \ge t\,\Omega\Omega^\dagger$ then $\varepsilon(T) \le 1 - t$, and every input gives a lower bound. The sandwich is exact on replacement channels, block pinchings, isotropic channels and the flagged-cut attainers. It is *not* exact on every channel: amplitude damping has $t^* = 0$ but error below 1. The sheet also has a fibre lemma, a coefficient lemma, a monotone null coordinate and an ancestor lemma.

A background section presents the dark-matter analogy as structural motivation only. Every physical sentence in it is marked heuristic and not claimed, and points to its finite-channel certificate where one exists. A user draft (t272u) was checked claim by claim and its verified results are adopted. No finite Hilbert space, temperature, entropy, particle species, clustering mechanism, Einstein dynamics or Page curve is derived. No quantum coefficient is admitted.

Version 2 (plan t273u; user draft t275u). Version 2 keeps every statement of version 1 unchanged. By addition only, it adds a robustness subsection after each main theorem, a section of results integrated from the user's t275u

draft, a cross-cutting section, checks DC-17 to DC-31, an updated plan audit, a verdict appendix for the draft, and new documentary edges. *I-R:* a code that corrects every error of weight $\le t$ has distance $\ge 2t+1$, so $\ln K \le (n-4t)\ln q$; the form $(n-2t)\ln q$ is the bound for t erasures. Approximate correctability gives $(1-\varepsilon_A-\varepsilon_B)\ln K \le \ln\dim \mathscr{H}_C + \frac{1}{2}(g(\varepsilon_A)+g(\varepsilon_B))$ with $g(\varepsilon)=(1+\varepsilon)h(\varepsilon/(1+\varepsilon))$, which costs a dimension-dependent term $2\varepsilon\ln K$ as well as $O(\varepsilon\ln(1/\varepsilon))$. *II-R:* if the receiver channel is within half-diamond distance ε of a replacement, the error is at least $1-1/d^2-\varepsilon$, and this is tight. Per-observer darkness does not imply joint darkness. A cut with L classical labels and an m-dimensional register has error at least $1-\min\{m/d, Lm^2/d^2\}$, attained with a shared seed when $m \mid d$. Every error in $[0, 1-1/d^2]$ is realized by a thermal operation at every $\beta > 0$. *III-R:* a residual state τ gives installed error $1-1/(1+\operatorname{Tr}\tau^{-1})$, with an exact entropy-constrained envelope, but optimized recovery has error $1-1/(1+j^2)$ whatever the entropy. Amplitude damping has error exactly γ. *IV-R:* explicit moduli for R_0 hold under tail hypotheses, and local regularity alone gives none. Quasi-analytic futures are rigid, while Gevrey ($s>1$), $C^{k,\alpha}$, Sobolev and C^∞ futures realize every R_0. A null-type condition $\dot{H} \le 0$ restricts R_0 to $[c/H_1, \infty)$ without removing the dark-event floor. Junction radii are affinely tied to later ones, and the status-flip region has an exact 4-volume.

Version 3 (plans t276u and t277u; user drafts t279u). Version 3 keeps every statement of version 2 unchanged. By addition only, it adds an early-universe comparison host with a thermodynamic interface (Section), packaging results and further expansions (Section), two corrections to version-2 wording (Section), checks DC-32 to DC-43, plan audits, a verdict appendix for the drafts, and documentary edges. *Early host:* a radiation-type prefix $a=\sqrt{1+2H_0 t}$ is prescribed, not derived. Its causal relation, arrival times and deadlines are explicit. Radiation labels $T_{\mathrm{r}} \propto \kappa^{3/4}/a$ and $S_{\mathrm{r}} \propto \kappa^{-3/4}$ are not functions of the admitted data, but an exact calibrated label reveals κ. With the terminal rate H_1 fixed, the settled late radius is c/H_1, and the *early-junction* radius ranges over $(0,\infty)$, or over $(c/H_0, c/H_1)$ under $\dot{H} \le 0$; a settling deadline shrinks these ranges further. The status-flip four-volume has a closed polynomial form. Early-source loss is a replacement floor independent of every label. A late Gibbons–Hawking label is not determined by early data unless H_1 and κ are supplied, and $G+\Lambda g$ conservation is vacuous on a prescribed host. *Packaging:* the declared-data monoid acts uniformly only through its spectator lift. The seeded (L,m) optimum is $1-F_{\max}$, with explicit sign-correct witnesses. It equals $1-\min\{m/d, Lm^2/d^2\}$ exactly when $Lm \le d$ or when I_d is a weighted sum of L rank-m projectors, and fails for $(3,2,2)$. A mean-energy promise supplies no dimension. The cutoff-and-reset error is exactly $(q+\sqrt{4q-3q^2})/2$. The information defect is additive, and its allowance keeps a $(\varepsilon_A+\varepsilon_B)\ln K$ term.

Version 4 (request t280u; derived agenda). Version 4 keeps every statement of version 3 unchanged. No plan was attached to the request, so the agenda was derived from the open, partial and refuted items of version 3 (Appendix). By addition only, version 4 adds:

• (L,m) *values.* A Ky Fan branch bound gives the exact variational formula $d^2 F_{\max} = \max \sum_x \|\sqrt{E}_x\|_{(m)}^2$ over L -outcome POVMs. For two labels the closed form is $d^2 F_{\max} = 2r^2 + 2\sqrt{2}rk + k^2$, so $e_{\mathrm{seed}}(3,2,2) = (6-2\sqrt{2})/9$ exactly.

• *Energy floor.* The energy-constrained replacement floor is exactly $(v+\sqrt{4v-3v^2})/2$ for $v = E/\Delta \le \frac{1}{3}$ in any number of modes. For one mode it is $3v(1-v)$ on $[\frac{1}{3}, 2-\sqrt{22}/3]$, with an explicit dual certificate, and it is bracketed beyond. Optimized decoding does not improve cutoff-and-reset at $q \le \frac{1}{3}$.

• *Coding cost.* A flag code shows that the coefficient $\varepsilon_A+\varepsilon_B$ of $\ln K$ is tight to first order for the information defect, which replaces the refuted dimension-free cost; the dimension form stays open.

• *Early host.* Power-law prefixes give general-w labels with Jacobian $-3w/(1+w)$ and a flip four-volume for every exponent. Rate-bounded futures give the early-junction range $(B, \overline{R})$ with $\overline{R} = B[\gamma\rho^{1-1/\gamma}-1]/(\gamma-1)$, and

decelerating futures have no event horizon.

• *Checks and re-check.* Checks DC-44 to DC-53 pass, and the re-check of version 3 found no erratum.

Version 5 (plan t283u; user draft t285u). Version 5 keeps every statement of version 4 unchanged and adds, by insertion only:

• (L, m) *decompositions.* Rank conditions, complement and Naimark moves give an unconditional classification for three labels: I_d is a weighted sum of three rank-m projectors iff $d/m \in \{1, \frac{3}{2}, 2, 3\}$. For every L, a finite Horn–Klyachko linear program decides the question and a finite second-order-cone program gives $F_{\max}$; both rest on a cited spectral theorem.

• *Energy.* For M modes, the optimized replacement error is exactly $\min_{u,v>0,\, A_M<1} (u + vv)/(1 - A_M(u, v)^2)$ at every energy, with matching primal and dual certificates. Vacuum replacement is optimal up to $v = M^2/(M^2 + M + 1)$. Cutoff-and-reset recovery is a certified limit of finite SDPs.

• *Coding.* A Bell-twirled approximate Singleton bound sharpens the version-2 allowance. The version-4 allocation code is small only in Bell form: its half-diamond error is at least $\frac{1}{2}$. The dimension form remains open.

• *Geometry.* Simultaneous minimax for several junctions, a window-plus-tail modulus with sharp tail constant, an exact radiation–de Sitter benchmark volume $341479\pi/80640$, and a classification of κ-free label combinations.

• *Checks.* DC-54 to DC-69 are executed and pass. The re-check of version 4 found no erratum.

Version 6 (plan t287u, items 3 and 6). Version 6 keeps every statement of version 5 unchanged and adds, by insertion only:

• *Closed* (L, m) *values.* A meeting-subspace inequality and a two-parameter certificate give the seeded optimum below m/d in closed form, without the cited Horn–Klyachko theorem: for every L in the band $(L - 1)m \le d \le Lm$ and in the co-band $(L - 1)d \le Lm$, and for every d when $L = 3$. The version-5 numerical surds $5 + 2\sqrt{6}$, $7 + 2\sqrt{6}$, $8 + 4\sqrt{3}$ and $13 + 4\sqrt{3}$ become theorems, and $Q_{7,4,5} = 18 + 12\sqrt{2}$.

• *Early labels.* For any supplied system Hamiltonian, the identity, eigenbasis dephasing and Gibbs replacement are thermal operations at the label temperature, with recovery errors 0, $1 - 1/d$ and $1 - 1/d^2$ that do not depend on κ. $\mathscr{R}_{I/d}$ is thermal only for scalar Hamiltonians. Otherwise the undecoded Gibbs-replacement error depends on the label, with exact minimax error $1/(2d^2)$.

• *Checks.* DC-70 to DC-78 are executed and pass. The re-check of version 5 found no erratum.

Version 7 (plans t333u and t334u; user draft t335u). Version 7 keeps every statement of version 6 unchanged and adds, by insertion only:

• *Split-fibre non-selection (plan t333u, user draft t335u).* Every dark-capacity certificate of this sheet factors through the map that forgets the ordinary/dark/vacuum split of the total source, so all of them, and every adaptive rule built from them, are constant on each fixed-account split fibre. On explicit fibres with nonnegative energy densities, every such rule has exact minimax error $\frac{1}{2}$ on a dominance indicator (1 for deterministic yes/no answers), $\frac{2}{3}$ on naming the dominant component, and infinite error on the dark-to-ordinary ratio and on the absolute vacuum term (Theorem , Proposition). The certificate package depends on the declared account, while R_0 and the access floor are genuine functionals of the full prescribed future (Proposition). This matters because it shows, from this sheet's own definitions, that operational darkness cannot be read as dark matter or vacuum energy, and it says precisely in what sense dark capacity is ``not a host functional''.

• *Horizons.* Without an energy condition R_0 ranges over $(0, \infty]$; under $H \le 0$ it ranges over $[c/H_1, \infty]$, which is exactly the Big05-A range, so the two records agree and neither strengthens the other. Past geometry, past splits, past experiments and any finite ledger of causal statuses leave R_0 in an interval of positive length, except in one boundary case; the drift identity $\dot{R}_e = HR_e - c$ selects no horizon (Propositions , , Corollary).

• *Coefficient blindness and rank.* No coefficient-blind adaptive experiment, even with quantum memory and stopping, estimates κ or a label with bounded worst-case error, while one exact label value identifies κ; the injectivity of Theorem is unchanged. A constant recovery floor or split independence does not make a process coefficient-blind, and full rank of a tensor ambiguity selects no parameter (Theorem , Propositions ,).

• *Conditional cosmological predictions.* On every flat FLRW future the least access error is $a_d 1[r \geq \chi_e(t)]$. A declared band $\varepsilon_- \leq -\dot{H}/H^2 \leq \varepsilon_+ < 1$ gives $c/(H(1-\varepsilon_-)) \leq R_e \leq c/(H(1-\varepsilon_+))$, sharp, together with certified visible and dark regions. Each comoving source crosses to dark exactly once, at a computable time, and the accessible population decays at rate $3c/R_e$. A declared capacity promise gives a critical radius $\sqrt{(\ln K + 2r\ln q)/C_\kappa}$ that grows with $\sqrt{\kappa}$. An optional, supplied holographic closure (Li's model, outside the ledger) forces $w_X = -\frac{1}{3} - \frac{2\sqrt{\Omega_X}}{3b}$, a constant diagnostic $\mathscr{B} \equiv b$, and three futures: de Sitter ($b = 1$), power law ($b > 1$) and a finite-time rip ($b < 1$). Redshift drift and $H(z)$ are the observables. These results matter because they turn the non-identification theorems into stated interval predictions and falsifiers under declared assumptions, without fits and without a value of κ (Section , Table).

• *Audit.* All three inputs (t333u, t334u, t335u) receive per-item verdicts. A plan pre-check claiming that the critical-radius bound should use $n - r$ is rejected: $n - 2r$ is the quantum Singleton bound and is attained. All cited labels were re-read from the site records, and every mismatch is recorded (Appendices –). Checks DC-79 to DC-90 are executed and pass. The re-check of version 6 found no erratum. Standing ledger (verbatim): No quantum coefficient is admitted. Host A and Host B stay separate; Host C is not created. O.1 stays NOT_CLOSED. XVI-N is not discharged; N-Ins.4 stays unresolved. XV-B is not opened. H2, I3, J2 and T3 stay unresolved. U6-sel is not moved. No Einstein dynamics, derived metric, spacetime emergence, Page curve, or quantized relational time without an external clock is claimed. The last reading bars stay open, and continuum acquisition stays a negative certificate. No Route-B successor is created. Res02-A Q1–Q4 and A1–A2 stay open. Gate 0 and ST4b stay NOT_SUPPLIED. Citations identify source records only; no proof transfer. No compilation is asserted.

Bibliographic record

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Scope box. DSS_H is a prescribed classical host. T_{GH}, S_{GH} and $N_{\mathrm{GH}} = \exp(S_{\mathrm{GH}}/k_B)$ are external labels only, and their coefficient slot κ is never filled by an admitted datum. Host A and Host B stay separate, and Host C is not created. Citations identify source records only; no proof transfer. The cross-sheet edges in Appendix are documentary and not registered. Big01-A version 5 and every other sheet are source records only; none of them is edited. The title page names Ngo Cheung. Authorship requires confirmation. Citation does not by itself assign authorship. No compilation is asserted.

Background: the dark-matter analogy (motivation only)

This section restates the user-supplied analogy (file background_t272u.md, sha256 in Appendix) point by point. It is a *structural analogy and motivation*. It is not a premise of any theorem. Each physical sentence is marked *heuristic, not claimed*. These sentences cover T_{GH}, a horizon-area entropy bound, scrambling, thermalization,

packing density, operation rate and expansion history. Where a finite-channel counterpart exists, we point to the theorem of this sheet that certifies it, and to the source record it parallels, tagged (source record; not a premise). Where a sentence is false as a finite-channel statement, we say so.

The lead definition

Analogy (heuristic, not claimed) (Lead). Consider the fixed-size quantum-computer analogue of a de Sitter static patch. In it, ``dark matter'' means any contribution to the total Hilbert-space dimension $\sim \exp(S)$, or to the effective noise and capacity ceiling, that influences the dynamics of the usable subsystem yet stays outside the controllable, readable logical subspace.

Status. The finite Hilbert space of dimension $\sim \exp(S)$ is not derived. Theorem shows that the label N_{GH} is not a function of admitted data. Theorem (d) shows that, with unrestricted declared data, the register dimension is unbounded at fixed host. This parallels Big01-A v5 thm:D (source record; not a premise). What we *can* certify is access-relative. Definition splits a *declared* finite register, together with a declared access channel, into four categories. Each certificate is about the channel, not about a physical dimension.

The four realizations, feature by feature

Analogy (heuristic, not claimed) (Realization 1: error-correction redundancy and gauge degrees of freedom). Most physical qubits protect a smaller logical code space. They raise the dimension, suppress logical error below the bare thermal floor, and enforce stabilizer or gauge commutation. Listed features: (1a) a large fraction of the dimension carries zero net logical information; (1b) they appear as syndrome measurement or gauge fixing and are invisible until an error occurs; (1c) the overhead is hard: reliable logical operations scale more slowly than the raw dimension; (1d) in holographic encodings they are the redundancy of a holographic encoding (the plan's ``bulk-to-boundary'' wording is rephrased; see Appendix ); (1e) they are ``cold'': they dilute rather than thermalize logical information if the distance suffices.

Counterparts. (1a)–(1c): Theorem (a)–(c) proves the exact dimension count and the Singleton overhead of $2(\delta - 1)$ sites. The overhead is a *fraction* only relative to n, so ``most qubits'' is false in general: the $[[n, n-2, 2]]$ family spends 2 of $n$ sites. (1c) holds only in the sense of a log-dimension deficit. A bound on ``reliable logical operations'' needs a declared schedule, as in Remark . ``Below the bare thermal floor'' is heuristic, not claimed: a label temperature fixes no channel (Theorem (e)). (1d) Heuristic, not claimed. A holographic code [ADH2015] is not constructed here. (1e) Heuristic, not claimed. Its finite counterpart is the exactness of recovery under a fixed, even mixed, gauge state (Theorem (b)). Redundancy cannot cancel a replacement channel (Theorem (e)).

Analogy (heuristic, not claimed) (Realization 2: horizon or environment entanglement and leakage). Residual interior–horizon correlations, or leakage into an effective bath at T_{GH}, consume dimension and impose a noise floor. The causal structure generates them: modes redshift, stretch and leave the patch, leaving unrecoverable entanglement. Listed features: (2a) nonzero logical error without control noise; (2b) a strictly positive minimum half-diamond error, which is the horizon floor $1 - 1/d^2$ in the simplest models; (2c) scrambled or thermal character, with interior entropy bounded by the horizon area; (2d) limits on packing density and operation rate; (2e) permanently dark to the interior; (2f) cumulative and epoch-dependent through the horizon radius.

Counterparts. (2a), (2b), (2e): Theorem (a)–(c) proves them. A located source with $u \le 0$ gives a replacement channel with least error exactly $1 - 1/d^2$, which is causally permanent. This parallels Big01-A v5 thm:B, thm:Bsharp and thm:Bexact (source record; not a premise). The bath at T_{GH}, redshift and mode stretching are heuristic, not claimed. The replacement output is a product with the reference (Theorem (b)), so ``unrecoverable entanglement'' of the payload is *not* what the certificate says. (2c) Heuristic, not claimed. ``Bounded by the horizon area'' is rephrased as follows: the entropy of a state of the declared register is at most $\ln d$ of that register, and it is related to S_{GH} only under the promise P_{η} (Assumption). (2d) Heuristic, not claimed. A latency bound exists (Remark , parallel to Big01-A v5 prop:echo (source record; not a premise)), but there is no gate-rate or packing theorem. (2f) The epoch dependence is Theorem . ``Cumulative'' is not claimed.

Analogy (heuristic, not claimed) (Realization 3: unaccounted or ``junk'' states). Non-code subspaces count

against $\exp(S)$): scrambled states, preparation residue, and stretched modes not yet departed. They are (near-)orthogonal to the logical subspace. Listed features: (3a) they occupy dimension without protected power, and may act as an extra reservoir; (3b) their entropy is hidden from logical observables, though it is counted in the total entropy (rephrased from ``von Neumann entropy of the patch''); (3c) scrambling raises the effective noise temperature; (3d) preparation, cooling and error correction reduce them, but not below a floor set by the horizon temperature and the code distance; (3e) they are analogous to the non-clustering, weakly interacting dark-matter fraction.

Counterparts. (3a) The finite counterpart is Theorem (a): an orthogonal complement forces no error. ``Reservoir'' is heuristic, not claimed. (3b) The exact entropy ledger is the counterpart; there is no patch entropy. (3c) Heuristic, not claimed. The finite counterpart is Theorem (e): scrambling the complement costs exactly $1 - 1/(1 + (d - k)^2)$, which is strictly below the horizon floor. (3d) is *false* as a finite-channel statement. Junk forces no floor (Theorem (a)). The cut floor $1 - m/d$ lies *below* the horizon floor (Theorem (f)). No floor depends on a temperature label or on the code distance (Theorem (e)). (3e) Heuristic, not claimed.

Analogy (heuristic, not claimed) (Realization 4: information that has left the causal patch). After crossing the horizon, information is permanently inaccessible, yet it is part of the global accounting that fixed the bound earlier. Listed features: (4a) it contributes to the historical capacity ceiling, the integral that determines R_0 and the instantaneous horizon radius, with zero present resource; (4b) recovery is impossible inside the patch, because an interior decoder sees a replacement channel whose error is bounded away from zero; (4c) it may stay entangled, giving a mixed interior state whose entropy interior operations cannot lower; (4d) it is the purest dark capacity; (4e) its amount depends on the expansion history, since the epoch results show that some smooth future realizes every positive R_0.

Counterparts. (4a) is corrected. $R_0 = c\int_0^\infty \mathrm{d}s/a(s)$ is a functional of the prescribed *future*, not of history. It is the event-horizon radius at the junction (Theorem (b)). (4b) is Theorem applied to a located source outside $\bar{J}(\gamma)$; Theorem (c) shows that dark status is future-dependent. (4c) Heuristic, not claimed. For the payload the certificate gives a product with the reference. (4d) Heuristic, not claimed. (4e) The finite counterpart is Theorem (b): every positive R_0 is realized by a smooth future with the same past. This parallels Big01-A v5 prop:Esmooth (source record; not a premise). ``Expansion history'' is heuristic: the dependence is on the future.

The five shared structural features

Analogy (heuristic, not claimed) (Shared features). (S1) capacity consumption without logical yield; (S2) invisibility to direct logical readout, with coupling only through noise, syndromes or ceilings; (S3) thermal or scrambled character; (S4) causal permanence, so they cannot be undone unitarily inside the patch; (S5) an overhead and floor-setting role for packing density, error rate and operation speed near the holographic bound.

Status. None of the five is universal across the four realizations. Section records an explicit counterexample for each. (S1) A pure syndrome ancilla has yield zero but costs nothing in recovery. (S2) A one-time pad has maximally mixed marginals, yet its recovery is exact (Theorem (f)). (S3) Thermal character is heuristic, and a temperature label fixes no channel. (S4) This holds for Realizations 2 and 4 by Theorem (a), but not for junk or gauge sectors. (S5) The holographic bound is heuristic, not claimed. The only certified ordering is the floor chain of Theorem (f).

Closing scope paragraph

Analogy (heuristic, not claimed) (Closing). The analogy is strictly structural. It supplies no particle species, no clustering mechanism and no derivation of dynamics from the classical host. It only identifies the portions of a finite, horizon-bounded Hilbert space that constrain usable computation while lying outside the controllable logical subspace.

Status. We adopt this scope with one change. A finite, horizon-bounded Hilbert space is not given by the host, so the identification is made only for a *declared* register with a declared access channel (Definition). The words ``horizon-bounded'' then refer to the promise P_η, which is supplied and not derived. par *Non-claim.* The background section is motivation. Nothing in it is a premise of a theorem, and no physical statement in it is

claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Early-universe narrative (version 3; heuristic, not claimed)
The plan t276u adds a Big-Bang thermodynamic narrative. Each sentence below is heuristic and not claimed, and it names the certificate that replaces it.

Analogy (heuristic, not claimed) (Hot early phase). ``A hot radiation era cools as the universe expands." Replacement: the prescribed prefix of Definition and the declared dictionary of Theorem , where aT_{r} is constant as an identity of declared labels.

Analogy (heuristic, not claimed) (Information left during the hot phase). ``Information emitted early is lost behind the late horizon." Replacement: Theorem , a replacement floor fixed by the causal relation and independent of every label.

Analogy (heuristic, not claimed) (Entropy handed from radiation to horizon). ``Early radiation entropy becomes late horizon entropy." Replacement: the negative certificates Theorem and Proposition .

Contribution and relation to sources

What is new

• *A code-capacity lemma with proof* (Lemma). Knill–Laflamme detection of every error on $\delta - 1$ sites implies decoupling of the reference, and subadditivity then gives the quantum Singleton bound. The subsystem logical factor obeys the same bound through a gauge-fixed subspace. This corrects the plan's ``$\log N - \Omega(d_{\mathrm{code}})$ from the monoid and commutation".

• *A sandwich lemma with exactness classes and a counterexample* (Lemma , Proposition , Proposition). This corrects the plan's ``exactness for every channel".

• *A placement certificate for the horizon floor* (Theorem). The least error over located circuits is the dark indicator times $1 - 1/d^2$. It is κ-independent and not a function of placement-blind data. This corrects the plan's ``constant on fibres of any map ignoring causal structure".

• *Junk sectors* (Theorem). Exact values and the correct direction of the floor chain. This corrects the plan's junk floor.

• *Dark events on comparison futures* (Theorem). This replaces the plan's undefined ``residual correlations differ".

• *Adopted from the user draft t272u* (Appendix): the access-relative definition of dark capacity, the coefficient and fibre propositions, the replacement dual certificate, the flagged m-cut lemma with its finite attainer, the same-temperature separation, the one-time-pad counterexample, the junk theorem, the futures theorem with analytic rigidity, the area proxies and inverse-source vacuity.

Source records

Big01-A version 5 [Big01A] is the parent of this sheet. Its site record (https://enter.grok.me/api/manuscript/Big01-A, retrieved 2026-10-23) is byte-identical to the local version 5 source. Other source records are the I-C family [IC,ICcert], the Hawk07 family [Hawk07], the de Sitter certificate sheets [DSScert], DS-OB-1, VC-OB-1 and PG-OB-1 [OBcert], and the sheets cited by the user draft [XIVKST,En02,Red06G,Red10A]. Citations identify source records only; no proof transfer. Every theorem of this sheet carries its own proof. A citation of the form ``parallels X label (source record; not a premise)" records only that a statement of the same shape appears in that record.

Changes from version 1

Version 2 is made under plan t273u, which is kept read-only as PLAN_t273u.md (sha256 b44b0a574e1363a12668c6690ad0c8c75d6f3a6cd74b771bfb1e7c25dae58fd8). Every statement, label and proof of version 1 is retained without change. The version-1 delivery is a frozen source record (sha256 in Appendix). The additions are:

• robustness subsections Sections , , and , with Theorems , , and and Propositions –;

• Section : results of the user's t275u draft that verified and do not overlap the robustness subsections (where they overlap, the stronger verified form was placed in those subsections);

• the cross-cutting Section : a category table, a schedule ledger, the separation from Page-curve readings and the emergence-page note;

• the non-claims of plan Section 8 (Section);

• the plan audit for t273u reconciled with t275u (Appendix), the verdict for each item of the t275u draft (Appendix), checks DC-17 to DC-31 (Appendix), new documentary edges (Appendix) and provenance (Appendix). Version 2 cites the version-6 certificate sheets of the I-C and Hawk07 families in addition to the version-5 sheets [ICcertv6]. All of them are source records; not premises.

Changes in version 3

Version 3 freezes version 2 (sha256 in Appendix) and inserts, by addition only:

• a heuristic early-universe narrative (Section), whose sentences point to certificates;

• Section : the early-universe comparison host and thermodynamic interface (plan t276u, items 2.1–2.6);

• Section : the four packaging items of plan t277u and the corpus expansions of plan t276u, Section 3;

• Section : two corrections to version-2 wording, recorded without editing version 2;

• Section : the version-3 non-claims and emergence-page sentences (not registered);

• new rows in the status, open-item and check tables (DC-32 to DC-43, in a single numbering);

• plan audits for t276u (Appendix) and t277u (Appendix), verdicts for the t279u drafts (Appendix), the check-number map (Appendix), documentary edges (Appendix) and provenance (Appendix). The newest delivered companions read as source records are Hawk07 v6, I-C v6, DSS-CERT v7 and OB v5. DSS-CERT v8 was in progress and was not read.

Changes in version 4

Version 4 freezes version 3 (sha256 in Appendix) and inserts, by addition only:

• Section : the Ky Fan branch bound, the variational formula, and the exact two-label law, including $(3, 2, 2)$ (targets P1 and P4);

• Section : the energy-constrained replacement floor (target P2);

• Section : the scaling of the approximate coding cost (target P3);

• Section : power-law prefixes, general-w labels, flip volumes and rate-bounded futures (target P5);

• Section : the re-check of version 3 (target P6; no erratum), and Section ;

• new rows in the status, open-item and check tables (DC-44 to DC-53);

• the derived agenda and its audit (Appendix), documentary edges (Appendix) and provenance (Appendix). No companion sheet was newly read for version 4. The documentary edges reuse labels verified in version 3.

Changes in version 5

Version 5 freezes version 4 (sha256 in Appendix) and inserts, by addition only:

• Section : decomposition criteria, the three-label classification, the finite linear and conic programs, and joint observers (plan item 1);

• Section : the exact all-energy replacement value and cutoff recovery (plan item 2);

• Section : the Bell-twirled Singleton bound and the metric of the allocation code (plan item 3);

• Section : several epochs, moduli, a benchmark volume and label invariants (plan item 4);

• Section : the spectator monoid and the new certificates (plan item 5); Sections and ;

• new rows in the status, open-item and check tables (DC-54 to DC-69);

• the plan audit (Appendix), the per-item verdicts on the user draft t285u (Appendix), documentary edges (Appendix) and provenance (Appendix). The plan's cited labels were checked in the read-only corpus copies, and nuADaM01-A version 1 was read from its delivery directory.

Changes in version 6

Version 6 freezes version 5 (sha256 in Appendix) and inserts, by addition only:

• Section : meeting subspaces, the value certificate, direct-sum constructions, and closed (L, m) values (plan item 3);

• Section : thermal operations at a common temperature, and the early-label separation for any system Hamiltonian (plan item 6);

• Sections and ;

• new rows in the status, open-item and check tables (DC-70 to DC-78);

• the plan audit (Appendix), documentary edges (Appendix) and provenance (Appendix). Plan items 2, 4 and 5 are DSS-sheet items and are placed in DSS-CERT. The plan's cited labels were checked in read-only corpus copies, and nuADaM01-A version 3 was read from its delivery directory.

Changes in version 7

Version 7 freezes version 6 (sha256 in Appendix) and inserts, by addition only:

• Sections –: accounts and fixed-account split fibres, the factorization lemma, split blindness with explicit fibres and the vacuum slot, the host qualification, future-horizon alignment with finite status ledgers and the drift identity, coefficient-blind adaptive experiments, rank versus selection, and documentary separations (plan t333u, items 1–4, integrated with the user draft t335u, whose section structure is followed);

• Section : the prediction interface, Theorems A–F of the plan t334u in corrected form, the optional closure package, observables, and the three-level table;

• Sections and , and a version-7 ledger paragraph after Section ;

• new rows in the status and check tables (DC-79 to DC-90); the open list is unchanged;

• the audits of the three inputs (Appendices , and), the label and citation verification (Appendix), the crosswalk (Appendix), documentary edges (Appendix), provenance (Appendix) and the standing ledger at the end (Appendix). Every cited label was re-read from https://enter.grok.me/api/manuscript/<ID> on 25 October 2026 (read-only; sha256 in Appendix). New citations are tagged ``source record; not a premise''.

Setting: prescribed host, named labels, declared registers

The prescribed static patch

Fix $H > 0$, $c > 0$ and $\ell = c/H$. Let $\mathrm{DSS}_H = \{X \in \mathbb{R}^{1,4} : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2\}$ carry the induced metric, time-oriented by ∂_{X_0}. The comoving observer is $\gamma(\tau) = (\ell\sinh(\tau/\ell), 0, 0, 0, \ell\cosh(\tau/\ell))$. Set $u = X_4 - X_0$; then $u(\gamma(\tau)) = \ell e^{-\tau/\ell} > 0$.

Lemma (Monotone null coordinate). *u is nonincreasing along every future-directed causal curve in* DSS_H. *Hence, if $u(p) \le 0$, then no future causal curve from p meets γ, and $p \notin J^-(\gamma)$.*

Proof. Let $n = (1, 0, 0, 0, 1)$, a future-directed null vector of $\mathbb{R}^{1,4}$. With $\langle V, W\rangle = -V_0 W_0 + \sum_{i \ge 1} V_i W_i$, we have $u(X) = \langle X, n\rangle$. A future-directed causal tangent vector V of DSS_H is a future-directed causal vector of $\mathbb{R}^{1,4}$. Two future-directed causal vectors have $\langle V, n\rangle \le 0$. Hence $\frac{\mathrm{d}}{\mathrm{d}s} u(c(s)) = \langle \dot{c}, n\rangle \le 0$. Since $u > 0$ on γ, a future causal curve starting at $u \le 0$ never reaches γ.

The converse, $J^-(\gamma) = \{u > 0\}$, parallels Big01-A v5 thm:A (source record; not a premise). It is not used below.

Named labels and the coefficient slot

The named labels carry a positive coefficient κ in the slot of $\hbar$:

$$T^{(\kappa)} = \frac{\kappa H}{2\pi k_B}, \qquad S^{(\kappa)} = \frac{\pi k_B c^5}{\kappa G H^2}, \qquad N^{(\kappa)} = \exp(S^{(\kappa)}/k_B).$$

The admitted data are the host (H, c), G, k_B, the declared data and every report. They do not contain κ. The usual T_{GH}, S_{GH} and N_{GH} are the values at $\kappa = \hbar$, and this sheet never supplies that value.

Lemma (Fibre lemma). *Let $F : \Omega \to \mathscr{Y}$ and $G : \Omega \to \mathbb{R}$. (a) There is g with $G = g \circ F$ if and only if G is constant on every fibre of F. (b) If $F(\omega_1) = F(\omega_2)$, then every $g : \mathscr{Y} \to \mathbb{R}$ has $\max_i |g(F(\omega_i)) - G(\omega_i)| \geq \frac{1}{2}|G(\omega_1) - G(\omega_2)|$. The midpoint attains equality on the pair. (c) If G is unbounded on one fibre, then every g has infinite worst-case error on that fibre.*

Proof. (a) If G is constant on fibres, set $g(y) = G(\omega)$ for any $\omega \in F^{-1}(y)$; the converse is immediate. (b) Write $y = F(\omega_i)$. Then $|G(\omega_1) - G(\omega_2)| \leq \sum_i |g(y) - G(\omega_i)| \leq 2\max_i |g(y) - G(\omega_i)|$. (c) follows from (b).

Part (a) parallels XIV-K-ST v8 thm:fibre and Big01-A v5 thm:fibre (source record; not a premise). par *Non-claim.* The lemma is pure logic. It identifies no physical parameter. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Coefficient certificate). *Fix all admitted data. Then $\partial_\kappa T^{(\kappa)} = H/(2\pi k_B) > 0$, $\partial_\kappa S^{(\kappa)} = -S^{(\kappa)}/\kappa < 0$ and $\partial_\kappa \ln N^{(\kappa)} = -S^{(\kappa)}/(k_B \kappa) < 0$. Each label is therefore injective in κ, and its range over $\kappa > 0$ is $(0, \infty)$ (for $N^{(\kappa)}$, the range is $(1, \infty)$). By Lemma , every rule built from admitted data has infinite worst-case error on each label. By contrast, the floors $1 - 1/d^2$, $1 - 1/d$ and $1 - m/d$ of Section have κ-derivative zero.*

Proof. Differentiate . The admitted data are constant on the family $\kappa \in (0, \infty)$, which is therefore contained in one fibre, and Lemma (c) applies. The floors depend only on d and m.

This parallels Big01-A v5 thm:Hcert and DS-OB-1-CERT v5 thm:DS-coef6 (source record; not a premise). Check DC-1. par *Non-claim.* No value of κ is supplied; in particular $\kappa = \hbar$ is not admitted. The labels are not temperatures or entropies of the host. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Declared registers, the promise, and dark capacity

Assumption (Optional capacity promise P_η). For a supplied $\eta > 0$ and a supplied κ, a declared register of n sites of dimension q satisfies $n \ln q \leq \eta S^{(\kappa)}/k_B$.

The promise is additional data. With unrestricted declared data, the register dimension is unbounded on a fixed host. This parallels Big01-A v5 thm:D (source record; not a premise), and Theorem (d) proves it again for codes of fixed distance.

Definition (Access-relative dark capacity; adopted from the t272u draft). A *logical account* specifies a declared finite register, an encoding, a transmission channel, the complete decoder access, and a target logical system. Relative to an account, *dark capacity* is any of the following: (D1) coding, syndrome or gauge allocations not counted as the target logical system; (D2) information excluded from the complete accessible channel; (D3) unused sectors or residual factors; (D4) information excluded from present access by a specified causal cut.

This is an operational classification. It is not a substance and not a canonical decomposition of a Hilbert space. The four categories need not be disjoint. Information outside a causal patch is not a summand of the present register.

Reorganizations

A *reorganization* is a map of declared data that fixes the host. Every host functional, including each label , is invariant under reorganizations, so reorganizations form a monoid under composition. The capacity-growing monoid $\rho_{n,\nu}(H, q, f) = (H, q + n, f + \nu)$ adjoins declared qubits and metadata. This parallels Big01-A v5 thm:D4 (source record; not a premise). It does *not* act by host-invariant unitaries, which is a plan correction (Appendix). The figure-of-merit ceiling is the fibre supremum, and with unrestricted declared data the declared dimension has infinite supremum. The draft's Proposition (reorganization ceiling) states the same thing, and we adopt it without change. par *Non-claim.* The reorganization monoid does not generate unitaries on a physical Hilbert space. Its ceiling belongs to a restricted declared class, not to the host. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Finite-channel tools

Conventions

For a channel T on M_d we use the half-diamond error $\varepsilon(T) = \frac{1}{2}\|T - \mathrm{id}\|_\diamond$. The Choi operator puts the output factor first: $J(T) = \sum_{ij} T(|i\rangle\langle j|) \otimes |i\rangle\langle j|$. The unnormalized vector is $\Omega = \sum_i |ii\rangle$, and $\varphi = \Omega/\sqrt{d}$. The entanglement fidelity is $F_e(T) = \langle\varphi|(T \otimes \mathrm{id})(\varphi\varphi^\dagger)|\varphi\rangle = \langle\Omega|J|\Omega\rangle/d^2$. For pure ψ we write $m_\psi(T) = \langle\psi|(T \otimes \mathrm{id})(\psi\psi^\dagger)|\psi\rangle$ and $m(T) = \max_\psi m_\psi(T)$, where ψ ranges over unit vectors of $\mathbb{C}^d \otimes \mathbb{C}^d$. The replacement channel with state σ is $\mathscr{R}_\sigma(X) = \mathrm{Tr}\,(X)\sigma$. The recovery error of a channel $\mathcal{N}$ is $\varepsilon^*(\mathcal{N}) = \inf_{\mathcal{D}} \varepsilon(\mathcal{D}\mathcal{N})$, with $\mathcal{D}$ ranging over all channels.

Lemma (Primal–dual sandwich). *Let T be a channel on M_d.*

• *(Primal.) For every unit $\psi \in \mathbb{C}^d \otimes \mathbb{C}^d$, $\varepsilon(T) \geq 1 - m_\psi(T)$.*

• *(Rank-one dual.) If $0 \leq t \leq 1$ and $J(T) - t\,\Omega\Omega^\dagger \geq 0$, then $T = t\,\mathrm{id} + (1-t)T'$ for a channel T' (any channel if $t = 1$), and $\varepsilon(T) \leq 1 - t$. The rank-one dual witness is $(1-t)\Omega$.*

• *Let $t^*(T) = \max\{t \in [0,1] : J(T) \geq t\,\Omega\Omega^\dagger\}$. Then $t^* \leq F_e \leq m$ and $1 - m \leq \varepsilon(T) \leq 1 - t^*$. If $t^* = m_\psi$ for some ψ, then $\varepsilon(T) = 1 - t^*$ (exactness).*

• *If $\Omega \in \mathrm{ran}\,J$, then $t^* = \min\{1, 1/\langle\Omega|J^+|\Omega\rangle\}$; otherwise $t^* = 0$.*

Proof. (a) Let $\rho = (T \otimes \mathrm{id})(\psi\psi^\dagger)$. Then $\frac{1}{2}\|T - \mathrm{id}\|_\diamond \geq \frac{1}{2}\|\rho - \psi\psi^\dagger\|_1 \geq \mathrm{Tr}\,[\psi\psi^\dagger(\psi\psi^\dagger - \rho)] = 1 - m_\psi$. The last step uses $\frac{1}{2}\|X\|_1 \geq \mathrm{Tr}\,(PX)$ for traceless Hermitian X and a projector P. (b) If $t < 1$, set $J' = (J - t\Omega\Omega^\dagger)/(1-t) \geq 0$. Since $\mathrm{Tr}_{\mathrm{out}}\,J = I$ and $\mathrm{Tr}_{\mathrm{out}}\,\Omega\Omega^\dagger = I$, we get $\mathrm{Tr}_{\mathrm{out}}\,J' = I$, so J' is the Choi operator of a channel T'. Then $T - \mathrm{id} = (1-t)(T' - \mathrm{id})$ and $\|T' - \mathrm{id}\|_\diamond \leq 2$. (c) If $J \geq t\Omega\Omega^\dagger$, then $\langle\Omega|J|\Omega\rangle \geq td^2$, so $F_e \geq t^*$. Also $F_e = m_\varphi \leq m$. The two bounds follow from (a) and (b). (d) Positive semidefiniteness of $J - t\Omega\Omega^\dagger$ is equivalent to $\Omega \in \mathrm{ran}\,J$ and $t\langle\Omega|J^+|\Omega\rangle \leq 1$ (Schur complement).

This parallels Hawk07-A-CERT v5 lem:v2-sandwich, Hawk07-A-CERT v5 prop:v4-rankone, I-C-Ex-CERT v5 thm:v4-rankone and En08-ST-DSS-CERT v6 lem:EC-sandwich (source record; not a premise). The t272u draft states a different dual form ($Y \pm J \geq 0 \Rightarrow \|L\|_\diamond \leq \|\mathrm{Tr}_{\mathrm{out}}\,Y\|$), which in general rests on the semidefinite program of [Watrous2009]. Lemma is elementary and is the only form used here. par *Non-claim.* Exactness is claimed only on the classes of Proposition , not for every channel. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Exactness classes). *Lemma is exact, with $\varepsilon(T) = 1 - t^*(T)$, in each of the following cases.*

• *Replacement: $T = \mathscr{R}_\sigma$ with $\sigma > 0$ and $S_\sigma = \mathrm{Tr}\,\sigma^{-1}$. Then $t^* = 1/S_\sigma$ and $\varepsilon = 1 - 1/S_\sigma \geq 1 - 1/d^2$, with equality iff $\sigma = I/d$. If σ is singular, $\varepsilon = 1$.*

• *Block pinching $\Delta(X) = \sum_{i=1}^m P_i X P_i$ with ranks k_i (orthogonal projectors summing to I, possibly unequal ranks): $t^* = 1/m$ and $\varepsilon = 1 - 1/m$. The case $m = d$ is complete dephasing, with error $1 - 1/d$.*

• *Isotropic: $T_{d,k} = \alpha\,\mathrm{id} + (1-\alpha)\mathscr{R}_{I/d}$ with $\alpha = (kd-1)/(d^2-1)$ and $1 \leq k \leq d$: $t^* = F_e = k/d$ and $\varepsilon = 1 - k/d$.*

• *Flagged-cut attainer (t272u draft): $\Lambda_m(X) = \frac{d-1}{m-1}^{-1} \sum_{|S|=m} P_S X P_S$, where P_S is the coordinate projector onto $S \subseteq \{1,\ldots,d\}$. Then $\Lambda_m = \alpha\,\mathrm{id} + (1-\alpha)\Delta_d$ with $\alpha = (m-1)/(d-1)$, where Δ_d is complete dephasing; moreover $t^* = F_e = m/d$ and $\varepsilon = 1 - m/d$.*

Proof. (a) Here $J = \sigma \otimes I$. If $\sigma > 0$, then $\langle\Omega|(\sigma^{-1} \otimes I)|\Omega\rangle = \mathrm{Tr}\,\sigma^{-1}$, so $t^* = 1/S_\sigma$ by Lemma (d). For the primal, take $\psi = (A \otimes I)\Omega$ with $A = \sigma^{-1/2}/\sqrt{S_\sigma}$. It has unit norm, and its reference marginal is $\tau = (A^\dagger A)^T = (\sigma^{-1})^T/S_\sigma$. The output is $\sigma \otimes \tau$, and with $\langle\Omega|X \otimes Y|\Omega\rangle = \mathrm{Tr}\,(XY^T)$ we get $m_\psi = \mathrm{Tr}\,(A\sigma A\,(\sigma^{-1}/S_\sigma)) = \mathrm{Tr}\,(\sigma^{-1})/S_\sigma^2 = 1/S_\sigma$. By Cauchy–Schwarz,

$d^2 = (\sum_i \lambda_i^{1/2}\lambda_i^{-1/2})^2 \le \sum_i \lambda_i \sum_i \lambda_i^{-1} = S_\sigma$, with equality iff all eigenvalues are equal. If σ is singular, take $\psi = v \otimes w$ with a unit $v \in \ker\sigma$ and any unit w. The output is $\sigma \otimes ww^\dagger$, so $m_\psi = \langle v|\sigma|v\rangle = 0$ and $\varepsilon \ge 1$; always $\varepsilon \le 1$. (b) Choose a basis adapted to the blocks and let $\Omega_i = \sum_{a \in \text{block}i} |aa\rangle$, so $\|\Omega_i\|^2 = k_i$. Then $J = \sum_i \Omega_i\Omega_i^\dagger$ with orthogonal Ω_i, and $\Omega = \sum_i \Omega_i$. So $\langle\Omega|J^+|\Omega\rangle = \sum_i k_i^2/k_i^2 = m$, and $t^* = 1/m$. For the primal, take $\psi = \sum_i (mk_i)^{-1/2}\Omega_i$, which has unit norm. The cross terms between blocks are annihilated, so $(\Delta \otimes \mathrm{id})(\psi\psi^\dagger) = \sum_i (mk_i)^{-1}\Omega_i\Omega_i^\dagger$ and $m_\psi = \sum_i (mk_i)^{-1}|\langle\psi, \Omega_i\rangle|^2 = \sum_i (mk_i)^{-2}k_i^2 = 1/m$. (c) $J = \alpha\Omega\Omega^\dagger + (1-\alpha)(I \otimes I)/d$, and $J - t\Omega\Omega^\dagger \ge 0$ iff $(\alpha - t)d + (1-\alpha)/d \ge 0$. Hence $t^* = \alpha + (1-\alpha)/d^2 = F_e = k/d$, and the primal φ attains it. The eigenvalues of J are $(1-\alpha)/d \ge 0$ and $\alpha d + (1-\alpha)/d$, and the second is nonnegative because $\alpha \ge -1/(d^2-1)$, so $T_{d,k}$ is a channel. (d) Each i lies in $\frac{d-1}{m-1}$ of the sets S, and each pair $i \ne j$ lies in $\frac{d-2}{m-2}$ of them. Hence $\Lambda_m(|i\rangle\langle j|) = \alpha|i\rangle\langle j|$ for $i \ne j$ and $\Lambda_m(|i\rangle\langle i|) = |i\rangle\langle i|$, which gives $\Lambda_m = \alpha\mathrm{id} + (1-\alpha)\Delta_d$. Then $J = \alpha\Omega\Omega^\dagger + (1-\alpha)D$, where $D = \sum_i |ii\rangle\langle ii|$. On span $\{|ii\rangle\}$, D is the identity, and $J - t\Omega\Omega^\dagger \ge 0$ iff $(\alpha - t)d + (1-\alpha) \ge 0$. So $t^* = \alpha + (1-\alpha)/d = m/d = F_e$.

Part (a) parallels Big01-A v5 thm:Bexact, Hawk07-A-CERT v5 thm:s2-exact and Red06-G-DSS-CERT v6 thm:RC-repdual; part (b) parallels I-C-Ex-CERT v5 prop:v5-pinch; part (c) parallels I-C-Ex-CERT v5 lem:attainer; all (source record; not a premise). Checks DC-3, DC-4 and DC-13. par *Non-claim.* These are statements about declared finite comparison channels, not about a host-generated channel. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (The sandwich is not exact on every channel). *Let A_γ be amplitude damping with Kraus operators $K_0 = \mathrm{diag}(1, \sqrt{1-\gamma})$ and $K_1 = \sqrt{\gamma}|0\rangle\langle 1|$, with $0 < \gamma < 1$. Then $t^*(A_\gamma) = 0$, yet $\varepsilon(A_\gamma) \le \gamma/2 + 1 - \sqrt{1-\gamma} < 1$ for $0 < \gamma \le \frac{1}{2}$. Hence $\varepsilon \ne 1 - t^*$.*

Proof. Here $J = vv^\dagger + \gamma|01\rangle\langle 01|$ with $v = |00\rangle + \sqrt{1-\gamma}|11\rangle$, so $\mathrm{ran}\, J = \mathrm{span}\{v, |01\rangle\}$. This does not contain $\Omega = |00\rangle + |11\rangle$ when $\gamma > 0$, so $t^* = 0$ by Lemma (d). For any ρ on the system and a reference, write $\widetilde{K} = K \otimes I$. Then $\widetilde{K}_0\rho\widetilde{K}_0^\dagger - \rho = (\widetilde{K}_0 - I)\rho\widetilde{K}_0^\dagger + \rho(\widetilde{K}_0 - I)^\dagger$, which has trace norm at most $2\|K_0 - I\| = 2(1 - \sqrt{1-\gamma})$. Also $\|\widetilde{K}_1\rho\widetilde{K}_1^\dagger\|_1 \le \gamma$. At $\gamma = \frac{1}{2}$ the bound is $0.543 < 1$, and it increases with γ.

A semidefinite computation gives $\varepsilon(A_\gamma) = \gamma$ (DC-6). This is a computed value, not proved here. The plan's ``strengthen the dual-certificate exactness to every channel'' is therefore false. Exactness over every *decoder* after a replacement channel is Proposition (a), which parallels Big01-A v5 thm:Bexact (source record; not a premise). par *Non-claim.* No claim is made about amplitude damping beyond the stated inequalities. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Lemma (Flagged *m*-cut floor; adopted from the t272u draft). *Suppose every message-bearing quantum access passes through an m-dimensional quantum system C, and all other accessible data form a finite classical register X. That is, $T = \mathscr{D} \circ \mathscr{E}$ with $\mathscr{E}(\rho) = \sum_x |x\rangle\langle x| \otimes \mathscr{E}_x(\rho)$, where each $\mathscr{E}_x : M_d \to M_m$ is completely positive and $\sum_x \mathscr{E}_x$ is trace preserving. Then $\varepsilon(T) \ge 1 - m/d$ for every decoder $\mathscr{D}$, with equality for Λ_m of Proposition (d).*

Proof. $(\mathscr{E} \otimes \mathrm{id})(\varphi\varphi^\dagger) = \sum_x |x\rangle\langle x| \otimes \omega_x$, where each ω_x is a positive operator on $\mathbb{C}^m \otimes \mathbb{C}^d$. It is a sum of pure terms of Schmidt rank at most m. Applying $\mathscr{D}$ (which reads x) and local Kraus operators does not raise the Schmidt rank of any pure term. For a pure χ with Schmidt coefficients $\mu_1, \ldots, \mu_r$ ($r \le m$), $|\mathrm{ip}\,\varphi\chi|^2 \le (\sum_i \sqrt{\mu_i})^2/d \le r/d$. So $F_e(T) \le m/d$, and Lemma (a) gives the bound. Λ_m factors through such a cut: the instrument $\{P_S/\sqrt{\frac{d-1}{m-1}}\}_S$ records S classically and leaves an m-dimensional quantum state. Proposition (d) gives the value.

For $m = 1$, this is the classical-cut floor $1 - 1/d$, which parallels I-C v2 thm:D (site record) and I-C-CERT v5

lem:floor (source record; not a premise). For general m, the I-C-Ex2 dimension floor $\varphi(d,k) = 1 - k/d$ of I-C-Ex2-CERT v5 prop:v4-dim is parallel (source record; not a premise). Check DC-13. par *Non-claim.* The cut is a declared access model, not a derived property of the patch. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Located circuits and the ancestor lemma

Definition (Located circuit). A *located circuit* on DSS_H is a finite circuit of channels. Each gate has a spacetime position, and each wire from a gate at p to a gate at p' requires $p' \in J^+(p)$. The source is a gate at s that inserts the payload. The receivers are gates on γ. The *receiver channel* maps the payload to the joint state of all receiver outputs.

Lemma (Ancestor lemma). *If no receiver lies in $J^+(s)$, then the receiver channel is a replacement channel $\mathscr{R}_\sigma$ for some state σ, whatever the circuit.*

Proof. Call a gate a *descendant* if it is the source or receives a wire from a descendant. Since wires respect J^+, every descendant lies in $J^+(s)$, so no receiver is a descendant. A non-descendant receives no wire from a descendant. Schedule all non-descendant gates first and the descendants afterwards; this respects the wire order. After the first stage, the state is $\omega \otimes \rho$, where ω does not depend on the payload ρ and the receiver outputs are already among the wires of ω. The second stage applies a trace-preserving map to ρ and to some wires of ω that are not receiver outputs, and all its outputs are traced. Trace preservation shows that the receiver marginal equals $\mathrm{Tr}\,(\rho)\,\mathrm{Tr}_{\mathrm{rest}}\,\omega =: \mathrm{Tr}\,(\rho)\sigma$.

With Lemma : if $u(s) \le 0$, then $J^+(s) \cap \gamma = \varnothing$ and the receiver channel is a replacement. This parallels Big01-A v5 thm:B (source record; not a premise). par *Non-claim.* Located circuits are declared finite comparison objects. No field theory or host-generated channel is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Main Theorem I: coding redundancy and gauge allocations

A *declared factorization* is a register $(\mathbb{C}^q)^{\otimes n}$ of n sites. A *subspace code* is a subspace $\mathscr{C}$ of dimension $K \ge 2$ with projector P. It *has distance δ* (Knill–Laflamme form [KL1997]) if $PEP = c(E)P$ for every operator E supported on at most $\delta - 1$ sites. Equivalently, it corrects the erasure of any $\delta - 1$ sites. A *subsystem code* is a subspace $\mathscr{H}_L \otimes \mathscr{H}_G$ with $\dim \mathscr{H}_L = k$ and $\dim \mathscr{H}_G = r$. It has distance δ if $PEP = I_L \otimes G_E$ for every such E.

Lemma (Knill–Laflamme decoupling). *Let $\mathscr{C}$ have distance δ. Let $|\psi\rangle = K^{-1/2}\sum_i |i\rangle_R |c_i\rangle$, where (c_i) is an orthonormal basis of $\mathscr{C}$, and let A be any set of at most $\delta - 1$ sites. Then $\rho_{RA} = \rho_R \otimes \rho_A$ with $\rho_R = I/K$. Moreover, every code state has the same marginal on A.*

Proof. For F on R and E on A: $\langle\psi|F \otimes E|\psi\rangle = K^{-1}\sum_{ij} F_{ij}\langle c_i \mathrm{rvertE}\, c_j\rangle = c(E)\mathrm{Tr}\,F/K$. Taking $F = I$ gives $\mathrm{Tr}\,(\rho_A E) = c(E)$, so the expectation factorizes. For a code state $\rho = P\rho P$, $\mathrm{Tr}\,(\rho E) = \mathrm{Tr}\,(\rho PEP) = c(E)$.

Lemma (Quantum Singleton bound). *If a subspace code on $(\mathbb{C}^q)^{\otimes n}$ has distance δ and $K \ge 2$, then $n \ge 2(\delta - 1) + 1$ and $\ln K \le (n - 2(\delta - 1))\ln q$. The same bound holds for the logical dimension k of a subsystem code of distance δ.*

Proof. If $\delta = 1$, then $K \le q^n$. If $n \le \delta - 1$, Lemma with A the whole register gives $\rho_R \otimes \rho_A$ pure, so $K = 1$. Otherwise, suppose first that $n \ge 2(\delta - 1)$, and split the sites into disjoint sets A and B of $\delta - 1$ sites each and the rest C. The state ψ of Lemma on $RABC$ is pure, so $S(RA) = S(BC)$ and $S(RB) = S(AC)$. By Lemma , $S(RA) = S(R) + S(A)$ and $S(RB) = S(R) + S(B)$. Subadditivity gives $S(R) + S(A) \le S(B) + S(C)$ and $S(R) + S(B) \le S(A) + S(C)$. Adding the two, $\ln K = S(R) \le S(C) \le |C| \ln q$. If C were empty, we would get $K \le 1$, a contradiction; and if $n < 2(\delta - 1)$, take A of size $\delta - 1$ and B the rest, so that $|B| \le \delta - 1$, and the same argument with $C = \varnothing$ gives $K \le 1$. For a subsystem code, fix a unit $|g\rangle \in \mathscr{H}_G$ and let P_g project onto $\mathscr{H}_L \otimes |g\rangle$. Then $P_g E P_g = P_g(I_L \otimes G_E)P_g = \langle g|G_E|g\rangle\, P_g$, so $\mathscr{H}_L \otimes |g\rangle$ is a subspace code of dimension k and distance δ.

This is the standard quantum Singleton bound (cf. [KL1997,Rains1999]); the proof above is self-contained. A

bound on $k + r$ is *not* claimed: impure subsystem codes can exceed such bounds [KS2007]. Check DC-2 verifies decoupling and attainment for [[5, 1, 3]], [[4, 2, 2]] and [[6, 4, 2]].

Main Theorem (Redundancy). *Fix a declared factorization and a code of distance δ.*

• *(Dimension count; t272u draft.) If a declared register of dimension D decomposes orthogonally as $(\mathscr{H}_L \otimes \mathscr{H}_G) \oplus \mathscr{H}_J$ with dimensions k, r and j, then $D = kr + j$ and $\ln k \le \ln D - \ln r$. If s mutually orthogonal syndrome sectors each contain a copy of $\mathscr{H}_L \otimes \mathscr{H}_G$, then $skr \le D$.*

• *(Code-capacity lemma.)* $\ln K \le (n - 2(\delta - 1))\ln q$. *The dimension deficit* $\ln(q^n/K)$ *is at least* $2(\delta - 1)\ln q$, *and the same holds for the logical factor of a subsystem code.*

• *(Under the promise* P_η*.)* $\ln K \le \eta S^{(\kappa)}/k_B - 2(\delta - 1)\ln q$, *and in (a),* $\ln k + \ln r (+\ln s) \le \eta S^{(\kappa)}/k_B$.

• *(No promise, no ceiling.) For every even $n \ge 4$, the qubit code fixed by $X^{\otimes n}$ and $Z^{\otimes n}$ has $K = 2^{n-2}$ and distance 2, and attains (b). On a fixed host, K is therefore unbounded at fixed distance.*

• *(Redundancy cannot cancel a replacement.) For every encoding channel $\mathscr{V}: M_k \to M_D$ and every replacement $\mathscr{R}_\sigma$, we have $\mathscr{R}_\sigma \circ \mathscr{V} = \mathscr{R}_\sigma$, and $\varepsilon^*(\mathscr{R}_\sigma \circ \mathscr{V}) = 1 - 1/k^2$.*

• *(Invisibility below the distance.) Every code state has the same marginal on any $\delta - 1$ sites.*

Proof. (a) Count dimensions of orthogonal summands; the syndrome sectors are orthogonal subspaces of dimension kr each. (b) is Lemma . (c) Combine (b) with $n\ln q \le \eta S^{(\kappa)}/k_B$, and (a) with $\ln D \le \eta S^{(\kappa)}/k_B$. (d) For even n the two stabilizers commute, and their joint +1 eigenspace has dimension $2^n/4$. Each single-site Pauli $E \ne I$ anticommutes with at least one stabilizer, so $PEP = 0$. Every single-site operator is a combination of Paulis, so $PEP = c(E)P$. Then $\log_2 K = n - 2 = n - 2(\delta - 1)$. (e) $\mathscr{R}_\sigma(\mathscr{V}(X)) = \mathrm{Tr}(\mathscr{V}(X))\sigma = \mathrm{Tr}(X)\sigma$. For every decoder $\mathscr{D}$, $\mathscr{D}\mathscr{R}_\sigma = \mathscr{R}_{\mathscr{D}(\sigma)}$, and Proposition (a) gives the minimum $1 - 1/k^2$ at $\mathscr{D}(\sigma) = I/k$. (f) is Lemma .

Status: proved, conditional on the declared factorization and the Knill–Laflamme definition of distance. Part (c) is also conditional on the supplied promise. Part (b) corrects the plan's ``$\log N - \Omega(d_{\mathrm{code}})$ from the monoid and commutation'' (Appendix). Part (e) parallels Big01-A v5 thm:Bexact and PG-OB-1-CERT v5 cor:PG-ceiling6 (source record; not a premise). Checks DC-2, DC-3 and DC-11. par *Non-claim.* No holographic code, bulk reconstruction or physical overhead law is derived. A ``fraction'' of dark sites is $2(\delta - 1)/n$ only relative to a declared n, and it need not be large. The labels enter only through the supplied promise. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Robustness and expansions (version 2)

For an operator set $\mathscr{E}$ on $(\mathbb{C}^q)^{\otimes n}$, a code with projector P is *$\mathscr{E}$-correcting* if $PE^\dagger FP = c(E,F)P$ for all $E, F \in \mathscr{E}$ (Knill–Laflamme [KL1997]). Let $\mathscr{E}_t$ be the operators of weight at most t. For a set A of sites, $M(A)$ denotes all operators supported on A. We write $h(x) = -x\ln x - (1 - x)\ln(1 - x)$ and $g(\varepsilon) = (1 + \varepsilon)h(\varepsilon/(1 + \varepsilon))$.

Theorem (I-R: weight-t errors, general operator sets, approximate correctability). *Let a subspace code of dimension $K \ge 2$ on $(\mathbb{C}^q)^{\otimes n}$ be given.*

• *If it is $\mathscr{E}_t$-correcting, then it has distance at least $2t + 1$, and $\ln K \le (n - 4t)\ln q$. If it corrects the erasure of every set of t sites, then $\ln K \le (n - 2t)\ln q$. For weight-t errors the second form is valid but not sharp:* [[5, 1, 3]] *with $t = 1$ attains $n - 4t = 1$, while $n - 2t = 3$.*

• *If it is $\mathscr{E}$-correcting, and A, B are disjoint site sets with $M(A) \cup M(B) \subseteq \mathrm{span}\{E^\dagger F : E, F \in \mathscr{E}\}$, then $\ln K \le \ln \dim \mathscr{H}_C$, where C is the complement of $A \cup B$.*

• *(Approximate correctability; heterogeneous form, adopted from the t275u draft.) Let $V: \mathbb{C}^K \to \mathscr{H}_A \otimes \mathscr{H}_B \otimes \mathscr{H}_C$ be an isometry (site dimensions may differ) and $\psi = K^{-1/2}\sum_i |i\rangle_R V|i\rangle$. Suppose there are channels $\mathscr{D}_A$ on BC and $\mathscr{D}_B$ on AC such that $\frac{1}{2}\|(\mathrm{id}_R \otimes \mathscr{D}_X)(\psi_{R\bar{X}}) - \Phi_K\|_1 \le \varepsilon_X \le 1$ for $X = A, B$; this is implied by half-diamond recovery errors*

$\varepsilon_A, \varepsilon_B$. *Then*

$$(1-\varepsilon_A-\varepsilon_B)\ln K \le \ln\dim \mathscr{H}_C + \tfrac{1}{2}(g(\varepsilon_A)+g(\varepsilon_B)).$$

For q-ary sites with $\varepsilon_A, \varepsilon_B \le \varepsilon < \frac{1}{2}$ *this gives* $\ln K \le (|C|\ln q + g(\varepsilon))/(1-2\varepsilon)$. *The deviation from* $|C|\ln q$ *is at most* $(2\varepsilon|C|\ln q + g(\varepsilon))/(1-2\varepsilon)$. *This consists of a term of order* $\varepsilon\ln(1/\varepsilon)$ *and a term* $2\varepsilon\ln K$ *that grows with the dimension. Moreover, there are states* τ_A, τ_B *with* $\frac{1}{2}\|\psi_{RX} - I_R/K \otimes \tau_X\|_1 \le \sqrt{\varepsilon}_X$ *for* $X = A, B$ *(approximate decoupling). Consequently* $\frac{1}{2}\|\psi_{RX} - I_R/K \otimes \psi_X\|_1 \le 2\sqrt{\varepsilon}_X$, *by the triangle inequality and contraction of the trace norm under* Tr_R.

Proof. (a) If $|\mathrm{supp}\, O| \le 2t$, split the support into two sets of at most t sites each. A product basis of $M(\mathrm{supp}\, O)$ consists of products $E^\dagger F$ with $E, F \in \mathscr{E}_t$, so $PEP = c(E)P$ for every E of weight $\le 2t$. That is distance $\ge 2t+1$, and Lemma with $\delta - 1 = 2t$ gives $n - 4t$. For erasures, the distance is $\ge t+1$. For $[[5,1,3]]$, the KL condition at weight ≤ 2 is check DC-22. (b) The proof of Lemma uses only $PEP = c(E)P$ for $E \in M(A)$, so $\rho_{RA} = \rho_R \otimes \rho_A$, and likewise for B. The entropy argument of Lemma gives $\ln K \le S(C) \le \ln\dim \mathscr{H}_C$. (c) Let $\omega = (\mathrm{id}_R \otimes \mathscr{D}_A)(\psi_{R\bar{A}})$. The R-marginal of ω is I/K. The Alicki–Fannes–Winter inequality [Winter2016] gives $|S(R|R')_\omega - S(R|R')_{\Phi_K}| \le 2\varepsilon_A \ln K + g(\varepsilon_A)$. Since $S(R|R')_{\Phi_K} = -\ln K$, we get $I(R:R')_\omega \ge 2\ln K - 2\varepsilon_A \ln K - g(\varepsilon_A)$. Data processing of mutual information gives $I(R:\bar{A})_\psi \ge I(R:R')_\omega$. For the pure state ψ_{RABC}, $I(R:A) + I(R:\bar{A}) = 2S(R) = 2\ln K$. Hence $I(R:A) \le 2\varepsilon_A \ln K + g(\varepsilon_A)$, and likewise for B. Now $S(RA) = S(BC) \le S(B) + S(C)$ and $S(RA) = S(R) + S(A) - I(R:A)$, and symmetrically for B, with ε_B in place of ε_A. Adding the two inequalities, $2\ln K \le 2S(C) + I(R:A) + I(R:B) \le 2\ln\dim \mathscr{H}_C + 2(\varepsilon_A+\varepsilon_B)\ln K + g(\varepsilon_A) + g(\varepsilon_B)$, which rearranges to . For approximate decoupling, dilate $\mathscr{D}_A$ to an isometry with environment F. The output Ξ_{RLFA} is pure and has Bell weight $\langle\Phi_K|\Xi_{RL}|\Phi_K\rangle \ge 1 - \varepsilon_A$. Its normalized projection onto $\Phi_K \otimes (\cdot)_{FA}$ is a product $\Phi_K \otimes \chi_{FA}$ at pure-state trace distance $\le \sqrt{\varepsilon}_A$. Tracing out L and F contracts the trace distance and gives $\tau_A = \mathrm{Tr}_F \chi_{FA}$.

Status: (a) and (b) proved. (c) proved, using two cited inequalities that are not reproved here: the Alicki–Fannes–Winter bound [Winter2016] and data processing for mutual information [Watrous2018]. At $\varepsilon = 0$, (c) reduces to Lemma . Check DC-17 tests the decoupling step and on perturbed $[[3,1,2]]_3$ codes with Petz recovery, check DC-31 tests the heterogeneous form and approximate decoupling, and check DC-22 tests (a). Parts (a)–(c) coincide with Theorems thm:operatorKL and thm:approxSingleton of the t275u draft (Appendix); the statement kept here is the stronger common form. The same conclusions hold for a subsystem code with $PE^\dagger FP = I_L \otimes G_{E,F}$, by fixing a gauge vector. par *Non-claim.* The bound concerns declared factorizations. No physical error model, threshold or overhead law is derived. The plan's ``additive $O(\varepsilon\log(1/\varepsilon))$'' is corrected to . Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (I-G: gauge-sector entropy ledger). *For a subsystem code* $(\mathscr{H}_L \otimes \mathscr{H}_G) \oplus \mathscr{H}_J$ *and a state* $\omega = (1-p)(\rho_L \otimes \gamma_G) \oplus p\tau_J$, *the entropy is given by , and the gauge contribution satisfies* $0 \le S(\gamma_G) \le \ln r$. *Recovery of* ρ_L *is exact for every fixed* γ_G *when* $p = 0$ *(Theorem (b)), and the recovery error does not depend on* γ_G. *The Singleton bound constrains k alone (Lemma); only* $kr \le q^n$ *constrains r. (Correlated ledger, adopted from the t275u draft.) For an isometry* $W : \mathscr{H}_L \otimes \mathscr{H}_G \to \mathscr{H}_P$ *and any joint state* ρ_{LG}, $S(W\rho_{LG}W^\dagger) = S(\rho_L) + S(\rho_G) - I(L:G)_\rho$. *For a classically flagged allocation* $\sum_x p_x \mathrm{proj}\, x \otimes W_x \rho^x_{LG} W_x^\dagger$ *the entropy is* $H(p) + \sum_x p_x (S(\rho^x_L) + S(\rho^x_G) - I(L:G)_{\rho^x})$. *If the decoded noise is* $\mathrm{id}_L \otimes \mathscr{F}_G$, *the logical system and its reference correlations are returned exactly. Recovery of all of LG needs a left inverse of* $\mathscr{F}_G$, *which logical recovery does not supply.*

Proof. The blockwise eigenvalue count of , and $S \le \ln(\mathrm{rank})$. The decoder of Theorem (b) traces $\mathscr{H}_G$, so γ_G never enters. Isometries preserve nonzero eigenvalues, which gives the correlated ledger; orthogonal flags add $H(p)$. Trace preservation of $\mathscr{F}_G$ leaves every RL marginal unchanged. Replacement of G with $\dim G > 1$ shows that

full-gauge recovery can fail.

par *Non-claim.* Gauge entropy is an entropy of a declared register state, not of a patch. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (I-CV: infinite-dimensional sites). *(a) Suppose the code lies in* $\mathrm{bigotimes}_i \mathscr{F}_i$ *with* $\dim \mathscr{F}_i \le q$ *(a finite-effective-dimension promise), and the KL condition holds for all operators on sets of at most* $\delta - 1$ *sites. Then* $\ln K \le (n - 2(\delta-1))\ln q$*. (b) (Partial.) Suppose every code state has mean excitation number at most* $\overline{N}$ *in each bosonic mode, and the KL condition holds for operators on sets of at most* $\delta - 1$ *modes. Then* $\ln K \le (n - 2(\delta-1))\,((\overline{N}+1)\ln(\overline{N}+1) - \overline{N}\ln\overline{N})$.

Proof. (a) The proof of Lemma applies, since all marginals of ψ are supported on the finite factors. (b) The maximally mixed code state obeys the same mean-number bound by linearity. The entropy of each single-mode marginal is at most that of the thermal state with the same mean number, and subadditivity gives $S(C) \le |C|\,(\overline{N}+1)\ln(\overline{N}+1) - |C|\,\overline{N}\ln\overline{N}$. The infinite-dimensional facts used (finiteness of the entropies, subadditivity, and the maximum-entropy property of thermal states) are cited and not reproved, so (b) is partial.

Hard cutoffs, soft cutoffs, energy-constrained floors and a global Gibbs bound are in Theorem . A fixed number of bosonic modes without a cutoff is not a finite dimension. par *Non-claim.* The energy promise is supplied. No energy scale is derived from the host. Parts of the plan cite ``energy-constrained statements in the Hawk07 family''; none was found (Appendix). Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (I-Rep: no code improves on an approximate replacement). *Let* $\mathscr{V}: M_k \to M_D$ *be any encoding,* $\mathscr{N}$ *a channel on* M_D *with* $\frac{1}{2}\|\mathscr{N} - \mathscr{R}_\sigma\|_\diamond \le \varepsilon$*, and* $\mathscr{D}$ *any decoder. Then* $\varepsilon(\mathscr{D}\mathscr{N}\mathscr{V}) \ge 1 - 1/k^2 - \varepsilon$*, and the optimized error of* $\mathscr{N}\mathscr{V}$ *is at most* $1 - 1/k^2$ *(decoder preparing I/k). This matches the horizon floor of Theorem (a), and coincides with Corollary cor:encodedreplacement of the t275u draft.*

Proof. $\mathscr{D}\mathscr{R}_\sigma\mathscr{V} = \mathscr{R}_{\mathscr{D}(\sigma)}$ on M_k, and its error is at least $1 - 1/k^2$. The diamond norm is contractive under composition with channels, so $\frac{1}{2}\|\mathscr{D}(\mathscr{N} - \mathscr{R}_\sigma)\mathscr{V}\|_\diamond \le \varepsilon$. Apply the triangle inequality.

Proposition (I-M: cost of redundancy under composition). *For a code* C *on n sites of dimension q, set* $\mathrm{cost(C)} = n\ln q - \ln K \ge 0$.

• *Cost is additive under tensor products of codes.*

• $\mathrm{cost(C)} \ge 2(\delta-1)\ln q$.

• *Adjoining m idle sites in a fixed product state keeps the distance and adds* $m\ln q$ *to the cost. The overhead* $2(\delta-1)\ln q$ *is additive in* $\ln K$*; in K it is the factor* $q^{-2(\delta-1)}$*. The plan's ``multiplicative cost* $2(\delta-1)\ln q$*'' is corrected accordingly.*

Proof. (a) $\ln(K_1K_2)$ and $n_1 + n_2$ add. (b) is Lemma . (c) For E on the old sites and the new sites, $(P \otimes |0\rangle\langle 0|^{\otimes m})E(P \otimes \mathrm{proj}\,0^{\otimes m})$ is $\widetilde{P}\widetilde{E}\widetilde{P} \otimes \mathrm{proj}\,0^{\otimes m}$, where $\widetilde{E}$ is the partial expectation in $|0\rangle^{\otimes m}$. $\widetilde{E}$ is supported on the old part of the support of E, so the KL condition is inherited.

This is a monoid of codes under $\otimes$ with an additive cost. It is not the declared-data monoid Big01-A v5 thm:D4 (source record; not a premise), which adjoins declared qubits and does not use subadditivity (Appendix , T4). par *Non-claim.* Costs are counted in declared dimensions; no physical resource price is assigned. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Main Theorem II: complete causal-access loss

Payloads have dimension $d \ge 2$. A decoder acts on the receiver outputs and returns M_d. For a source position s, let $\varepsilon_{\min}(s)$ be the infimum of ε over all located circuits with source s and all decoders.

Main Theorem (Complete causal-access loss).

• *If $s \notin J^-(\gamma)$, in particular if $u(s) \le 0$, then for every located circuit and every decoder, the end-to-end channel is $\mathscr{R}_{\sigma'}$, and its error is exactly $1 - 1/\mathrm{Tr}\,\sigma'^{\wedge -1} \ge 1 - 1/d^2$ (or 1 if σ' is singular). Equality holds iff $\sigma' = I/d$. If an implemented channel lies within δ in half-diamond distance of such a replacement, its error is at least $1 - 1/d^2 - \delta$.*

• *For every state ρ_{AR}, $(\mathscr{R}_{\sigma'} \otimes \mathrm{id})(\rho_{AR}) = \sigma' \otimes \rho_R$: the output carries no correlation with any reference.*

• $\varepsilon_{\min}(s) = (1 - 1/d^2)\,1[s \notin J^-(\gamma)]$.

• *$\varepsilon_{\min}$ has κ-derivative zero, and it is not a function of placement-blind data. If a data map F takes equal values at $s_1 \in J^-(\gamma)$ and $s_2 \notin J^-(\gamma)$, then every rule $g \circ F$ has worst-case error at least $\frac{1}{2}(1 - 1/d^2)$ on $\varepsilon_{\min}$.*

• *(Same temperature; t272u draft.) For every β, with zero system and bath Hamiltonians, the identity, complete dephasing and replacement by I/d are thermal operations. Their errors are 0, $1 - 1/d$ and $1 - 1/d^2$. For a Hamiltonian H_S, Gibbs replacement has error $1 - 1/(Z_\beta Z_{-\beta})$, which equals $1 - 1/d^2$ for some $\beta \neq 0$ iff H_S is scalar.*

• *(One-time pad; t272u draft.) The channel $\rho \mapsto d^{-2}\sum_{a,b} |ab\rangle\langle ab| \otimes W_{ab}\rho W_{ab}^\dagger$, with Weyl operators $W_{ab} = X^a Z^b$, has maximally mixed key and carrier marginals for every input, yet admits exact recovery.*

Proof. (a) By Lemmas and , the receiver channel is $\mathscr{R}_\sigma$, and $\mathscr{D}\mathscr{R}_\sigma = \mathscr{R}_{\mathscr{D}(\sigma)}$. Apply Proposition (a). Robustness follows from the triangle inequality: $\varepsilon(\mathscr{D}T) \ge \varepsilon(\mathscr{D}\mathscr{R}) - \frac{1}{2}\|\mathscr{D}(T - \mathscr{R})\|_\diamond \ge 1 - 1/d^2 - \delta$. (b) Tr_A of ρ_{AR} is ρ_R. (c) If $s \notin J^-(\gamma)$, use (a) and the decoder that prepares I/d. If $s \in J^-(\gamma)$, there is a point $\gamma(\tau) \in J^+(s)$. The circuit with one wire from s to a receiver at $\gamma(\tau)$, followed by the identity decoder, has error 0. (d) The value depends only on d and the placement indicator. Apply Lemma (b) to the pair s_1, s_2. (e) Take bath $\mathbb{C}^d$ in state I/d. The identity uses $U = I$; dephasing uses $U = \sum_r Z^r \otimes |r\rangle\langle r|$; replacement uses $U =$ SWAP. With zero Hamiltonians each U commutes with the total Hamiltonian. Apply Proposition . For the Gibbs state $g_\beta = e^{-\beta H_S}/Z_\beta$, we have $\mathrm{Tr}\, g_\beta^{-1} = Z_\beta \sum_i e^{\beta E_i} = Z_\beta Z_{-\beta} \ge d^2$ by Cauchy–Schwarz, with equality iff all E_i are equal, when $\beta \neq 0$. (f) The Weyl twirl gives $d^{-2}\sum W_{ab}\rho W_{ab}^\dagger = I/d$, and the key marginal is I/d^2. The decoder $|ab\rangle\langle ab| \otimes Y \mapsto W_{ab}^\dagger Y W_{ab}$ inverts the channel.

Status: proved. Part (a) parallels Big01-A v5 thm:B, thm:Bsharp, thm:Bexact and prop:Bblocks, En08-ST-DSS-CERT v6 lem:EC-replacement and Hawk07-A-CERT v5 prop:s2-ceiling. Part (e) parallels Big01-A v5 thm:Bthermal and thm:Bdephase. All are (source record; not a premise). Thermal operations are used in the sense of [HO2013], and the measure-and-prepare reading of replacement follows [HSR2003]. Checks DC-3, DC-7, DC-15 and DC-16. par *Non-claim.* No bath at T_{GH}, no redshift mechanism, no Hawking or Gibbons–Hawking radiation and no thermal state of the patch is derived. ``Entanglement with the interior'' of the payload is excluded by (b), not asserted. The placement indicator is a property of a declared source position, not of a physical mode. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Robustness and expansions (version 2)

Located circuits (Definition) make sense on any prescribed time-oriented Lorentzian host M with a set R of receiver positions. Lemma uses only the causal relation J^+.

Theorem (II-R: approximate influence, general hosts, many observers, side channels, temperature, steps). *Let the payload have dimension $d \ge 2$.*

• *(Approximate influence; normalization.) If the receiver channel $\mathscr{N}$ satisfies $\frac{1}{2}\|\mathscr{N} - \mathscr{R}_\sigma\|_\diamond \le \varepsilon$, then $\varepsilon(\mathscr{D}\mathscr{N}) \ge 1 - 1/d^2 - \varepsilon$ for every decoder. With the full diamond norm, $\|\mathscr{N} - \mathscr{R}_\sigma\|_\diamond \le \eta$ gives $1 - 1/d^2 - \eta/2$. The bound is tight: for $\mathscr{N}_\lambda = (1 - \lambda)\mathscr{R}_{I/d} + \lambda\,\mathrm{id}$ we have $\frac{1}{2}\|\mathscr{N}_\lambda - \mathscr{R}_{I/d}\|_\diamond = \lambda(1 - 1/d^2) =: \varepsilon$ and $\varepsilon^*(\mathscr{N}_\lambda) = 1 - 1/d^2 - \varepsilon$. More generally, the optimized error is 1-Lipschitz: $|\varepsilon^*(\mathscr{N}) - \varepsilon^*(\widetilde{\mathscr{N}})| \le \frac{1}{2}\|\mathscr{N} - \widetilde{\mathscr{N}}\|_\diamond$. If a complete circuit differs from a replacement comparison circuit gate by gate by half-diamond allowances $\delta_1, \ldots, \delta_M$, then $\varepsilon^* \ge [1 - 1/d^2 - \sum_a \delta_a]_+$.*

• *(General hosts.) On any prescribed time-oriented host, if* $s \notin J^{-}(\mathrm{R})$*, then every located circuit gives a replacement receiver channel and* $\varepsilon_{\min}(s) = 1 - 1/d^2$*. If* $s \in J^{-}(\mathrm{R})$*, then* $\varepsilon_{\min}(s) = 0$*.*

• *(Many observers.) If* $\mathrm{R} = \bigcup_i \mathrm{R}_i$ *and* $s \notin J^{-}(\mathrm{R}_i)$ *for every* i*, then the joint receiver channel is a replacement. The converse direction fails for marginals: in the* $((2,3))$ *qutrit threshold encoding* $V|j\rangle = 3^{-1/2}\sum_a |a, a+j, a+2j\rangle$ *(mod* 3 *), each single share's channel is exactly* $\mathscr{R}_{I/3}$*, yet every pair of shares recovers the payload exactly.*

• *(Classical side channel and joint record/register cuts.) Suppose the payload reaches the receivers only through an m-outcome measurement, so the channel is* $T(\rho) = \sum_{x=1}^{m} \mathrm{Tr}\,(M_x \rho)\,\omega_x$*. Then* $\varepsilon(\mathscr{D}T) \geq 1 - \min(m,d)/d^2$ *for every decoder, also for mixtures over an independent shared random seed. If* $d = \sum_x k_x$ *and* $B(X) = \sum_x \mathrm{Tr}\,(P_x X) P_x/k_x$ *for block projectors* P_x *of rank* k_x*, then* $\varepsilon(B) = 1 - 1/\sum_x k_x^2$*. Without a seed, the optimum is therefore exactly* $1 - m/d^2$ *when* $m \mid d$ *(equal blocks) and* $1 - 1/d$ *when* $m \geq d$*; for* $m$nmid d*,* $m < d$*, it lies in* $[1 - m/d^2,\ 1 - 1/\min \sum_x k_x^2]$ *(partial, B2-O8). With an independent shared seed, the value* $1 - \min(m,d)/d^2$ *is attained for every m (Theorem , the case of quantum width* 1*); for* $d = 3$*,* $m = 2$ *the seed lowers the best block value* 0.8 *to* $7/9$*.*

• *(Same temperature; simplex adopted from the t275u draft.) For every* $\beta > 0$ *and* $p, q \geq 0$ *with* $p + q \leq 1$*, the channel* $T_{p,q} = p\,\mathrm{id} + q\,\Delta_d + (1 - p - q)\mathscr{R}_{I/d}$ *is a thermal operation with zero system Hamiltonian at inverse temperature* β*, using a supplied finite bath, and* $\varepsilon(T_{p,q}) = 1 - p - q/d - (1 - p - q)/d^2$*. Every value in* $[0, 1 - 1/d^2]$ *is realized; the edges* $q = 0$ *and* $p + q = 1$ *are* $(1 - q')\mathrm{id} + q'\mathscr{R}_{I/d}$ *and* $(1 - q')\mathrm{id} + q'\Delta_d$ *with errors* $q'(1 - 1/d^2)$ *and* $q'(1 - 1/d)$*.*

• *(A step, not a curve.) On* DSS_H*, restrict the receivers to* $\gamma([\tau_0, \tau_1])$*. Then* $\tau_1 \mapsto \varepsilon_{\min}(s; \tau_1)$ *takes only the values* $1 - 1/d^2$ *and* 0*. It is nonincreasing, and it jumps exactly once, at the first* τ *with* $\gamma(\tau) \in J^{+}(s)$*, if there is one.*

Proof. (a) The first statement is the robustness part of Theorem (a); the factor $\frac{1}{2}$ is the normalization. For $\mathscr{N}_\lambda$, $\mathscr{N}_\lambda - \mathscr{R}_{I/d} = \lambda(\mathrm{id} - \mathscr{R}_{I/d})$, so by homogeneity and Proposition (a) the distance is $\lambda(1 - 1/d^2)$. The identity decoder has error $\frac{1}{2}\|(1-\lambda)(\mathscr{R}_{I/d} - \mathrm{id})\|_\diamond = (1-\lambda)(1 - 1/d^2) = 1 - 1/d^2 - \varepsilon$, which meets the lower bound. For the Lipschitz bound, apply an optimal decoder of one channel to the other, use contraction of the diamond norm under composition and the triangle inequality, and exchange the channels. Replacing the gates one at a time and telescoping gives $\frac{1}{2}\|\mathscr{N} - \mathscr{N}_0\|_\diamond \leq \sum_a \delta_a$. (b) The proofs of Lemma and Theorem (a),(c) use only $J^{\pm}$. (c) The first part is (b) applied to R, since $J^{-}(\mathrm{R}) = \bigcup_i J^{-}(\mathrm{R}_i)$. For the threshold encoding, take share 1. The other two shares $(a + j, a + 2j)$ determine (a, j), so $\mathrm{Tr}_{23}\, V|j\rangle\langle j'|V^\dagger = \delta_{jj'} I/3$, and the channel is $\mathscr{R}_{I/3}$. Shares 2 and 3 work the same way, because 2 is invertible mod 3. Purify with a reference R to get ψ_{R123}. Then $\rho_{R3} = (I/3) \otimes (I/3)$ is maximally mixed on 9 dimensions, and shares 12 have dimension 9. By the Schmidt decomposition, $\psi = (I_{R3} \otimes U)\,\Phi_{R,R'} \otimes \Phi_{3,3'}$ for a unitary $U : \mathbb{C}^3_{R'} \otimes \mathbb{C}^3_{3'} \to \mathbb{C}^9_{12}$, and the decoder $U^\dagger$ followed by $\mathrm{Tr}_{3'}$ recovers the payload exactly. The other pairs follow by symmetry. This is the scheme of [CGL1999]; check DC-18. (d) $\mathscr{D}T$ is again of the form $\sum_x \mathrm{Tr}\,(M_x \cdot)\omega'_x$. Using $\langle\Omega|A \otimes B^T|\Omega\rangle = \mathrm{Tr}\,(AB)$, we get $F_e = d^{-2}\sum_x \mathrm{Tr}\,(\omega'_x M_x) \leq d^{-2}\sum_x \|M_x\|_\infty \leq d^{-2}\min(m, \sum_x \mathrm{Tr}\, M_x) = \min(m,d)/d^2$. Lemma (a) gives the bound. For B, $J = \sum_x (P_x/k_x) \otimes P_x$ and $\langle\Omega|J^{+}|\Omega\rangle = \sum_x k_x \cdot k_x$, so $t^* = 1/\sum k_x^2$. For the primal, take $\psi = \sum_x c_x \Omega_x/\sqrt{k_x}$. The output is $\sum_x c_x^2 k_x^{-2} P_x \otimes P_x$, so $m_\psi = \sum_x c_x^4/k_x^2$. This is minimized at $c_x^2 = k_x^2/\sum k^2$ with value $1/\sum k_x^2 = t^*$, so the sandwich is exact. With equal blocks, $\sum k_x^2 = d^2/m$. (e) $T_{p,q}$ is a mixture of Weyl conjugations with identity weight $p + q/d + (1 - p - q)/d^2$ (Δ_d is the uniform mixture of the d diagonal Weyl operators and $\mathscr{R}_{I/d}$ that of all d^2). Its Choi operator is diagonal in the Bell basis, so the Bell test and the triangle inequality give error one minus the identity weight. For the implementation, a selector bath with Gibbs weights $p, q, 1 - p - q$ (levels $E_a = -\beta^{-1}\ln w_a$, zero weights omitted) controls the three vertex implementations below, which commute with the total Hamiltonian.

Vertex implementations: use a bath $\mathbb{C}^m$ with Hamiltonian $H_B = \sum_r E_r \mathrm{proj}\, r$, $E_r = -\beta^{-1}\ln p_r$, whose Gibbs state is $\sum_r p_r \mathrm{proj}\, r$. A controlled unitary $U = \sum_r U_r \otimes \mathrm{proj}\, r$ commutes with $0 \otimes I + I \otimes H_B$ and gives $\sum_r p_r U_r \rho U_r^\dagger$. With $U_r = Z^r$ and $p_0 = 1 - q + q/d$, $p_r = q/d$, this is $(1-q)\mathrm{id} + q\Delta_d$. A second, zero-Hamiltonian bath $\mathbb{C}^d$ in state I/d, with $U = \mathrm{proj}\, 0 \otimes I + \mathrm{proj}\, 1 \otimes \mathrm{SWAP}$ and $p_1 = q$, gives $(1-q)\mathrm{id} + q\mathscr{R}_{I/d}$. The edge errors also follow by homogeneity and Proposition . (f) $\bar{J}(\gamma([\tau_0, \tau_1]))$ increases with τ_1. Apply (b).

Status: proved, except that (d) for *m*nmid *d* is a bracket only. Parts (a) and (d) parallel Big01-A v5 prop:Bblocks. Part (d) at $m = d$ parallels I-C-CERT v6 lem:floor. The mixtures in (e) parallel I-C-Ex-CERT v6 prop:v6-mixpinch. Parts (a)–(c) and (e) contain Theorem thm:network, Corollary cor:influence and Theorem thm:thermalSimplex of the t275u draft (Appendix). The exact replacement witness parallels Hawk07-B-CERT v6 thm:s2-exact. All are (source record; not a premise). Checks DC-18, DC-21, DC-23 and DC-25. par *Non-claim.* (b) needs a supplied causal relation. For black-hole exteriors or conical products no causal criterion is proved here, so those cases are heuristic, not claimed. (e) is not a physical noise model at T_{GH}, and (f) is not a Page curve. The threshold encoding is a declared code, not a model of horizon modes. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Main Theorem III: unused sectors, residual factors and junk

Main Theorem (Junk). *Let* $2 \le k \le D$.

• *(t272u draft.) A k-dimensional logical system embeds isometrically in a D-dimensional register with exact recovery. An orthogonal complement of dimension* $D - k$ *forces no positive lower bound on recovery error.*

• *(t272u draft.) In* $(\mathscr{H}_L \otimes \mathscr{H}_G) \oplus \mathscr{H}_J$*, the encoding* $\rho_L \mapsto (\rho_L \otimes \gamma_G) \oplus 0_J$ *with fixed* γ_G *permits exact recovery, even when* γ_G *is mixed.*

• *(Flagged loss; t272u draft.) For* $\mathscr{E}_p(X) = (1-p)X \oplus p\mathrm{Tr}\,(X)\tau_J$ *on a k-dimensional success sector,* $\varepsilon^*(\mathscr{E}_p) = p(1 - 1/k^2)$.

• *(Flagged cut; t272u draft.) If all message-bearing quantum access passes through a flagged m-dimensional cut, then* $\varepsilon^* \ge 1 - m/k$*, with equality for* Λ_m*. This is a bottleneck-dimension statement, not a statement about the complement.*

• *(Scrambled complement.) Let P have rank* $k < d$ *in* $\mathbb{C}^d$*, let* $Q = I - P$ *and* $j = d - k$*, and let* $\mathscr{S}(X) = PXP + \mathrm{Tr}\,(QX)Q/j$*. Then* $\varepsilon(\mathscr{S}) = 1 - 1/(1 + j^2) < 1 - 1/d^2$*, and* $\mathscr{S}$ *restricted to the code is the identity.*

• *(Floor chain.) For* $1 \le m \le d$*,* $1 - m/d \le 1 - 1/d < 1 - 1/d^2$*, and* $(1 - 1/d^2) - (1 - m/d) = (md - 1)/d^2 \ge (d-1)/d^2$*, with equality iff* $m = 1$*. Also* $p(1 - 1/k^2) \le 1 - 1/k^2$.

Proof. (a) For an isometry V with $P = VV^\dagger$, the decoder $\mathscr{D}(Y) = V^\dagger Y V + \mathrm{Tr}\,[(I-P)Y]\tau_L$ is a channel and inverts $V \cdot V^\dagger$, also after tensoring with a reference. (b) Project onto the code sector, trace out $\mathscr{H}_G$, and use a fixed output on the rest. (c) Test any decoder on φ. The success branch contributes overlap at most $1 - p$, and the failure branch is a replacement contributing exactly p/k^2. So $\varepsilon \ge p(1 - 1/k^2)$ by Lemma (a). The decoder that is the identity on success and prepares I/k on failure gives $(1-p)\mathrm{id} + p\mathscr{R}_{I/k}$, whose error is $p(1 - 1/k^2)$ by homogeneity and Proposition (a). (d) is Lemma with $d = k$. (e) In an adapted basis, let Ω_P and Ω_Q be the parts of Ω in $P \otimes P$ and $Q \otimes Q$. Cross blocks are annihilated, so $J = \Omega_P \Omega_P^\dagger + (Q \otimes Q)/j$ and $\langle \Omega | J^+ | \Omega \rangle = 1 + j \cdot j = 1 + j^2$. Hence $t^* = 1/(1 + j^2)$. For the primal, take $\psi = x\,\Omega_P/\sqrt{k} + y\,\Omega_Q/\sqrt{j}$ with $x^2 + y^2 = 1$. The output is $x^2 \widehat{\Omega}_P \widehat{\Omega}_P^\dagger + y^2 (Q \otimes Q)/j^2$, so $m_\psi = x^4 + y^4/j^2$. This is minimized at $x^2 = 1/(1 + j^2)$ with value $1/(1 + j^2) = t^*$. Since $j \le d - 1$, $1 + j^2 < d^2$. (f) is arithmetic.

The entropy ledger (t272u draft): for $\omega = (1-p)(\rho_L \otimes \gamma_G) \oplus p\tau_J$,

$$S(\omega) = h(p) + (1-p)(S(\rho_L) + S(\gamma_G)) + pS(\tau_J), \qquad h(p) = -p\ln p - (1-p)\ln(1-p),$$

by listing the eigenvalues blockwise. The junk entropy is counted in the declared register only, not in a ``patch''. *Status: proved.* The plan's junk certificate is corrected in Appendix . Tracing out a complement is not an operation on a subspace, an isometric code has error zero, scrambling the complement does not reach the horizon floor, and the floor $1 - k/d$ lies *below* the horizon floor. Part (d) parallels I-C-Ex2-CERT v5 prop:v4-dim, and the shared-randomness register floor $1 - m^2/d^2$ of PG-OB-1-CERT v5 thm:v5-PG-shared is a different access model; both are (source record; not a premise). Checks DC-5, DC-10, DC-13 and DC-14. par *Non-claim.* Junk sectors are declared subspaces. No reservoir, temperature or cooling law is derived, and no floor depends on a temperature label or a code distance. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Robustness and expansions (version 2)

A *flagged channel* is $\mathcal{N}(X) = \text{bigoplus}_i p_i \mathcal{N}_i(X)$ with orthogonal flag sectors, probabilities p_i and channels $\mathcal{N}_i$ on M_k. Let $F_i^* = \max_{\mathcal{D}} F_e(\mathcal{D}\mathcal{N}_i)$. For a state τ on the range of $Q = I - P$ (rank j), put $\mathcal{S}_\tau(X) = PXP + \operatorname{Tr}(QX)\tau$.

Theorem (III-R: convex bound, residual error, partial access, floors).

• *(Flagged convex bound.)* $\varepsilon^*(\mathcal{N}) \geq 1 - \sum_i p_i F_i^*$. *If each branch has an optimal decoder* $\mathcal{D}_i$ *with* $t^*(\mathcal{D}_i\mathcal{N}_i) = F_e(\mathcal{D}_i\mathcal{N}_i) = F_i^*$, *then* $\varepsilon^*(\mathcal{N}) = \sum_i p_i \varepsilon^*(\mathcal{N}_i)$. *Flagged loss (Theorem (c)) is a special case. For example, an input-independent mixture of replacement access (weight* $1 - p$*) and a flagged m-cut attainer (weight p) has exact error* $(1 - p)(1 - 1/d^2) + p(1 - m/d)$. *If the implemented channel differs by* δ *in half-diamond distance, both bounds move by at most* δ.

• *(Installed error versus optimized recovery.) For the installed channel: if* $\tau > 0$ *on* $\operatorname{ran} Q$, *then* $\varepsilon(\mathcal{S}_\tau) = 1 - 1/(1 + \operatorname{Tr}\tau^{-1})$; *if* τ *is singular on* $\operatorname{ran} Q$, *then* $\varepsilon(\mathcal{S}_\tau) = 1$. *Moreover* $\operatorname{Tr}\tau^{-1} \geq e^{2S(\tau)}$, *with equality iff* $\tau = Q/j$, *so* $\varepsilon(\mathcal{S}_\tau) \geq 1 - 1/(1 + e^{2S(\tau)}) \geq 1 - 1/(1 + j^2)$. *The maximally mixed residual gives the least installed error. The optimized recovery error, over all decoders to the full* $(k + j)$*-dimensional target, is* $\varepsilon^*(\mathcal{S}_\tau) = 1 - 1/(1 + j^2)$ *for every* τ, *whatever its entropy (t275u draft, Theorem thm:residualspectrum). On the k-dimensional code in* $\operatorname{ran} P$ *recovery stays exact. So no entropy-only recovery law exists in this model.*

(b') (Installed-error envelope; adopted from the t275u draft.) Let $E_j(s)$ *be the largest installed error at residual entropy s, for* $j \geq 2$. *Then* $E_j(s) = 1$ *for* $0 \leq s \leq \ln(j - 1)$, *and* $E_j(\ln j) = 1 - 1/(1 + j^2)$. *For* $\ln(j - 1) < s < \ln j$, *let* $x_r \in (0, 1/j)$ *be the unique solution of* $-rx_r \ln x_r - (1 - rx_r)\ln\frac{1 - rx_r}{j - r} = s$, *for* $r = 1, \ldots, j - 1$. *Then* $E_j(s) = 1 - 1/(1 + A_j(s))$ *with* $A_j(s) = \max_r \{r/x_r + (j - r)^2/(1 - rx_r)\}$.

• *(Partial access to the lost branch.) In flagged loss with probability p on a k-dimensional payload, suppose the failure branch carries an m-outcome classical record of the payload. Then* $\varepsilon^* = p(1 - m/k^2)$ *for* $m \mid k$, *and for every* $m \leq k$ *when an independent shared seed is allowed; without a seed and for* $m \operatorname{nmid} k$ *the bracket of Theorem (d) applies. If the failure branch passes through a flagged m-dimensional quantum cut, then* $\varepsilon^* = p(1 - m/k)$. *Classical records reach at best* $p(1 - 1/k)$; *zero error needs quantum access (*$m = k$*). A different partial-access model, where the quantum carrier survives and a classical transcript of a Weyl key is read, has the exact law of Theorem .*

• *(Floor chain.) For* $1 \leq m \leq d$: $1 - m/d \leq 1 - m/d^2 \leq 1 - 1/d^2$, *and* $1 - 1/(1 + (d - k)^2) < 1 - 1/d^2$ *for* $1 \leq k < d$. *These are, respectively, the quantum cut, the classical m-symbol channel (for* $m \mid d$*), the replacement channel, and the scrambled complement.*

• *(Processing.) The classes ``replacement'', ``factors through a flagged m-dimensional quantum cut'' and ``factors through an m-outcome classical record'' are closed under pre- and post-composition with arbitrary channels. Every floor of (d) therefore persists under encoders and decoders, and declared-data reorganizations do not act on channels.*

• *(Unbounded payload.) For a replacement channel on an infinite-dimensional separable payload space, the unconstrained half-diamond error is* 1. *The energy-constrained value is open.*

Proof. (a) A decoder acts on each flag sector separately, $\mathscr{D}$ = bigoplus $\mathscr{D}_i$ in effect, so $F_e(\mathscr{D}\mathscr{N}) = \sum_i p_i F_e(\mathscr{D}_i\mathscr{N}_i) \le \sum p_i F_i^*$; apply Lemma (a). Under the hypothesis, $J(\sum p_i \mathscr{D}_i\mathscr{N}_i) \ge \sum p_i F_i^* \Omega\Omega^\dagger$, and Lemma (b) gives the matching upper bound. (b) In an adapted basis, $J = \Omega_P\Omega_P^\dagger + \tau \otimes Q$, and $\langle\Omega|J^+|\Omega\rangle = 1 + \operatorname{Tr}\tau^{-1}$. For the primal, take $\psi = x\widehat{\Omega}_P + y(A \otimes I)\Omega_Q$ with $A = \tau^{-1/2}/\sqrt{\operatorname{Tr}\tau^{-1}}$. Cross terms vanish, and the computation of Proposition (a) gives $m_\psi = x^4 + y^4/\operatorname{Tr}\tau^{-1}$. This is minimized at the value $1/(1 + \operatorname{Tr}\tau^{-1}) = t^*$. If $v \in \operatorname{ran} Q$ with $\tau v = 0$, the input $v \otimes w$ gives overlap 0. For eigenvalues λ_i of τ, Jensen's inequality for exp gives $\sum_i \lambda_i^{-1} = \sum_i \lambda_i e^{-2\ln\lambda_i} \ge e^{-2\sum_i \lambda_i \ln\lambda_i} = e^{2S(\tau)}$, with equality iff all λ_i are equal. Also $e^{2S} \le j^2$. For the optimized value, the decoder that keeps the P block and resets the Q block to Q/j gives $\mathscr{S}_{Q/j}$, with error $1 - 1/(1 + j^2)$. Conversely, test with $\psi_w = \sqrt{w}\,\varphi_k + \sqrt{1-w}\,\varphi_j$, where φ_k, φ_j are normalized Bell vectors on the system–reference blocks of P and Q. The output is $w\,\varphi_k\varphi_k^\dagger + (1-w)\,\tau \otimes I_{R_Q}/j$, with cross terms removed. For any decoder the first term contributes at most w^2 to the overlap with ψ_w (its reference support is R_P), and the second at most $(1-w)^2/j^2$ (because $\langle\varphi_j$rvertA$\otimes I/j|\varphi_j\rangle \le 1/j^2$ for every state A). At $w = 1/(1 + j^2)$ the overlap is at most $1/(1 + j^2)$, and the primal bound gives the claim. (b′) A rank-$(j-1)$ state can have any entropy in $[0, \ln(j-1)]$, which gives $E_j = 1$ there. For $s > \ln(j-1)$ the level set is compact and consists of full-rank states, so $\sum_i \lambda_i^{-1}$ attains its maximum. The entropy and normalization gradients are independent away from the uniform spectrum. Lagrange multipliers give $-\lambda_i^{-2} + \mu\ln\lambda_i = \nu$ for all i. The left side has derivative $(2 + \mu x^2)/x^3$ in $x = \lambda_i$, with at most one positive zero, so at most two distinct eigenvalues occur. Let the smaller value x have multiplicity r and the larger be $y = (1 - rx)/(j - r)$. The entropy of this spectrum has derivative $r\ln(y/x) > 0$ in x and increases from $\ln(j-r)$ to $\ln j$, so each r gives exactly one x_r. Substitute into the installed formula. Checks DC-27 (optimized values by a joint decoder SDP; envelope by constrained optimization) and the draft's symbolic listing. (c) Use (a) with branches id (where $t^* = F_e = 1$) and, on failure, either the block channel of Theorem (d) (with $t^* = F_e = m/k^2$ for equal blocks) or Λ_m (with $t^* = F_e = m/k$, Proposition (d)). The lower bounds on the failure branch are Theorem (d) and Lemma . (d) $m/d \ge m/d^2 \ge 1/d^2$, and $1 + (d-k)^2 \le 1 + (d-1)^2 < d^2$. (e) For example, $\mathscr{D}(\sum_x |x\rangle\langle x| \otimes \mathscr{E}_x)\mathscr{V}$ again has this form with $\mathscr{E}_x\mathscr{V}$, and $\mathscr{D}\mathscr{R}_\sigma\mathscr{V} = \mathscr{R}_{\mathscr{D}(\sigma)}$. The floors were proved for the whole classes. (f) Restricting to d-dimensional subspaces gives $\varepsilon \ge 1 - 1/d^2$ for every d.

Status: proved, except that the energy-constrained value in (f) is open. The worst-case installed frontier left open in an earlier draft of this version is closed by (b′), adopted from the t275u draft. Part (c) for $m$nmid k is a bracket. Part (b) corrects heuristic (3c): scrambling the junk toward maximal entropy *lowers* the installed error of the scrambled-complement channel and leaves the optimized error unchanged. Checks DC-14, DC-19, DC-21, DC-26 and DC-27. Part (a) parallels I-C-Ex-CERT v6 thm:cert-every, and the exactness test parallels I-C-CERT v6 thm:v5-exact; both are (source record; not a premise). par *Non-claim.* Residual states are declared. No reservoir, cooling law or noise temperature is derived. The plan's ``junk forces no additional curvature term'' (V-DST operator-class vacuity) is heuristic, not claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Main Theorem IV: information that has left the patch

Prescribed comparison futures

Fix $H_1 > 0$ and $c > 0$. Let $f(u) = e^{-1/u}$ for $u > 0$ and $f(u) = 0$ otherwise. Let $\chi(t) = f(t)/(f(t) + f(1-t))$, a smooth step equal to 0 for $t \le 0$ and to 1 for $t \ge 1$, positive on $(0, \infty)$. Let $X(t) = \int_0^t \chi$. For $K > 0$, the prescribed flat FLRW comparison host is

$$g_K = -c^2\mathrm{d}t^2 + a_K(t)^2\mathrm{d}\mathbf{x}^2, \qquad a_K(t) = \exp\left(H_1 t + (K - H_1)X(t)\right),$$

with Hubble profile $H_1 + (K - H_1)\chi(t)$, which is monotone between H_1 and K. For $t \le 0$, $a_K = e^{H_1 t}$: all members share the entire past, which is a flat-slicing de Sitter region of rate H_1. For $t \ge 1$ the Hubble rate is exactly K. The draft's family with a general transition time τ is the same construction rescaled; we fix $\tau = 1$. The comoving observer is $\gamma = \{\mathbf{x} = 0\}$. Located circuits (Definition) and Lemma apply verbatim on any prescribed time-oriented host. Let $\chi_e(t) = c\int_t^\infty \mathrm{d}s/a_K(s)$, let $R_e(t) = a_K(t)\chi_e(t)$, and let $R_0(K) = R_e(0)$.

Main Theorem (Information that has left the patch).

• *On the slice $t = t_p$, we have $p \in J^-(\gamma)$ iff $|\mathbf{x}_p| < \chi_e(t_p)$. For $t \le 0$, $\chi_e(t) = (c/H_1)(e^{-H_1 t} - 1) + R_0(K)$ and $R_e(t) = c/H_1 + (R_0(K) - c/H_1)e^{H_1 t}$.*

• *R_0 is continuous and strictly decreasing in K, and $R_0(H_1) = c/H_1$. For $K \le H_1$, $R_0(K) \ge c\,e^{-H_1}/K$. Also $R_0(K) \to 0$ as $K \to \infty$, so the range is $(0, \infty)$. Under a promise $R_0 \in [R_-, R_+]$, the exact minimax error of a past-data estimate of $R_e(t)$, $t \le 0$, is $\frac{1}{2}(R_+ - R_-)e^{H_1 t}$. Without a promise it is infinite.*

• *(Dark events.) Let $K_1 < K_2$ and let $p = (0, \mathbf{x})$ with $R_0(K_2) \le |\mathbf{x}| < R_0(K_1)$. Then $p \in J^-(\gamma)$ for g_{K_1}, $p \notin J^-(\gamma)$ for g_{K_2}, and $\varepsilon_{\min}(p)$ equals 0 and $1 - 1/d^2$ respectively. Every rule of past data, meaning the metric on $\{t \le 0\}$ together with declared data and reports located there, has worst-case error at least $\frac{1}{2}(1 - 1/d^2)$ on $\varepsilon_{\min}(p)$.*

• *(Labels, proxies and conservation.) The late label $N^{(\kappa)}(K) = \exp(\pi c^5/(\kappa G K^2))$ and the area proxies $\mathcal{S}_A(R) = \pi k_B c^3 R^2/(G\kappa)$ and $\mathcal{N}_A(R) = \exp(\pi c^3 R^2/(G\kappa))$ at $R = R_0(K)$ are not functions of past data, and each has infinite worst-case error over the family. With $\Lambda = 3H_1^2/c^2$, $\Theta = G[g_K] + \Lambda g_K$ is divergence-free for every K, and $\Theta_{tt} = 3(K^2 - H_1^2)$ on $t \ge 1$, whose two-point minimax is $\frac{3}{2}|K_1^2 - K_2^2|$.*

• *(Analytic rigidity; t272u draft.) A positive real-analytic scale factor on a connected interval containing $(-\infty, 0]$ that agrees with $e^{H_1 t}$ for $t \le 0$ equals $e^{H_1 t}$, and its junction radius is c/H_1. The dark-event separation (c) therefore requires non-analytic futures.*

Proof. (a) A future causal curve has $|\mathrm{d}\mathbf{x}/\mathrm{d}t| \le c/a_K$. If $|\mathbf{x}_p| \ge \chi_e(t_p)$, then for every finite $t > t_p$, $|\mathbf{x}(t)| \ge |\mathbf{x}_p| - c\int_{t_p}^t \mathrm{d}s/a_K > |\mathbf{x}_p| - \chi_e(t_p) \ge 0$, so γ is never reached. If $|\mathbf{x}_p| < \chi_e(t_p)$, the radial null ray reaches $\mathbf{x} = 0$ at a finite time. The formulas follow from $a_K = e^{H_1 t}$ on $t \le 0$. (b) $\partial_K \ln a_K(s) = X(s) > 0$ for $s > 0$, which gives strict decrease. Continuity and $R_0 \to 0$ follow by dominated convergence: for $K \ge H_1$, $1/a_K(s) \le e^{-H_1 s}$, and $X(s) > 0$ for $s > 0$. For $K \le H_1$, $a_K(s) \le e^{H_1} e^{K(s-1)}$ for $s \ge 1$, so $R_0(K) \ge c\int_1^\infty e^{-H_1 - K(s-1)}\mathrm{d}s = c\,e^{-H_1}/K$. For $K = H_1$, $a = e^{H_1 t}$. The minimax statement is Lemma (b) applied to the affine formula in (a), attained by the midpoint; without a promise, Lemma (c) applies. (c) At $t = 0$, $a_K = 1$, and (a) gives the membership. Theorem (c), via Lemma , gives the two values. Past data coincide on the two hosts, so Lemma (b) applies. (d) $N^{(\kappa)}(K)$ is strictly decreasing in K and unbounded as $K \to 0$; the proxies are strictly increasing in R, which ranges over $(0, \infty)$. Apply Lemma (c). Divergence-freeness is the contracted Bianchi identity together with $\nabla g = 0$ [HawkingEllis1973]. For g_K, $G_{tt} = 3\dot{a}^2/a^2 = 3K^2$ on $t \ge 1$, and $\Lambda g_{tt} = -3H_1^2$. (e) Identity theorem for real-analytic functions on a connected interval.

Status: proved. Part (b) parallels Big01-A v5 thm:Efuture and prop:Esmooth; part (d) parallels prop:Dyncert and Hawk07-C-Dyn-CERT v5 thm:v5-frw; part (e) parallels Big01-A v5 prop:Eanalytic. All are (source record; not a premise). Checks DC-8, DC-9 and DC-12. Numerically, at $H_1 = 2$ and $c = 1$: $R_0(1) = 0.68925$ and $R_0(4) = 0.398836$, and the event $|\mathbf{x}| = 0.544043$ is dark for $K = 4$ and visible for $K = 1$. par *Non-claim.* R_0 is a functional of the prescribed future, not an integral of lost messages or a ``historical capacity''. No amount of information that has left, no residual entanglement, no Einstein dynamics and no source law is derived. Θ is a conserved tensor of every prescribed metric and selects nothing. No spacetime emergence is claimed. Ledger

unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Robustness and expansions (version 2)

For a positive scale factor a on $[0,\infty)$ with $a(0)=1$, Hubble profile $h=\dot a/a$ and finite $R_0[a]=c\int_0^\infty \mathrm{d}s/a(s)$, the comparison host is $g_a=-c^2\mathrm{d}t^2+a^2\mathrm{d}\mathbf{x}^2$, with $a=e^{H_1 t}$ for $t\le 0$. Call a class of positive functions *quasi-analytic* if each of its members that is flat (all derivatives zero) at a point of a connected interval vanishes identically there.

Theorem (IV-R: modulus, regularity, null condition, multiple junctions, flip region).

• *(Modulus of continuity.)* $|\ln R_0[a]-\ln R_0[b]|\le \sup_{t\ge 0}|\ln a-\ln b|\le \|h_a-h_b\|_{L^1(0,\infty)}$. *If* $h_a,h_b\ge h_0>0$ *and* $\sup|h_a-h_b|\le\varepsilon<h_0$, *then* $|R_0[a]-R_0[b]|\le c\varepsilon/(h_0(h_0-\varepsilon))$, *with equality for constant profiles. If the profiles agree after time T, then for every k and α,* $|\ln R_0[a]-\ln R_0[b]|\le T\|h_a-h_b\|_{C^{k,\alpha}[0,T]}$.

• *(Regularity classes.) The family is* C^∞. *So within* $C^{k,\alpha}_{\mathrm{loc}}$, H^s_{loc} *and* C^∞ *futures sharing the past,* R_0 *takes every value in* $(0,\infty)$. *If* $\ln a-H_1 t$ *lies in a quasi-analytic class on a connected interval containing* $(-\infty,0]$, *then* $a=e^{H_1 t}$ *and* $R_0=c/H_1$. *Real-analytic futures are the case of Theorem (e). For every Gevrey order* $s>1$ *(not quasi-analytic) the full range* $(0,\infty)$ *is attained by locally* G^s *futures that are exactly exponential after a finite transition (adopted from the t275u draft, Theorem thm:Gevrey).*

• *(Null-type condition.) For* $k=\partial_t+(c/a)\partial_x$, $G[g_a](k,k)=-2h$ *and* $\Theta(k,k)=G(k,k)$. *Under the condition* $\Theta(k,k)\ge 0$ *for all null k on* $t\ge 0$, *that is* $h\le 0$, *every future sharing the past either has no event horizon or has* $R_0\in[c/H_1,\infty)$, *with* $R_0=c/H_1$ *iff* $h\equiv H_1$. *Every value in* $[c/H_1,\infty)$ *is attained by the members* $K\le H_1$ *of , and* $h=H_1/(1+H_1X(t))$ *gives a future without event horizon. If in addition* $h\ge h_*$ *with* $0<h_*<H_1$, *the exact finite range is* $[c/H_1,c/h_*)$. *If instead a supplied autonomous equation* $\dot h=-\frac{3}{2}(1+w)(h^2-H_\Lambda^2)$, $w>-1$, *is imposed, a de Sitter past forces* $H_1=H_\Lambda$ *and then* $h\equiv H_1$ *and* $R_0=c/H_1$; *otherwise no solution exists. The dark-event separation of Theorem (c) persists for* $K_1<K_2\le H_1$, *so the two-point floor* $\frac{1}{2}(1-1/d^2)$ *survives the condition.*

• *(Multiple junctions.) Let the metric on* $(-\infty,t_j]$ *be the restriction of a smooth* $\ell=\ln a$ *on* $(-\infty,t_j+\eta)$. *Then the junction radius* $R_j=c\,a(t_j)\int_{t_j}^\infty \mathrm{d}s/a(s)$ *of smooth futures agreeing with it on* $(-\infty,t_j]$ *takes every value in* $(0,\infty)$. *Each junction radius is therefore not a function of the data before its own junction. The radii are not mutually independent: if the scale factor is fixed on* $[t_0,t_N]$, *then* $R_i=c\,a(t_i)\int_{t_i}^{t_{i+1}}\mathrm{d}s/a+\frac{a(t_i)}{a(t_{i+1})}R_{i+1}$, *so* $R_i=C_i+\frac{a(t_i)}{a(t_N)}R_N$ *with* $C_i=c\,a(t_i)\int_{t_i}^{t_N}\mathrm{d}s/a$. *If* $R_N\in[u,v]$, *the exact minimax error for* R_i *from the fixed prefix is* $\frac{a(t_i)}{2a(t_N)}(v-u)$ *(affine law adopted from the t275u draft).*

• *(Flip region.) For* $t\le 0$, *let* R_0 *range over* $[R_-,R_+]$. *The events of the slice t whose dark or visible status depends on* R_0 *form the comoving shell* $A(t)+R_-\le|\mathbf{x}|<A(t)+R_+$, *where* $A(t)=(c/H_1)(e^{-H_1 t}-1)$. *Its comoving thickness is* R_+-R_- *for every t. Its proper volume is* $V(t)=\frac{4\pi}{3}e^{3H_1 t}[(A+R_+)^3-(A+R_-)^3]\sim 4\pi(c/H_1)^2(R_+-R_-)e^{H_1 t}$ *as* $t\to-\infty$. *With* $\alpha=c/H_1$ *and* $\beta_\pm=R_\pm-\alpha$, $V(t)=\frac{4\pi}{3}[(\alpha+\beta_+e^{H_1 t})^3-(\alpha+\beta_-e^{H_1 t})^3]$, *and its 4-volume over* $t_a\le t\le t_b\le 0$ *is*

$$\frac{4\pi c}{3H_1}\Big[\;3\alpha^2(\beta_+-\beta_-)(e^{H_1 t_b}-e^{H_1 t_a})+\frac{3}{2}\alpha(\beta_+^2-\beta_-^2)(e^{2H_1 t_b}-e^{2H_1 t_a})$$
$$+\frac{1}{3}(\beta_+^3-\beta_-^3)(e^{3H_1 t_b}-e^{3H_1 t_a})\Big],$$

which over all of $t\le 0$ *is* $\frac{4\pi c}{3H_1}[3\alpha^2(\beta_+-\beta_-)+\frac{3}{2}\alpha(\beta_+^2-\beta_-^2)+\frac{1}{3}(\beta_+^3-\beta_-^3)]$. *For every source in the open shell, the exact minimax error of a past-only predictor of the least located-circuit error is* $\frac{1}{2}(1-1/d^2)$.

• *(Double fibre.)* $N^{(\kappa)}(K)=\exp(\pi c^5/(\kappa G K^2))$ *is injective in* κ *at fixed K and in K at fixed* κ. *It is therefore not a*

*function of admitted data together with past data, and every such rule has infinite worst-case error. Under the condition of (c) it stays unbounded as K*downarrow 0*, and if a supplied equation fixes K, the unfilled slot still gives* $\partial_\kappa \ln N^{(\kappa)}(K) = -\pi c^5/(GK^2\kappa^2) \neq 0$.

Proof. (a) $1/b \le e^{\sup|\ln a - \ln b|}/a$ pointwise; integrate, and use symmetry. Also $\ln a(t) - \ln b(t) = \int_0^t (h_a - h_b)$. Under the second hypothesis, $1/b(s) \le e^{\varepsilon s}/a(s)$ and $1/a(s) \le e^{-h_0 s}$, so $R_0[b] - R_0[a] \le c\int_0^\infty (e^{\varepsilon s} - 1)e^{-h_0 s}\mathrm{d}s = c\varepsilon/(h_0(h_0 - \varepsilon))$, and symmetrically. For $h \equiv h_0$ and $h \equiv h_0 - \varepsilon$ this is an equality. The last bound follows from $\|\cdot\|_{L^1(0,T)} \le T\|\cdot\|_{C^0} \le T\|\cdot\|_{C^{k,\alpha}}$. (b) χ is C^∞, so the proof of Theorem (b) applies in each class. In the quasi-analytic case, $\ln a - H_1 t$ is flat at 0, hence zero. For Gevrey order $s > 1$, put $v = 1/(s-1)$ and $f_v(t) = e^{-t^{-v}}$ for $t > 0$, 0 otherwise. On a disk of radius θt about $t > 0$, $\mathrm{Re}\, z^{-v} \ge Ct^{-v}$ for suitable $\theta, C > 0$, so Cauchy's inequality gives $|f_v^{(n)}(t)| \le n!\,(\theta t)^{-n} e^{-Ct^{-v}} \le AB^n(n!)^{1+1/v}$, uniformly in $t > 0$. So f_v is G^s. Replace f by f_v in χ; the quotient and the integral stay G^s by the standard closure of Gevrey classes under products, quotients by nonvanishing functions and composition [Rodino1993], which is cited, not reproved. The proof of Theorem (b) then applies verbatim, because it uses only $\chi \ge 0$, $\chi = 1$ on $[1,\infty)$ and $X > 0$ on $(0,\infty)$. (c) The identity $G(k,k) = 2(\ddot{a}a - \dot{a}^2)/a^2 = -2\dot{h}$ is check DC-22. Also $g(k,k) = 0$. If $\dot{h} \le 0$ and $h(0) = H_1$, then $h \le H_1$, so $\ln a(t) \le H_1 t$ and $R_0 \ge c\int e^{-H_1 s} = c/H_1$, with equality iff $\ln a = H_1 t$. For $K \le H_1$, $\dot{h}_K = (K - H_1)\chi \le 0$, and $R_0(K)$ decreases continuously from ∞ (as $K \to 0$) to c/H_1 (at $K = H_1$) by Theorem (b). For $h = H_1/(1 + H_1 X)$, $\dot{h} \le 0$ and $h = 1/(t + C)$ for $t \ge 1$, so a grows linearly and $\int^\infty \mathrm{d}s/a$ diverges. If $h \ge h_*$, then $R_0 \le c/h_*$, with equality only for $h \equiv h_*$, which is excluded by $h(0) = H_1$. The futures $h_\tau = H_1 + (h_* - H_1)\chi(t/\tau)$ are nonincreasing, increase pointwise with τ, and have radii varying continuously and strictly from c/h_* (τdownarrow 0) to c/H_1 ($\tau \to \infty$). A constant past solves the autonomous equation only if $H_1^2 = H_\Lambda^2$, and the right side is locally Lipschitz, so the equilibrium is the unique solution. Apply Theorem (c) with $K_1 < K_2 \le H_1$. (d) Put $\hat{\ell}(t) = \ell(t)$ for $t \le t_j$ and $\hat{\ell}(t) = (1 - \chi(t - t_j))\ell(t)$ for $t > t_j$, which is smooth. Let $\ln a_K = \hat{\ell} + KX(\cdot - t_j)$. Then $R_j(K)$ is continuous and strictly decreasing, because $X > 0$ on $(0,\infty)$. As $K \to \infty$, $R_j \to 0$ by dominated convergence: $\hat{\ell}$ is bounded on $[t_j, t_j + 1]$ and zero afterwards, and $X(s) \ge s - 1$. As $K \to 0$, $R_j \ge c\,a(t_j)e^{-KX(1)}/K \to \infty$. For the affine law, split the integral defining R_i at t_{i+1} and multiply by $a(t_i)$; iterate. The attainable set of R_i is an interval affine in R_N, and the midpoint attains the half-width. (e) Apply Theorem (a) on the slice t. Then $e^{3H_1 t}(A + R)^3 = (\alpha + \beta e^{H_1 t})^3$; substitute $w = e^{H_1 t}$ and integrate $c\,V\mathrm{d}t$ over $[t_a, t_b]$. Inside the open shell two admissible futures give least errors 0 and $1 - 1/d^2$ with the same past, so the midpoint is the minimax predictor. (f) Theorem together with Theorem (d). *Status: proved*; the Gevrey clause of (b) is proved conditional on the cited Gevrey closure properties. The plan's ``intermediate classes fill the gap'' is corrected: the dichotomy is quasi-analytic (rigid) versus not (full range $(0,\infty)$ ). The plan's ``a null-energy-condition future shrinks the range'' is made precise: the range shrinks to $[c/H_1, \infty)$, which is still unbounded. The condition is placed on the prescribed future data through Θ; no matter model or Einstein equation is admitted. Part (c) parallels VC-OB-1-CERT v5 prop:s2-vac and Big01-A v5 prop:Dyncert; part (b) parallels Big01-A v5 prop:Eanalytic; both are (source record; not a premise). Checks DC-20, DC-22 and DC-30. Parts (b)–(f) contain Theorems thm:Gevrey, thm:NEC, thm:junctions, thm:darkvolume and Proposition prop:latecoefficient of the t275u draft (Appendix). The plan's ``each junction radius is independent'' is corrected: radii are not functions of earlier data, but they are affinely tied to later ones. The Denjoy–Carleman background for quasi-analytic classes is in [Hormander2003]. par *Non-claim.* No energy condition is imposed on a matter model, and no stress tensor is supplied. The flip region is a coordinate region of the prescribed metric, not an amount of information. No Page curve, no Einstein dynamics and no emergence is derived. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Shared features as negatives

The plan presents five shared features as permanent negatives that all four realizations satisfy. Each holds only in a qualified, realization-specific form.

• *Capacity without logical yield.* Theorem (b) gives a deficit of $2(\delta - 1)$ sites for codes. Junk forces no error (Theorem (a)), and a pure syndrome ancilla in a fixed state costs nothing in recovery. ``Yield'' is relative to a declared account (Definition).

• *Invisibility to logical readout.* Below the distance, marginals are logical-blind (Theorem (f)). But maximally mixed marginals do not imply loss (Theorem (f)). The plan's ``the logical reduced state is maximally mixed independently of the dark component'' is true only for the optimal decoder after a replacement.

• *Thermal or scrambled character.* A temperature label fixes no channel (Theorem (e)), and the same zero Hamiltonian realizes all three floors.

• *Causal permanence.* This holds for a source outside $J^-(\gamma)$, by Theorem (a),(c), but it is future-relative (Theorem (c)). It is false for gauge and junk sectors, which are undone exactly (Theorem (a),(b)).

• *Overhead and floor-setting role.* The only certified ordering is Theorem (f). The horizon floor is the worst of the floors here, and the gap to the shredder floor is $(d-1)/d^2$; this parallels Big01-A v5 prop:shredsharp (source record; not a premise). ``Near the holographic bound'' is heuristic, not claimed. par *Non-claim.* The shared features are not universal laws, and no particle species, clustering mechanism or dynamics is implied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Further results integrated from the t275u draft (version 2)

The user supplied a draft addendum (request t275u; Appendix gives its sha256 and a verdict for each item). Where the draft and Sections – overlap, the stronger verified form was placed there. The remaining results are re-proved below. Draft labels are quoted in typewriter font. Citations identify source records only; no proof transfer. For a channel $\mathscr{N}$ on $\mathbb{C}^d$ with any output, $\varepsilon^*(\mathscr{N})$ denotes the optimized half-diamond recovery error over all decoders.

Joint record/register cuts

An (L, m) *cut* sends a classical label x with at most L values and, given x, a quantum system of dimension at most m. The encoder is an instrument $\{\mathscr{E}_x : M_d \to M_m\}$, and the decoder may depend on x. There is no other message-bearing path and no shared entanglement. An independent shared random seed, if allowed, is declared separately.

Theorem (Joint record/register ceiling; draft thm:Lm). *Every* (L, m) *cut with* $1 \le m \le d$ *satisfies* $F_e(\mathscr{D}\mathscr{E}) \le f_{d,L,m} := \min\{m/d, Lm^2/d^2\}$, *so* $\varepsilon^* \ge 1 - f_{d,L,m}$. *The bound survives independent shared randomness. If* $d = gm$, *the bound is attained with a finite independent seed, so* $\inf \varepsilon = 1 - \min\{1/g, L/g^2\}$. *For* $m = 1$ *this is* $1 - \min\{L, d\}/d^2$, *and for* $L = 1$ *it is* $1 - m^2/d^2$.

Proof. Let $A_{x,a} : \mathbb{C}^d \to \mathbb{C}^m$ and $B_{x,b} : \mathbb{C}^m \to \mathbb{C}^d$ be Kraus operators of $\mathscr{E}_x$ and of the decoder for x, and put $E_x = \sum_a A_{x,a}^\dagger A_{x,a}$, so $\sum_x E_x = I$. The branch contribution to $d^2 F_e$ is $F_x = \sum_{a,b} |\mathrm{Tr}\,(B_{x,b} A_{x,a})|^2$. Since $B_{x,b} A_{x,a}$ has rank at most m, $|\mathrm{Tr}\, C|^2 \le m \mathrm{Tr}\,(C^\dagger C)$ gives $F_x \le m \sum_a \mathrm{Tr}\,(A_{x,a}^\dagger A_{x,a}) = m \mathrm{Tr}\, E_x$, using $\sum_b B_{x,b}^\dagger B_{x,b} = I_m$. Applied instead to the $m \times m$ matrix $A_{x,a} B_{x,b}$, it gives $F_x \le m \sum_b \mathrm{Tr}\,(B_{x,b}^\dagger E_x B_{x,b}) \le m \mathrm{Tr}\, I_m = m^2$. Summing over at most L branches gives $d^2 F_e \le \min\{md, Lm^2\}$. The primal bound of Lemma (a) gives the error bound, and averaging over a seed preserves it. For attainment, write $\mathbb{C}^d = \mathbb{C}^g \otimes \mathbb{C}^m$ and send the second factor unchanged. On $\mathbb{C}^g$, if $L < g$, partition a basis into L nonempty groups, measure the group and prepare one basis vector of it. This channel has $F_e = L/g^2$. Average its conjugates under a shared Haar-random unitary on $\mathbb{C}^g$. The average is isotropic with the same F_e, and an isotropic channel has error $1 - F_e$ (Proposition (c)). The Haar average lies in the convex hull of a compact orbit in a finite-dimensional space, so by Carathéodory's theorem a finite seed suffices. If $L \ge g$, complete

measure-and-prepare on $\mathbb{C}^g$ has error $1 - 1/g$. Tensoring with id_m does not change the half-diamond error.

Status: proved. Check DC-23 (random cuts below the ceiling; seeded attainers equal $1 - f$ by SDP). Without a seed, the block channel of Theorem (d) gives $0.8 > 7/9$ at $d = 3$, $L = 2$, $m = 1$, so the seed matters; the no-seed value for $m$nmid d stays open (B2-O8). par *Non-claim. L* and *m* are declared access dimensions. The seed is neither shared entanglement nor message-dependent side information. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Adaptive Weyl-syndrome law; draft thm:syndrome). *Let W_g, $g = 1, \dots, d^2$, be the Weyl unitaries, drawn with probabilities π_g. A d-dimensional carrier undergoes W_g; a classical reader obtains a transcript z with law $p(z|g)$ (finite adaptive querying included); a decoder acts on z and the carrier; no other key-dependent quantum interaction exists. Then $\varepsilon^* = 1 - \sum_z \max_g \pi_g p(z|g)$. For a uniform key and a deterministic partition into L nonempty classes, $\varepsilon^* = 1 - L/d^2$. With at most L transcript values the best success term is the sum of the* $\min\{L, d^2\}$ *largest priors; for b syndrome bits and a uniform key the best error is* $1 - \min\{2^b, d^2\}/d^2$.

Proof. For a decoder $\mathscr{D}_z$ put $a_{z,g} = \langle \Phi_d | (\mathscr{D}_z \otimes \mathrm{id})(\Phi_g) | \Phi_d \rangle \geq 0$, where $\Phi_g = (W_g \otimes I)\Phi_d(W_g \otimes I)^\dagger$ are the Bell projectors. They sum to I, so $\sum_g a_{z,g} = \langle \Phi_d | \mathscr{D}_z(I) \otimes I/d | \Phi_d \rangle = 1$. Hence $F_e = \sum_{z,g} \pi_g p(z|g) a_{z,g} \leq \sum_z \max_g \pi_g p(z|g)$, and the primal bound applies. Conversely, for each z apply $W_{g_z}^\dagger$ with g_z a maximizer. The decoded channel is a Weyl mixture whose identity weight is the success term, and a Weyl mixture has error one minus its identity weight (its Choi operator is diagonal in the Bell basis). For L transcripts, $\sum_z \pi_{g_z} p(z|g_z) \leq \sum_{g \in \{g_z\}} \pi_g$, and a partition isolating the largest priors attains it.

Status: proved. Check DC-24 (joint decoder SDP equals the formula). This is partial access to a classical key while the quantum carrier survives; it is not a classical-only channel and differs from Theorem (c). par *Non-claim.* The key, the reader and the carrier are declared; no syndrome of horizon modes is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Direct sums, products and a non-rank-one certificate

Theorem (Harmonic direct-sum certificate; draft thm:directsum). *Let $\mathscr{H} = \mathrm{bigoplus}_{a=1}^{s} \mathscr{H}_a$, and let channels T_a on the blocks have $t_a > 0$ with $J(T_a) \geq t_a \Omega_a \Omega_a^\dagger$ and unit vectors ψ_a with overlap t_a (exact rank-one certificates). For $T(X) = \mathrm{bigoplus}_a T_a(P_a X P_a)$, $\varepsilon(T) = 1 - (\sum_a t_a^{-1})^{-1}$. In particular, r identity blocks and replacement blocks with full-rank states σ_b give* $1 - 1/(r + \sum_b \mathrm{Tr}\, \sigma_b^{-1})$, *and a singular replacement state gives error* 1. *For two channels with exact certificates, $\varepsilon(T_1 \otimes T_2) = 1 - t_1 t_2$.*

Proof. Put $Z = \sum_a t_a^{-1}$. $J(T)$ is block diagonal, and for every vector v, $\sum_a t_a |\langle \Omega_a, v \rangle|^2 \geq |\sum_a \langle \Omega_a, v \rangle|^2 / Z$ by weighted Cauchy–Schwarz, so $J(T) \geq Z^{-1} \Omega\Omega^\dagger$ and $\varepsilon \leq 1 - 1/Z$. With $w_a = t_a^{-1}/Z$ and $\psi = \sum_a \sqrt{w_a}\, \psi_a$ (each ψ_a in its system–reference block), cross-block terms are annihilated and the overlap is $\sum_a w_a^2 t_a = 1/Z$. Identity blocks have $t_a = 1$, and replacement blocks have $t_a = 1/\mathrm{Tr}\, \sigma_a^{-1}$ (Proposition (a)). A singular block admits an input orthogonal to its output. For products, tensor the certificates: $J_1 \otimes J_2 \geq t_1 t_2\, \Omega\Omega^\dagger$, and the product input has overlap $t_1 t_2$.

Status: proved. It contains Theorem (b) (installed), the block channel of Theorem (d) ($\sigma_x = P_x/k_x$), and the pinching formula. Check DC-26. par *Non-claim.* The blocks are declared; no physical sector decomposition is derived. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Amplitude damping; draft prop:amplitude). *For the qubit amplitude-damping channel A_γ, $0 \leq \gamma \leq 1$, $\varepsilon(A_\gamma) = \gamma$.*

Proof. The input proj 1 gives $\varepsilon \geq \gamma$. For $\gamma > 0$ put $a = 1 - \sqrt{1 - \gamma} \leq \gamma$. In the basis $00, 01, 10, 11$ (output first),

$J = J(\mathrm{id} - A_\gamma)$ has entries $J_{00,11} = J_{11,00} = a$, $J_{01,01} = -\gamma$, $J_{11,11} = \gamma$, and zeros elsewhere. Let Y agree with J except $Y_{00,00} = 2a^2/\gamma$ and $Y_{01,01} = \gamma$. Then $Y - J = \mathrm{diag}\,(2a^2/\gamma, 2\gamma, 0, 0) \geq 0$, and $Y + J$ has the single nonzero block $\begin{pmatrix} 2a^2/\gamma & 2a \\ 2a & 2\gamma \end{pmatrix}$ with determinant 0, so $Y + J \geq 0$. Also $\mathrm{Tr}_{\mathrm{out}}\, Y = \mathrm{diag}\,(2a^2/\gamma, 2\gamma) \leq 2\gamma I$. The Watrous bound [Watrous2009] gives $\frac{1}{2}\|\mathrm{id} - A_\gamma\|_\diamond \leq \frac{1}{2}\|\mathrm{Tr}_{\mathrm{out}}\, Y\|_\infty \leq \gamma$.

Status: proved. This closes, by addition, the amplitude-damping case of open item B2-O3; the elementary certificate for a general channel stays open. Here $F_e = (1 + \sqrt{1-\gamma})^2/4$ while $\varepsilon = \gamma$, so one Choi overlap does not determine the diamond error. Checks DC-6 and DC-28. par *Non-claim.* The damping channel is a declared comparison channel, not a model of horizon emission. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Continuous-variable sites under explicit energy promises

Let M bosonic modes have total number operator N, with supplied gap $\Delta > 0$ and energy ΔN; no relation between Δ and κ is assumed. The energy-constrained half-diamond distance ε_E takes the supremum over referenced inputs with $\mathrm{Tr}\,(\Delta N \rho) \leq E$ [Shirokov2018,WinterECD].

Theorem (Hard cutoff, soft cutoff and photon-number floors; draft thm:energy).

• The hard promise $N \leq n$ gives the finite space of dimension $D_{M,n} = \frac{n+M}{M}$. With a hard cutoff on each site, the counting bounds of Theorem apply with those local dimensions.

• A fixed number of modes without a cutoff is not finite-dimensional, and a mean-energy bound is not a hard cutoff.

• For $P_n = 1_{\{N \leq n\}}$ and $\mathscr{C}_n(X) = P_n X P_n + \mathrm{Tr}\,[(I - P_n)X]\mathrm{proj}\,0$, $\varepsilon_E(\mathscr{C}_n, \mathrm{id}) \leq \min\{1, \sqrt{q_n} + q_n/2\}$ with $q_n = \min\{1, E/(\Delta(n+1))\}$.

• The maximally entangled state on $P_n\mathscr{H}$ has input energy $E_{M,n} = \Delta M n/(M+1)$. For $E \geq E_{M,n}$, every decoded replacement channel has $\varepsilon_E \geq 1 - D_{M,n}^{-2}$, and every channel through a flagged m-dimensional quantum cut has $\varepsilon_E \geq [1 - m/D_{M,n}]_+$.

• If a K-dimensional code has maximally mixed encoded state of mean energy at most E, then $\ln K \leq M\, s_{\mathrm{osc}}(E/(M\Delta))$ with $s_{\mathrm{osc}}(u) = (u+1)\ln(u+1) - u\ln u$.

Proof. (a) Count occupation vectors (stars and bars); the untruncated basis is infinite, which gives (b). (c) By Markov's inequality the rejected weight is at most q_n, also with a reference. For a purification, $\|\psi\psi^\dagger - (P\psi)(P\psi)^\dagger\|_1 \leq 2\|\psi - P\psi\| \leq 2\sqrt{q_n}$, and the reset term has trace at most q_n; halve. (d) There are $\frac{r+M-1}{M-1}$ vectors with $N = r$, and $\sum_{r \leq n} r\frac{r+M-1}{M-1} = M\frac{n+M}{M+1}$, which gives $E_{M,n}$. On this input a decoded replacement has Bell overlap at most $D_{M,n}^{-2}$, and a Schmidt-rank-m output has overlap at most $m/D_{M,n}$; the primal bound applies even if outputs leave the cutoff space. (e) The encoded state ρ has finite rank and entropy $\ln K$. Nonnegativity of $D(\rho\|\gamma_\beta)$ for the product Gibbs state γ_β gives $\ln K \leq \beta E - M\ln(1 - e^{-\beta\Delta})$, and minimizing over $\beta > 0$ gives $M s_{\mathrm{osc}}(E/(M\Delta))$.

Status: (a)–(d) proved; (e) proved using the cited nonnegativity of relative entropy against an infinite-dimensional Gibbs state. Part (d) gives admissible-test lower bounds only; the exact energy-constrained values stay open (B2-O10, partial). Proposition (b) stays partial. Check DC-29. par *Non-claim. Δ, E* and *n* are supplied promises. No mode spectrum, temperature or energy scale is derived from the host, and a mean-photon-number promise is not converted into $d = e^S$. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Horizon modulus under a tail hypothesis

Theorem (Weighted horizon modulus; draft thm:modulus). *Write $b = \ln a$ and $\mathscr{R}(b) = c\int_0^\infty e^{-b}\mathrm{d}t$. If*

$b, \hat{b} \geq ht - B$ *on* $t \geq 0$ *for supplied* $h > 0$, $B \geq 0$, *then* $|\mathscr{R}(b) - \mathscr{R}(\hat{b})| \leq ce^{B}\int_0^\infty e^{-ht}|b - \hat{b}|\mathrm{d}t \leq ce^{B}\|e^{-h\cdot}\|_p\|b - \hat{b}\|_p$. *Through a window* $[0, T]$ *the bound is* $ce^{B}\int_0^T e^{-ht}|b - \hat{b}|\mathrm{d}t + 2ce^{B}e^{-hT}/h$. *If* $\|v\|_\infty \leq \delta$, *then* $e^{-\delta}\mathscr{R}(b) \leq \mathscr{R}(b + v) \leq e^{\delta}\mathscr{R}(b)$, *and* $D\mathscr{R}_b[v] = -c\int_0^\infty e^{-b}v\,\mathrm{d}t$ *with remainder at most* $\frac{1}{2}e^{\delta}\mathscr{R}(b)\|v\|_\infty^2$.

Proof. $|e^{-b} - e^{-\hat{b}}| \leq e^{-\min(b,\hat{b})}|b - \hat{b}| \leq e^{B-ht}|b - \hat{b}|$; integrate and apply Hölder. Each tail beyond T is at most $ce^{B}e^{-hT}/h$. The multiplicative bounds are immediate, and $|e^{-v} - 1 + v| \leq e^{|v|}v^2/2$ gives the derivative and remainder.

Proposition (No modulus from local regularity alone; draft prop:localdiscontinuity). *On strictly expanding smooth futures with finite event horizon,* C^∞_{loc} *convergence of scale factors does not imply convergence of* R_0.

Proof. Fix $R_* > c/H_1$. Let $a_n = e^{H_1 t}$ up to $t = n$, followed by $a_n(t) = e^{H_1 n}a_K(t - n)$ with a member a_K of ; its Hubble rate $H_1 + (K - H_1)\chi > 0$, and by Theorem (b) K can be chosen so that the junction radius at $t = n$ is $U_n = e^{H_1 n}[R_* - (c/H_1)(1 - e^{-H_1 n})] > 0$. Then $R_0(a_n) = (c/H_1)(1 - e^{-H_1 n}) + e^{-H_1 n}U_n = R_*$, while $a_n \to e^{H_1 t}$ in every local topology and $R_0(e^{H_1 t}) = c/H_1$.

Status: proved. Together with Theorem (a) this shows that a tail or agreement hypothesis is necessary for any modulus. Check DC-30. par *Non-claim.* No future is selected; the hypotheses are supplied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Access degradation, simultaneous ledger, algebraic lift

Theorem (Access degradation; draft thm:resource). *Fix the target dimension and the complete access interface. (a) If* $\mathscr{M} = \mathscr{F}\mathscr{N}$, *then* $\varepsilon^*(\mathscr{M}) \geq \varepsilon^*(\mathscr{N})$; *input-independent spectators and invertible coordinate changes leave* ε^* *unchanged. (b) Replacement access, flagged m-cuts and* (L, m) *cuts stay in their classes under output postprocessing, so their floors cannot be lowered. (c) The log-dimension overhead* $\ln D - \ln K$ *is additive under tensor products; known r-site erasure protection gives at least* $2r\ln q$ *and unknown weight-t protection at least* $4t\ln q$. *(d) Exact certificates multiply under* $\otimes$ *(Theorem), and factor allowances* δ_a *change the error by at most* $\sum_a \delta_a$. *These conclusions fail for arbitrary host-preserving declaration changes.*

Proof. (a) Every decoder after $\mathscr{F}$ is a decoder after $\mathscr{N}$; spectators can be prepared and discarded, and invertible maps can be undone. (b) Postprocessing adds no message-bearing path. (c) Theorem and Proposition . (d) Iterate and telescope. A host-invariant declaration can add a perfect quantum bypass or reveal the key of Theorem , lowering the error while fixing the metric.

Status: proved. No conversion rates between categories are claimed (B2-O16 below records this). par *Non-claim.* The monoid of access-degrading operations is not the declared-data monoid Big01-A v5 thm:D4 (source record; not a premise). Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (A simultaneous conditional ledger; draft thm:jointledger). *Suppose a declared device has: an n -site q-ary encoded memory with r-site erasure recovery of error* $\varepsilon < \frac{1}{2}$; *an* (L, m) *complete communication cut with permitted error* $e < 1$; *duration T, at most p counted gates per layer and layer time* $\tau_{\mathrm{tick}} > 0$; *comparison dynamics* $i\zeta\dot{U} = K(t)U$ *with a declared constant* $\zeta > 0$; *and a control budget* $\sum_a \inf_\lambda \|K_a(t) - \lambda I\|_\infty \leq E_{\mathrm{ctrl}}$ *over active gates. Assume each counted gate moves some test state to an orthogonal one. Then, for target dimension d, simultaneously*

$$\ln d \leq \frac{(n - 2r)\ln q + g(\varepsilon)}{1 - 2\varepsilon}, \qquad d \leq \min\{\frac{m}{1 - e}, m\sqrt{\frac{L}{1 - e}}\}, \qquad G_{\mathrm{gates}} \leq \min\{p\lfloor\frac{T}{\tau_{\mathrm{tick}}}\rfloor, \frac{2E_{\mathrm{ctrl}}T}{\pi\zeta}\}.$$

A distributed task in the static patch also needs $T \geq (2/H)\mathrm{artanh}\,(HR/c)$ *for a central round trip to radius* $R < c/H$.

Proof. The first inequality is Theorem (c), and the second is Theorem with $1 - e \leq f_{d,L,m}$. Layer counting gives the

first gate bound. The Fubini–Study speed under K is $\Delta K/\zeta \le \inf_\lambda \|K - \lambda I\|/\zeta$, and orthogonal endpoints are at Fubini–Study distance $\pi/2$, so each counted gate uses integrated control norm at least $\pi\zeta/2$; sum over gates. The latency bound is Remark .

Status: proved, conditional on the declared resources. It upgrades the conjunction of Remark . No trade-off beyond these necessary conditions, and no heat or Landauer bill, is derived; B2-O13 stays open for an energy–time ledger tied to the host. par *Non-claim.* ζ, the clock, the control Hamiltonians and the budget are declared comparison resources. ζ is not identified with any quantum coefficient. No quantum coefficient is admitted, and no relational time without an external clock is claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Finite targets with normal algebraic access; draft thm:algebra). *Let $\mathcal{M}$ be a von Neumann algebra of any type, $\mathcal{N}_* : M_d \to \mathcal{M}_*$ a normal-state channel, and decoders normal channels $\mathcal{M}_* \to M_d$. If $\mathcal{N}_*(\rho) = \omega$ for all inputs, then $\varepsilon^* = 1 - 1/d^2$; if $\mathcal{N}_*$ is within η of such a replacement in complete half-distance, then $\varepsilon^* \ge [1 - 1/d^2 - \eta]_+$. A normal conditional expectation alone implies neither replacement access nor a finite total dimension.*

Proof. After any normal decoder a constant channel is a replacement channel on M_d, with Bell overlap $1/d^2$; preparing I/d attains the bound, and contraction gives the robust form. For the last claim take a supplied $\mathcal{M} = M_d \,\overline{\otimes}\, \mathcal{A}$ with a normal state $\omega_{\mathcal{A}}$ ($\mathcal{A}$ possibly infinite-dimensional). The expectation $\mathrm{id} \otimes \omega_{\mathcal{A}}$ onto $M_d \otimes I$ preserves the logical algebra, and $\rho \mapsto \rho \otimes \omega_{\mathcal{A}}$ is exactly recoverable, while a scalar expectation erases the same access.

Status: proved for the finite-target lift. The observer algebra of [CLPW2023] stays a source record, and the algebra for this host is not constructed (B2-O12 open). par *Non-claim.* No Type II_1 or Type III algebra, observer state, or algebraic entropy is derived from the prescribed host. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (A horizon-error profile does not identify a Page curve; draft prop:noPage). *Fix a finite release experiment whose maps ignore an adjoined future metric. Adjoining two futures of Theorem (b) leaves every channel, recovery test and release entropy unchanged, while the junction radii differ arbitrarily. Conversely, with one future fixed, an identity channel and a replacement channel can both be supplied for the same target, with optimal errors* 0 *and* $1 - 1/d^2$.

Proof. In the first case the metric is an unused parameter, so all finite outputs agree, while R_0 varies by Theorem (b). The second case is immediate.

Status: proved. Neither a finite entropy history nor a future-dependent located-circuit floor identifies an evaporation channel or a Page curve. par *Non-claim.* This is a finite-channel non-identification, not a statement about a separately constructed gravitational entropy calculation. No Page curve is claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Cross-cutting additions (version 2)

Category table

Table collects the costs and floors of the four realizations. Reorganizations of declared data form the monoid of Big01-A v5 thm:D4 (source record; not a premise), and they act on none of these channels or futures. Costs and floors by category (version 2).

toprule Category Cost or floor (local result) Invariance

midrule endfirsthead toprule Category Cost or floor (local result) Invariance

midrule endhead I Redundancy Cost $n\ln q - \ln K \ge 2(\delta - 1)\ln q$, additive under $\otimes$; weight-t correction gives $\ln K \le (n - 4t)\ln q$; approximate form (Theorem , Proposition) Encoder-independent; adjoining m idle sites adds $m\ln q$

II Horizon Floor $1 - 1/d^2$; approximate influence $1 - 1/d^2 - \varepsilon$; (L, m) cuts $1 - \min\{m/d, Lm^2/d^2\}$ (Theorems ,) Access degradation (Theorem); not a function of placement-blind data

III Junk 0 (embedding); $p(1 - 1/k^2)$ (flagged loss); $1 - m/d$ (quantum cut); installed $1 - 1/(1 + \operatorname{Tr}\tau^{-1})$ but optimized $1 - 1/(1 + j^2)$ (residual state); harmonic direct sums (Theorems , ,) Processing-closed classes

IV Left the patch $R_0 \in (0, \infty)$ for smooth futures, $[c/H_1, \infty)$ under $\dot{h} \le 0$, $\{c/H_1\}$ for quasi-analytic futures (Theorem) Futures are prescribed data; no reorganization changes them

bottomrule *Status: partial.* The table records costs and floors category by category, and Theorem proves their invariance under access degradation. No conversion rates between categories are proved and no resource monotone beyond those listed is claimed (plan item T36; B2-O16). par *Non-claim.* The table is bookkeeping of proved bounds, not a resource theory with conversion rates. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Algebraic lift (documentary, open)

Remark (Type II_1 / Type III). The observer algebra of [CLPW2023] is a source record only. The finite-target lift for normal channels into any von Neumann algebra is Theorem ; it also shows that a conditional expectation alone does not supply replacement access. No algebra for this host is constructed, so that part stays open (B2-O12). Nothing in Sections – uses it.

Schedule ledger

Remark (Space, time and error as a conjunction). Fix a declared code of n sites with distance δ, a declared schedule with layer time τ_{tick} and at most p gates per layer, and a receiver set as in Theorem . Then, simultaneously: $\ln K \le (n - 2(\delta - 1))\ln q$; at most $p\lfloor T/\tau_{\text{tick}}\rfloor$ gates finish in time T (Remark); and every dark payload has error at least $1 - 1/d^2$. Each statement holds for its own data, so their conjunction holds. No trade-off between them is derived. An energy–time ledger would need energy-constrained statements. The I-C-Th records supply only exact heat ranges for energy-conserving unitaries (I-C-Th-CERT v6 thm:v3-allU) and the relative-energy statement (prop:th-vc) (source record; not a premise), which give no such ledger. It stays open (B2-O13). Under declared clock, control and action resources, Theorem turns this conjunction into explicit necessary inequalities.

Not a Page curve

Remark (Page-curve separation). The only time-indexed quantity in this sheet is the step $\tau_1 \mapsto \varepsilon_{\min}(s; \tau_1) \in \{0, 1 - 1/d^2\}$ of Theorem (f). It is nonincreasing, takes two values and involves no entropy of radiation. The dimension-ceiling statement Hawk07-B-CERT v6 cor:s2-ceiling (``A fixed resource reorganized up to its ceiling is not a Page curve'') and the refusal list of the catalogue record OpenAI rem:physics ([OpenAIcat], retrieved read-only; not a Page curve, an island, a firewall resolution, or a derivation of Einstein dynamics) are parallel (source record; not a premise). Proposition gives the finite-channel non-identification.

Emergence-page note (not registered)

The following note text is recorded for use if the sheet is later registered. It is not registered here. Big02-A is a separation on dark capacity relative to a prescribed host. None of its theorems is spacetime emergence, a Page curve, or a derived temperature.

What is deliberately not done (version 2)

• No derivation of a finite Hilbert space, a temperature, an entropy, a particle species, a clustering mechanism, Einstein dynamics, or a Page curve.

• No filling of the coefficient slot κ. No quantum coefficient is admitted.

• No discharge of O.1, XV-B, XVI-N or Gate 0.

• No revision of any parent sheet. Citations identify source records only; no proof transfer. The version-2 expansions stay inside the certificate style of version 1. Each new statement either strengthens a finite-channel

bound, records an explicit negative certificate, or is marked partial or open. The dark-matter analogy of Section remains motivational background only, and it is heuristic. No compilation is asserted.

Early-universe comparison host and thermodynamic interface (version 3)

This section implements items 2.1–2.6 of plan t276u. It merges the user's t279u drafts (Appendix) with the version-3 checks DC-32 to DC-43 (Appendix). Every statement is a local theorem about a *prescribed* metric and *declared* labels. The physical narrative it replaces is in Section and is heuristic.

Notation in this section. In Sections and , H_1 is the rate of the de Sitter *past*. In this section, H_1 is the prescribed *terminal* rate of a comparison future, and H_0 is the Hubble rate of the radiation-type prefix at the junction $t = 0$. We write $a_d = 1 - 1/d^2$, $B = c/H_0$ and $D = c/H_1$.

The prescribed early host

Definition (Radiation-type prefix and comparison futures). Fix $c, H_0 > 0$ and $t_i \in (-1/(2H_0), 0)$. Prescribe

$$a_r(t) = \sqrt{1 + 2H_0 t}, \qquad H_r(t) = \frac{H_0}{1 + 2H_0 t} \qquad (t_i \le t \le 0),$$

so $a_r(0) = 1$. A *comparison future* is a smooth $a : (t_i, \infty) \to (0, \infty)$ that equals a_r on $(t_i, 0]$, with metric $g_a = -c^2 \mathrm{d}t^2 + a(t)^2 \mathrm{d}\mathbf{x}^2$ on $(t_i, \infty) \times \mathbb{R}^3$, observer $\gamma(t) = (t, 0)$, and future orientation of increasing t. Let $\mathscr{F}(H_1)$ be the class of comparison futures with $H = \dot{a}/a > 0$ that equal H_1 after some finite time, and let $\mathscr{F}_\tau(H_1)$ require this by a fixed time $\tau > 0$. The junction radius is $R_0[a] = c\int_0^\infty \mathrm{d}s/a(s)$. ``Radiation-type" names the declared power law only. No state, equation of state, Einstein equation or radiation domination is inferred, and the excluded endpoint $t = -1/(2H_0)$ is not used as an initial condition.

Theorem (Finite-window causality and early-to-late arrival; draft t279u_2). *For every comparison future, $(t', \mathbf{x}') \in J^+(t, \mathbf{x})$ iff $t' \ge t$ and $|\mathbf{x}' - \mathbf{x}| \le c\int_t^{t'} \mathrm{d}s/a(s)$. Suppose $a(t) = a(\tau)e^{H_1(t-\tau)}$ for $t \ge \tau > 0$, and put $A(t, \tau) = c\int_t^\tau \mathrm{d}s/a(s)$ and $D_\tau = c/(H_1 a(\tau))$. For a source at $t \le 0$ and radius $r = |\mathbf{x}|$:*

- *it reaches some receiver on $\gamma([\tau, \infty))$ iff $r < A(t, \tau) + D_\tau$; equality is never received at finite time;*
- *if $A(t, \tau) < r < A(t, \tau) + D_\tau$, the first reception time is $t_{\mathrm{arr}} = \tau - H_1^{-1} \ln(1 - (r - A(t, \tau))/D_\tau)$, and if $r \le A(t, \tau)$ reception is possible by τ;*
- *with a receiver deadline $T \ge \tau$, reception is possible iff $r \le A(t, \tau) + D_\tau(1 - e^{-H_1(T-\tau)})$. The metric on a finite window $[t_i, t_f]$ fixes the causal relation between events in that window. It does not fix $J^-(\gamma)$ for a future-complete observer unless the continuation after t_f is supplied.*

Proof. A future causal curve has $|\mathbf{x}| \le ct/a$, which gives necessity. The straight spatial segment traversed at a constant fraction of the available conformal speed gives sufficiency. On the tail, $c\int_\tau^T \mathrm{d}s/a = D_\tau(1 - e^{-H_1(T-\tau)})$, whose supremum D_τ is not attained. This gives (a) and (c). Solving the equality for T gives (b); an earlier arrival can wait on γ. For the last sentence, the observer-past criterion uses $c\int_t^\infty \mathrm{d}s/a$, which depends on the continuation; Theorem gives distinct values with an identical prefix. Check DC-32.

par *Non-claim.* The prefix is prescribed. No Big-Bang initial condition, matter content or Einstein equation is derived. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Thermodynamic labels as external data

Definition (Radiation-label dictionary). Supply $U, \alpha, V_c > 0$ independently of the geometry, and keep the coefficient slot $\kappa > 0$ external. On the prefix put $u(t) = Ua(t)^{-4}$, $V(t) = V_c a(t)^3$,

$$T_{\mathrm{r}}^{(\kappa)}(t) = \frac{1}{k_B}\left(\frac{u(t)\kappa^3 c^3}{\alpha}\right)^{1/4}, \qquad S_{\mathrm{r}}^{(\kappa)}(t) = \frac{4u(t)V(t)}{3T_{\mathrm{r}}^{(\kappa)}(t)}.$$

These define labels only. The familiar equilibrium-radiation template has $\alpha = \pi^2 g_*/30$ and also supplies the degree count g_*; it is an external input [TongCosmology], recorded as a source record, not a premise. The coefficient-free early datum is $\mathfrak{E} = (g_a|_{[t_i,0]}, c, G, k_B, U, \alpha, V_c)$.

Theorem (Radiation-label coefficient certificate and calibration; drafts t279u_2 and this sheet). *At fixed* $\mathfrak{E}$*:*

• $T_{\mathrm{r}}^{(\kappa)} = \frac{(Uc^3/\alpha)^{1/4}}{k_B a(t)}\kappa^{3/4}$ *and* $S_{\mathrm{r}}^{(\kappa)} = C_{\mathrm{r}}\kappa^{-3/4}$ *with* $C_{\mathrm{r}} = \frac{4}{3}k_B\alpha^{1/4}U^{3/4}V_c c^{-3/4}$*. Hence* $\partial_\kappa T_{\mathrm{r}} = \frac{3T_{\mathrm{r}}}{4\kappa} > 0$ *and* $\partial_\kappa S_{\mathrm{r}} = -\frac{3S_{\mathrm{r}}}{4\kappa} < 0$*; each label ranges over* $(0,\infty)$*, is not a function of* $\mathfrak{E}$*, and every* $\mathfrak{E}$*-only estimator has infinite worst-case error.*

• *As identities of the dictionary,* aT_{r} *and* S_{r} *are constant in t, and* $T_{\mathrm{r}}S_{\mathrm{r}} = 4UV_c/(3a)$*. So, once an equation of state and adiabaticity are declared (here through* $u \propto a^{-4}$ *and* $V \propto a^3$*),* $S_{\mathrm{r}} \propto a^3 T_{\mathrm{r}}^3$ *becomes a function of the geometry up to one constant; that constant depends on U and* κ*. This is not a derivation of entropy conservation.*

• *An exact calibrated value* $T_* = T_{\mathrm{r}}^{(\kappa)}(t_*)$ *determines* $\kappa = [\alpha(k_B T_*)^4/(u(t_*)c^3)]^{1/3}$*, and an exact value* S_* *determines* $\kappa = (C_{\mathrm{r}}/S_*)^{4/3}$*.*

• *If U is not supplied, the rescaling* $(U,\kappa) \mapsto (r^{-3}U, r\kappa)$ *preserves the whole temperature function, so* T_{r} *alone does not determine* κ*. The pair* $(T_{\mathrm{r}}, S_{\mathrm{r}})$ *at one time does determine both: the Jacobian of* $(\ln T_{\mathrm{r}}, \ln S_{\mathrm{r}})$ *with respect to* $(\ln U, \ln\kappa)$ *has determinant* $-3/4 \neq 0$*, and* $U = 3aT_{\mathrm{r}}S_{\mathrm{r}}/(4V_c)$*.*

• *The alternative apparent-horizon declaration* $T_A^{(\kappa)} = \kappa H/(2\pi k_B)$*,* $S_A^{(\kappa)} = \pi k_B c^5/(G\kappa H^2)$ *has* $\partial_\kappa T_A = T_A/\kappa$ *and* $\partial_\kappa S_A = -S_A/\kappa$*. On the nonstationary prefix it is a declaration, not a Gibbons–Hawking temperature of a state.*

Proof. Substitute $u = Ua^{-4}$ and $V = V_c a^3$. Derivatives, products and inversions are direct (DC-33). A data-only rule takes one value on all κ, while the labels take every positive value, which gives (a). In (d), T_{r} depends on $U\kappa^3$ only, and the product identity of (b) gives U.

par *Non-claim.* The dictionary constructs no thermal state, equilibrium or noise channel. Calibration in (c) is a conditional statement about declared data. It does not admit a value of κ, and no quantum coefficient is admitted. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Horizon accounting across the junction

Theorem (Fixed-tail range and settling-time restriction; draft t279u_2, checked).

• $\{R_0[a] : a \in \mathscr{F}(H_1)\} = (0,\infty)$ *and* $\{R_0[a] : a \in \mathscr{F}_\tau(H_1)\} = (0, c\tau + D)$*.*

• *Under* $\dot{H} \leq 0$*: if* $H_1 \geq H_0$ *the class is empty. If* $0 < H_1 < H_0$*, the range is* (B,D)*, and with settling by* τ *it is* $(B + (D-B)e^{-H_0\tau}, D)$*. No endpoint is attained.*

• *These are ranges of the radius at the early junction* $t = 0$*. On a settled tail,* $R_e(t) = c\,a(t)\int_t^\infty \mathrm{d}s/a = D$ *for* $t \geq \tau$*. So with an exactly fixed eternal rate* H_1 *the late settled radius is fixed, and the future-dependent quantity is the early-junction radius.*

• *If the terminal rate is not fixed (futures exactly exponential with any rate* $K > 0$ *after a finite time), the range is* $(0,\infty)$*, and under* $\dot{H} \leq 0$ *it is* (B,∞)*. Among nonincreasing futures without a prescribed terminal rate, some have no event horizon.*

• *The ranges are unchanged for locally Gevrey futures of any order* $s > 1$*, conditional on the cited Gevrey closure [Rodino1993]. No positive real-analytic scale factor contains both an open radiation-type interval and an open exact de Sitter tail.*

Proof. (a) Strict expansion gives $a(s) > 1$ for $s > 0$, so $R_0 = c\int_0^\tau a^{-1} + Da(\tau)^{-1} < c\tau + D$. A high Hubble pulse after a short germ interval gives $R_0 \to 0$. Keeping H arbitrarily small away from short neighbourhoods of 0 and τ gives $R_0 \to c\tau + D$, and unboundedly large values when no deadline is fixed. Flat-ended smooth cutoffs keep the radiation germ. (b) Matching the germ gives $\dot{H}(0) = -2H_0^2 < 0$, so $H < H_0$ just after 0. With nonincrease this forces $H_1 < H_0$ and $H_1 \le H \le H_0$, whence $R_0 < D$ (strictly, as $\dot{H}(0) > \dot{H}_1$) and $R_0 > c\int_0^\infty e^{-H_0 s}\mathrm{d}s = B$. With a deadline, $R_0 \ge c\int_0^\tau e^{-H_0 s}\mathrm{d}s + De^{-H_0\tau} = B + (D - B)e^{-H_0\tau}$, strictly. For sharpness, let β_δ be a smooth cutoff equal to 1 near 0 and supported in $[0, 2\delta)$, put $w_\delta = \beta_\delta \cdot 2H_0^2/(1 + 2H_0 t)^2$ with mass $M_\delta < H_0 - H_1$ for small δ, add a smooth nonnegative bump of mass $H_0 - H_1 - M_\delta$, and set $H = H_0 - \int_0^t (w_\delta + \text{bump})$. This matches the germ, is nonincreasing, stays $\ge H_1$ and equals H_1 after the bump. A bump near 0 gives R_0uparrow$D$. A bump just before τ gives the deadline endpoint as δdownarrow 0, and a late bump without a deadline gives R_0downarrow$B$. Convex combinations of admissible profiles stay admissible, and R_0 is continuous along them, so each open interval is filled. (c) Direct integration of the exponential tail. (d) Keep the germ and switch to rate K after a short time; R_0 is continuous and strictly decreasing in K, tends to ∞ as Kdownarrow 0, and tends to 0 as $K \to \infty$ by dominated convergence. Under nonincrease, $h_{\delta,K} = H_0 - \int_0^t w_\delta - (H_0 - M_\delta - K)\chi_\delta$, with χ_δ a flat step reaching 1 at δ, covers (B, ∞) (draft t279u_1, Theorem thm:radfibre). Continuing the radiation law for all $t \ge 0$ is nonincreasing and gives $\int_0^\infty \mathrm{d}s/a = \infty$, so it has no event horizon. (e) Build every cutoff from $e^{-t^{-1/(s-1)}}$. For analyticity, H is analytic and equal to $H_0/(1 + 2H_0 t)$ on an open set, hence everywhere on the connected domain. Check DC-34.

par *Non-claim.* The tail rate, settling time and nonincrease condition are declared data. $\dot{H} \le 0$ is a geometric null-contraction condition (Proposition). It is a matter null-energy condition only under a separately supplied Einstein equation. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Radiation-prefix horizons, uncertainty propagation and sharp modulus). *Let $z = a_{\mathrm{r}}(t)$, $t_i \le t \le 0$.*

• *For every comparison future, $\chi_e(t) = B(1 - z) + R_0$, $R_e(t) = z[B(1 - z) + R_0]$ and $R_A(t) = c/H_{\mathrm{r}}(t) = Bz^2$. The event and apparent radii are distinct functionals.*

• *If R_0 ranges over an interval with infimum u and supremum v, the exact minimax error of an early-data-only estimate of $R_e(t)$ is $z(v - u)/2$. For supplied κ, that of the area proxy $C_\kappa R_0^2$, $C_\kappa = \pi k_B c^3/(G\kappa)$, is $C_\kappa(v^2 - u^2)/2$. These hold whether or not the endpoints are attained.*

• *(Sharp modulus; corrects Theorem (a), see Proposition .) If $a(0) = \widetilde{a}(0) = 1$ and the future Hubble profiles satisfy $H, \widetilde{H} \ge h > 0$ and $\|H - \widetilde{H}\|_\infty \le \delta$, then for every $\delta \ge 0$*

$$|R_0[a] - R_0[\widetilde{a}]| \le \frac{c\delta}{h(h + \delta)},$$

with equality for the constant profiles h and $h + \delta$. For a common radiation-type prefix, $|R_e(t) - \widetilde{R}_e(t)| \le zc\delta/(h(h + \delta))$.

Proof. (a) $\mathrm{d}t = z\,\mathrm{d}z/H_0$ gives $c\int_t^0 \mathrm{d}s/a_{\mathrm{r}} = B(1 - z)$; split the horizon integral at 0. (b) The targets are affine or monotone in R_0, and the midpoint attains half the width. An estimator takes one value on all futures with equal early data, which gives the matching lower bound. (c) With $b = \ln a$ and $\widetilde{b} = \ln \widetilde{a}$, we have $\min(b, \widetilde{b}) \ge ht$ and $|b - \widetilde{b}| \le \delta t$. Hence $|e^{-b} - e^{-\widetilde{b}}| = e^{-\min(b,\widetilde{b})}(1 - e^{-|b-\widetilde{b}|}) \le e^{-ht}(1 - e^{-\delta t})$. Integration gives $c/h - c/(h + \delta)$. Check DC-35.

par *Non-claim.* No entropy is assigned to the horizon. The area proxy uses an external κ. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Status-flip four-volume; plan t277u S 6, corrected; drafts t279u_1 and t279u_2). *Let a future promise supply radii* $0 < R_- < R_+$. *For a source slice* $t \le 0$ *on any prescribed prefix with* $a(0) = 1$, *put* $D(t) = c\int_t^0 \mathrm{d}s/a$. *The events whose dark or visible status differs across the promised futures form the comoving shell* $D(t) + R_- \le |\mathbf{x}| < D(t) + R_+$. *With* $\sqrt{-g} = c\,a^3$ *in the coordinates* $(t, \mathbf{x})$, *its invariant four-volume over* $[t_a, t_b]$ *is*

$$\mathscr{V}_4 = c\int_{t_a}^{t_b} a(t)^3\,\frac{4\pi}{3}[(D(t)+R_+)^3 - (D(t)+R_-)^3]\mathrm{d}t.$$

On the radiation-type prefix, with $C_\pm = B + R_\pm$, $z_{a,b} = a_{\mathrm{r}}(t_{a,b})$ *and* $\Delta C^j = C_+^j - C_-^j$,

$$\mathscr{V}_4 = \frac{4\pi c}{3H_0}\Big[\frac{\Delta C^3}{5}z^5 - \frac{B\,\Delta C^2}{2}z^6 + \frac{3B^2\Delta C}{7}z^7\Big]_{z_a}^{z_b}.$$

For $c = 1$, $H_0 = 2$, $R_- = \frac{3}{4}$, $R_+ = 1$, $t_a = -\frac{3}{16}$, $t_b = 0$, *this is* $\mathscr{V}_4 = 19049\pi/215040 \approx 0.278293$. *For every source in the open shell, the exact minimax error of a past-only predictor of the least located-circuit error is* $\frac{1}{2}(1 - 1/d^2)$. *The volume contains none of* κ, U, α, V_c *or the labels. It does depend on the early geometry, through* a *and* D.

Proof. An inward null ray from radius r at time t reaches γ iff $r < D(t) + R_0$ (Theorem), so varying R_0 in the promise gives the shell. Multiply the comoving volume by a^3 and integrate with $c\,\mathrm{d}t$. On the prefix, $D = B(1 - z)$ and $\mathrm{d}t = z\,\mathrm{d}z/H_0$. The z^3 terms of the cubic difference cancel, and termwise integration gives the polynomial. The instance and a direct quadrature agree (DC-36). For the minimax claim, two promised futures give errors 0 (a causal wire) and $1 - 1/d^2$ (joint replacement); a past-only predictor takes one value on both, and the midpoint attains half the gap.

Remark (Plan wording). The plan's $\int \mathrm{d}t\,a^3 \int \mathrm{d}^3x$ omits the factor c. It is the proper-volume–time convention, which equals the invariant four-volume divided by c. ``Independent of the entire early geometry'' is correct for the fibre statement: R_0 is not a function of past data (Theorem , Theorem). It is false for the volume, and under $H \le 0$ the admissible $R_\pm$ are tied to H_0. Successive junction radii stay affinely tied (Theorem (d)); on the radiation prefix the transfer coefficient is $a(t) = z$. The version-1 check DC-8 concerns an exponential prefix, so the instance above is a newly specified radiation-prefix computation.

par *Non-claim.* The volume is a geometric measure on a prescribed host. It is not a Page curve, an emitted-information count or an entropy. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Causal access across epochs

Theorem (Cross-epoch complete-access loss, influence envelope and interpolation). *Let a payload of dimension* d *be inserted at an early source, and let the complete late receiver set be a finite network of receivers or a subset of* $\gamma([\tau, \infty))$.

• *If no receiver is causally reachable from the source, the joint accessible channel is a replacement channel, even when its output is correlated across receivers, and* $\varepsilon^* = a_d$. *If the implemented channel is within half-diamond distance* η *of a replacement, then* $[a_d - \eta]_+ \le \varepsilon^* \le a_d$. *For the central observer the darkness criterion is* $r \ge B(1 - z) + R_0$. *None of these bounds depends on* $T_{\mathrm{r}}^{(\kappa)}$, $S_{\mathrm{r}}^{(\kappa)}$ *or* κ.

• *(Envelope.)* $\inf\{\varepsilon^*(\mathscr{N}) : \inf_\sigma \frac{1}{2}\|\mathscr{N} - \mathscr{R}_\sigma\|_\diamond \le \eta\} = [a_d - \eta]_+$, *attained by* $(1 - \lambda)\mathscr{R}_{I/d} + \lambda\,\mathrm{id}$ *with* $\lambda = \eta/a_d$ *for* $\eta \le a_d$, *and by* id *for* $\eta \ge a_d$.

• *(Interpolation without a flag.) For* $1 \le m \le d$, *let* $\Lambda_m(X) = \binom{d-1}{m-1}^{-1}\sum_{|S|=m} P_S X P_S$ *over coordinate projectors, and* $T_{p,m} = (1 - p)\mathscr{R}_{I/d} + p\Lambda_m$. *Then* $\varepsilon(T_{p,m}) = \varepsilon^*(T_{p,m}) = (1 - p)a_d + p(1 - m/d)$, *whether or not the branch label is delivered.*

Proof. (a) A gate affects a receiver output only along a directed descendant path, and such paths are future-causal (Lemma , Theorem). Pulling a joint receiver observable back through the channel adjoints leaves the identity on the payload, so the joint map is constant. A decoded replacement has Bell overlap $1/d^2$, and preparing I/d attains a_d . Contraction and the triangle inequality give the robust form. (b) For the stated channel, the distance to $\mathscr{R}_{I/d}$ is λa_d and the identity decoder has error $(1-\lambda)a_d$. (c) $J(\Lambda_m) = \alpha W_d + (1-\alpha)\sum_i |ii\rangle\langle ii|$ with $\alpha = (m-1)/(d-1)$, so $J(T_{p,m}) \geq f W_d$ with $f = (1-p)/d^2 + pm/d$, and Lemma gives $\varepsilon \leq 1 - f$. For any decoder, the replacement branch has Bell overlap $1/d^2$, and the cut branch has overlap at most m/d by the Schmidt-rank bound of Lemma . By linearity every decoded mixture has overlap at most f. Checks DC-37.

par *Non-claim.* Part (c) strengthens the example of Theorem (a): the flag is not needed. A positive influence allowance is an implementation model, not evidence of a temperature-induced signal outside J^+. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Three negative identification propositions

Proposition (One early label, three late errors). *Fix one comparison future, one early visible source and one evaluation of the early labels. Separately supplied late interfaces after ideal transport can have optimal errors* 0, $1 - 1/d$ *and* $1 - 1/d^2$ *(identity, complete dephasing, replacement by* I/d*). Each is implementable by a thermal operation at every inverse temperature with zero system Hamiltonian: no interaction, a controlled phase twirl, and a swap with a maximally mixed bath. For an exactly dark source, only the replacement value is available, so a temperature label does not open a causal bypass.*

Proof. Theorem (e) and Theorem give the three errors. A zero-Hamiltonian bath is maximally mixed at every β, and the three unitaries commute with the zero total Hamiltonian. Check DC-38.

par *Non-claim.* No bath located in the early universe is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (What early data and early labels do not determine). *Declare late labels at terminal rate* $K > 0$*:* $T_{\mathrm{late}}^{(\kappa)} = \kappa K/(2\pi k_B)$, $S_{\mathrm{late}}^{(\kappa)} = \pi k_B c^5/(G\kappa K^2)$ *and* $N_{\mathrm{late}}^{(\kappa)} = \exp(S_{\mathrm{late}}^{(\kappa)}/k_B)$.

• *At fixed* κ*, the early metric and the early label functions are identical across futures with the same prefix. If* K *is not fixed, these data determine none of the three late labels. Over* $K > 0$ *every early-data rule has infinite worst-case error.*

• *Under* $H \leq 0$ *the prefix allows exactly* $0 < K < H_0$*. Then the temperature label has exact minimax error* $\kappa H_0/(4\pi k_B)$*, while the entropy and exponential labels still have infinite worst-case error.*

• *(Countercase.) If* $K = H_1$ *is supplied and* κ *is supplied, or is recovered from an exact calibrated early label by Theorem (c), all three late labels are determined. So the plan's unconditional refusal is false. It holds for an unfixed terminal rate.*

• *Even in the countercase, the early-junction area proxy is not determined: its range and minimax error are those of Theorems and .*

Proof. The late labels are strictly monotone in K, and the early data do not depend on K (Theorem (d)). Under nonincrease the temperature range is $(0, \kappa H_0/(2\pi k_B))$, and the other labels diverge as $K \mathrm{downarrow} 0$. Part (c) is substitution (DC-38), and (d) follows from the fixed-tail fibre.

par *Non-claim.* Part (c) is a conditional calibration statement. It does not admit κ or identify any entropy. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Conserved inverse sources select no entropy bridge). *For any smooth prescribed* a*, constant* Λ *and* $k = \partial_t + (c/a)\partial_x$*:* $G_{tt} = 3H^2$, $G_{ij} = -(a^2/c^2)(2\dot{H} + 3H^2)\delta_{ij}$*, and* $(G + \Lambda g)(k,k) = G(k,k) = -2\dot{H}$*. The tensor* $\Theta = G[g_a] + \Lambda g_a$ *is conserved identically (contracted Bianchi identity), and its homogeneous coefficients*

$\rho = 3H^2 - \Lambda c^2$, $p = -(2\dot{H} + 3H^2) + \Lambda c^2$ *satisfy* $\dot{\rho} + 3H(\rho + p) = 0$ *for every* H. *The conservation law therefore holds on the prescribed host and is vacuous there: it selects no matter model and imposes no relation between* $S_{\mathrm{r}}^{(\kappa)}$ *and either an event-area proxy or* $S_{\mathrm{late}}^{(\kappa)}$. *With* κ *and the whole dictionary fixed, the futures of Theorem (b) have the same early entropy label and different early event-area proxies. At fixed geometry* $S_{\mathrm{r}} \propto \kappa^{-3/4}$ *and* $S_{\mathrm{late}} \propto \kappa^{-1}$, *so imposing numerical equality selects at most one* κ; *that is a choice, not a consequence of conservation.*

Proof. A symbolic computation of the Einstein tensor with c explicit, plus $g(k,k) = 0$, gives the identities (DC-38). The label parameters do not occur in the tensor identity.

par *Non-claim.* Calling the early dictionary ``radiation entropy" and the area proxy ``horizon entropy" produces no entropy-transfer law. No matter model is selected. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Epoch tags and a conditional ledger

Proposition (An epoch tag is not a fifth independent category). *Adjoining early-epoch geometry and labels as spectator metadata leaves the complete channel and* ε^* *unchanged. The proposed fifth category, ``early-universe loss", is an application of Theorem (a), with cost* a_d *independent of the early labels. Two serial replacement stages on one target still cost* a_d, *not* $2a_d$; *complete loss of independent targets of dimensions* d_1, d_2 *costs* $1 - 1/(d_1 d_2)^2 = a_{d_1} + a_{d_2} - a_{d_1} a_{d_2}$.

Proof. A composite of replacements is a replacement, and a joint replacement on $d_1 d_2$ dimensions has the stated floor.

par *Non-claim.* No conversion rate between categories is supplied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Early-source deadline and resource ledger; conditional). *Supply the resources of Theorem . For an early source at coordinate distance* r, *a receiver deadline* $T \geq \tau$ *and a target error* $e < a_d$, *the condition* $r \leq A(t,\tau) + D_\tau(1 - e^{-H_1(T-\tau)})$ *is necessary. Without an influence allowance its failure makes the target impossible, whatever the early labels. The finite-resource conditions remain*

$$\ln d \leq \frac{(n - 2r_{\mathrm{er}})\ln q + g(\varepsilon)}{1 - 2\varepsilon}, \qquad d \leq \min\{\frac{m}{1-e}, m\sqrt{\frac{L}{1-e}}\},$$

where r_{er} *counts correctable erased sites. With a supplied promise* $n \ln q \leq \eta S_{\mathrm{r}}^{(\kappa)}/k_B$ *at a supplied* κ, *the first bound becomes* $\ln d \leq (\eta S_{\mathrm{r}}^{(\kappa)}/k_B - 2r_{\mathrm{er}} \ln q + g(\varepsilon))/(1 - 2\varepsilon)$.

Proof. Theorem (c), Theorem (a), the approximate Singleton bound with $\varepsilon_A = \varepsilon_B = \varepsilon$, and $1 - e \leq F_e \leq \min\{m/d, Lm^2/d^2\}$ (Theorem).

par *Non-claim.* The capacity promise and κ are supplied, not derived. No identification of ζ, Δ or κ with a quantum coefficient is made, and adjoining labels supplies no heat bill. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Packaging results and further expansions (version 3)

This section implements the four packaging items of plan t277u and the corpus expansions of plan t276u, Section 3. Each statement merges the user's t279u drafts with this sheet's checks, keeping the stronger verified form. Duplicates of version 2 are not restated; they are listed in Appendices and . Choi operators are output-first, $W_d = \omega_d \omega_d^\dagger$ with $\omega_d = \sum_i |ii\rangle$ is an *operator*, $\Phi_d = W_d/d$, and $a_d = 1 - 1/d^2$.

A uniform action after the operational lift is fixed

Definition (Spectator lift of the declared-data monoid). The monoid $\mathscr{M}_{\mathrm{sp}} = \mathbb{N}_0 \times V$, $V = C^\infty([0,1])$, with $(n,v)(m,w) = (n+m, v+w)$, is the monoid of Big01-A v5 thm:D4 (source record; not a premise). It acts on an

account (host, source, complete receiver interface, active code, logical target, accessible channel $\mathscr{N}$, comparison-future class, past-data map, a idle qubits, metadata f) by $(n, v) \cdot (\dots, a, f) = (\dots, a + n, f + v)$. Its *operational lift* replaces $\mathscr{N}$ by $\rho \mapsto \mathscr{N}(\rho) \otimes \tau_n$ for a fixed idle state τ_n that is disconnected from every message-bearing operation, and leaves everything else unchanged.

Theorem (Uniform spectator-action certificate; draft t279u_1, merged). *The lifted action is spectator-blind and has the following properties.*

• $\varepsilon^*(\mathscr{N} \otimes \tau_n) = \varepsilon^*(\mathscr{N})$. *Every certified access floor indexed by the active dimensions is therefore invariant: replacement* a_d *(II), flagged cut* $1 - m/d$, (L, m) *cut, scrambled complement* $1 - 1/(1 + (d - k)^2)$ *and residual floors (III).*

• *For a code of ambient dimension D and logical dimension K, padding changes the log-dimension cost to* $\ln(2^n D) - \ln K = \ln D - \ln K + n \ln 2$. *The active Knill–Laflamme conditions, the logical dimension and the certified distance lower bound are preserved. The Singleton costs that are preserved are* $2r \ln q$ *for every r-site erasure and* $4t \ln q$ *for every unknown error of weight* $\le t$ *(I).*

• *Every host functional, every junction radius, every comparison future and every past-data prediction problem is unchanged; in particular the gap* $\frac{1}{2}(1 - 1/d^2)$ *is invariant (IV).*

• *The certificates are of different types. Log-dimension overheads add under* $\otimes$*; exact rank-one parameters multiply,* $t_{12} = t_1 t_2$*, so the error is* $1 - t_1 t_2$ *and not a sum; and a prediction gap is a decision-theoretic quantity. The action preserves each type and supplies no conversion rate between them.*

• *For a general declaration change that keeps the host, the floors of categories II and IV are unchanged, because they are defined by optimization over located circuits on the host and are functions of the causal relation. Floors attached to declared codes, declared channels or a declared access interface (categories I and III, or a declared bypass) can change (Proposition).*

Proof. (a) Appending τ_n is a channel $\mathscr{A}$ with left inverse $\mathscr{B}$ (discard), so decoders transfer in both directions. (b) Counting dimensions gives the cost. The partial expectation of an error in the fixed spectator state does not enlarge its active support, so the KL conditions persist (DC-39: padded $[[5, 1, 3]]$, maximal violation 0). The weight-t law is Theorem (a). (c) The action reads no geometric data. (d) $J(T_1 \otimes T_2) \ge t_1 t_2 W$ after reordering, with matching product input (DC-39). (e) Theorems and optimize over all located circuits on the host, and declarations do not enter.

par *Non-claim.* The monoid derives no finite Hilbert space, temperature, Einstein dynamics or Page curve. A finite capacity promise permits only the padding that stays admissible under it. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Declarations versus access degradation). *No theorem says that every host-preserving declaration change preserves the four floors. Changing a declared accessible channel from* $\mathscr{R}_{I/d}$ *to* id_d *at a fixed metric (a declared bypass, which alters the complete access contract and not the host) lowers* ε^* *from* a_d *to* 0. *Separately, the access-degradation monoid* $\mathscr{N} \mapsto \mathscr{F}\mathscr{N}$ *is monotone,* $\varepsilon^*(\mathscr{F}\mathscr{N}) \ge \varepsilon^*(\mathscr{N})$ *(Theorem (a)). So ``no reorganization reduces a certified floor'' holds for the spectator action and for degradation, and not for general reorganizations.*

Proof. Direct evaluation; the degradation half is Theorem (a).

par *Non-claim.* No resource theory with conversion rates is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Sign-correct duals and the exact scope of the hybrid-cut formula

Lemma (One- and two-sided certificates; draft t279u_1). *Let T be a channel on* M_d *with* $J(T) - tW_d \ge 0$, $0 \le t \le 1$. *Put* $C = J(T) - W_d$, $Y = J(T) - tW_d$, $Z = (1 - t)W_d$ *and* $H = Y + Z$. *Then* $Y, Z \ge 0$, $Y - C = Z$, $Z + C = Y$,

$H \mp C = 2Z, 2Y \geq 0$, $\mathrm{Tr}_{\mathrm{out}}\, Y = \mathrm{Tr}_{\mathrm{out}}\, Z = (1-t)I$ *and* $\mathrm{Tr}_{\mathrm{out}}\, H = 2(1-t)I$*. So* Y *is a majorant for* C*,* Z *for* $-C$*, and* H *for both signs, and* $\varepsilon(T) \leq 1 - t$*. If a pure input has overlap* t*, the bound is exact. Changing* $T - \mathrm{id}$ *to* $\mathrm{id} - T$ *exchanges* Y *and* Z *and fixes* H*. Changing to input-first order conjugates every operator by the swap; neither change is a partial transpose.*

Proof. Subtraction and partial trace, then Lemma . Exact symbolic check DC-40.

par *Non-claim.* A certificate for one channel class only. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Definition (Hybrid cut). An (L, m) cut of a d-dimensional payload is an instrument $\{\mathscr{E}_x : M_d \to M_m\}_{x=1}^{L}$ followed by channels $\mathscr{D}_x : M_m \to M_d$, with no other message-bearing path and no shared entanglement. An independent seed may select the whole cut. Let $F_{\max}(d, L, m) = \max F_e(\sum_x \mathscr{D}_x \mathscr{E}_x)$ over unseeded cuts, and $f_{d,L,m} = \min\{m/d, Lm^2/d^2\}$.

Theorem (Exact seeded law, attaining dual and exactness criterion). *Let* $d \geq 2$ *and* $1 \leq m \leq d$*.*

• $F_{\max}(d, L, m)$ *exists and* $F_{\max} \leq f_{d,L,m}$ *(Theorem).*

• *With a finite independent seed, the optimal error is exactly* $e_{\mathrm{seed}} = 1 - F_{\max}$*. It is attained by the isotropic channel* $J_t = dt\,\Phi_d + \frac{d(1-t)}{d^2-1}(I - \Phi_d)$*,* $t = F_{\max}$*, with explicit witnesses*

$$Y_t = \frac{d(1-t)}{d^2-1}(I - \Phi_d), \qquad Z_t = d(1-t)\Phi_d, \qquad H_t = Y_t + Z_t,$$

which satisfy Lemma , and the Bell input gives primal value $1 - t$*. At most* $d^4 + 1$ *seed values are needed.*

• *If* $Lm \leq d$*, then* $F_{\max} = Lm^2/d^2 = f$*.*

• *If* $Lm > d$ *(so* $f = m/d$*), then* $F_{\max} = m/d$ *iff* $I_d = \sum_{x=1}^{L} w_x P_x$ *for some rank-*m *orthogonal projectors* P_x *and weights* $w_x \geq 0$*. This holds if* $m \mid d$*, and also if* $L \geq d$ *(cyclic coordinate windows with* $w_x = 1/m$*). In those cases* $e_{\mathrm{seed}} = 1 - f$*; otherwise* $e_{\mathrm{seed}} > 1 - f$*.*

Proof. Write Kraus operators $A_{x,a} : \mathbb{C}^d \to \mathbb{C}^m$ and $B_{x,b} : \mathbb{C}^m \to \mathbb{C}^d$, and $E_x = \sum_a A_{x,a}^{\dagger} A_{x,a}$. The branch contribution to $d^2 F_e$ is $F_x = \sum_{a,b} |\mathrm{Tr}\, C_{ab}|^2$, with $C_{ab} = B_{x,b} A_{x,a}$. Since $|\mathrm{Tr}\, C|^2 = |\langle P_{\mathrm{ran}\, C}, C\rangle|^2 \leq \mathrm{rank}\,(C)\mathrm{Tr}\, C^{\dagger}C$, we get $F_x \leq m\mathrm{Tr}\, E_x$. Applying the same bound with $\mathrm{Tr}\,(B^{\dagger}E_x B) \leq \mathrm{Tr}\,(B^{\dagger}B)$ gives $F_x \leq m^2$. Summing gives (a), and compactness gives existence. (b) A decoded channel has error $\geq 1 - F_e$ (Bell test), and seed averaging is linear in F_e. Conversely, twirling a maximizer by $U \otimes \overline{U}$ uses encoder U and decoder $U^{\dagger}$, keeps L and m, preserves F_e, and gives J_t. Then $Y_t = J_t - tW_d$, $Z_t = (1-t)W_d$, and Lemma gives $\varepsilon = 1 - t$. Carathéodory's theorem bounds the number of seed values. (c) Take L orthogonal rank-m blocks, transmitted isometrically, and send the remainder to a fixed vector of block 1 by trace-zero Kraus operators. The composite Kraus operators are the L block projectors, so $F_e = Lm^2/d^2$. (d) If $I = \sum w_x P_x$, the instrument with Kraus $\sqrt{w_x} P_x$ followed by isometric transmission gives $d^2 F_e = \sum_x w_x m^2 = md$. Conversely, suppose $F_e = m/d$. Then $\sum_x F_x = m\sum_x \mathrm{Tr}\, E_x$, so every nonzero C_{ab} attains $|\mathrm{Tr}\, C|^2 = m\mathrm{Tr}\, C^{\dagger}C$. Hence $C_{ab} = c_{ab}P$ for a rank-m projector, with $A_{x,a}$ surjective and $B_{x,b}$ injective. In a branch with $E_x \neq 0$ every product is then nonzero. Fixing a and varying b shows that all $B_{x,b}$ are multiples of one B_x whose range is $\mathrm{ran}\, P_x$, and that all ranges agree. The decoder condition $\sum_b B_{x,b}^{\dagger} B_{x,b} = I_m$ makes B_x a multiple of an isometry, so each $A_{x,a} \propto B_x^{\dagger} P_x$ and $E_x = w_x P_x$. Trace preservation gives $\sum w_x P_x = I$. For $m \mid d$ use block projectors; for $L \geq d$ use the d cyclic windows of length m, each coordinate lying in exactly m of them. Checks DC-40.

par *Non-claim.* The cut opens no Page curve and identifies no temperature. All these finite-channel values have κ -derivative 0. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Two labels: complete law, and the $(3,2,2)$ **counterexample).** *For* $L = 2$*: if* $2m \le d$*, then* $e_{\mathrm{seed}} = 1 - 2m^2/d^2$*. If* $d < 2m < 2d$*, then* $F_{\max}(d,2,m) < m/d$*, so* $e_{\mathrm{seed}} > 1 - m/d = 1 - f$ *and the formula* $1 - f$ *is not exact. In particular* $\frac{1}{3} < e_{\mathrm{seed}}(3,2,2) \le \frac{6-2\sqrt{2}}{9} \approx 0.352397$*. The upper value is attained by the effects* $E_1 = \mathrm{diag}\,(1, \frac{1}{2}, 0)$, $E_2 = \mathrm{diag}\,(0, \frac{1}{2}, 1)$ *with isometric transmission of each rank-two square-root output, which gives* $F_e = (3 + 2\sqrt{2})/9$*, followed by a twirl.*

Proof. For $2m \le d$ apply Theorem (c). Suppose $w_1 P_1 + w_2 P_2 = I$ with rank-m projectors and $m < d$. Then P_2 is a function of P_1, so they commute. On $\ker P_1 \cap \ker P_2$ the identity fails, so that space is 0, which forces $2m \ge d$. Since the ranks are equal and $m < d$, the ranges differ, so $\mathrm{ran}\,P_1 \cap \ker P_2$ and $\ker P_1 \cap \mathrm{ran}\,P_2$ are both nonzero, which gives $w_1 = w_2 = 1$. If $\mathrm{ran}\,P_1 \cap \mathrm{ran}\,P_2 \neq 0$, then also $w_1 + w_2 = 1$, a contradiction. So the ranges are complementary and $d = 2m$. Theorem (d) then gives strictness for $d < 2m$. The construction is the draft's; its value is checked in DC-40. A seesaw search (DC-40, six random starts) found no larger value than $(3 + 2\sqrt{2})/9$. That is numerical evidence only, and the exact $F_{\max}(3,2,2)$ remains open (B2-O20).

par *Non-claim.* No universal closed formula for $F_{\max}$ is claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corollary (Joint replacement witness and tensor extension). *If no receiver descends from the source, the joint receiver channel is a replacement* $\mathscr{R}_\sigma$*, even with correlated outputs. With* $t_\sigma = (\mathrm{Tr}\,\sigma^{-1})^{-1}$ *for* $\sigma > 0$ *and* $t_\sigma = 0$ *otherwise, the witnesses* $Y_\sigma = \sigma \otimes I - t_\sigma W_d$, $Z_\sigma = (1 - t_\sigma) W_d$ *and* $H_\sigma = Y_\sigma + Z_\sigma$ *are exact, the error is* $1 - t_\sigma$*, and the decoder optimum is* a_d*. The plan's* $\sigma \otimes I - \Xi^{-1}\Omega$ *is* Y_σ *in the notation of Hawk07-B-CERT v6 thm:s2-exact* (source record; not a premise)*, where* Ω *denotes the operator* $|\Omega\rangle\langle\Omega| = W_d$ *and* $\Xi = \mathrm{Tr}\,\sigma^{-1}$*. It is the replacement witness, not a witness for the* (L,m) *attainers, which need* Y_t, Z_t, H_t *of Theorem . For an untouched system of dimension* s*, tensoring all three witnesses with* W_s *certifies* $\varepsilon(T \otimes \mathrm{id}_s) = \varepsilon(T)$*.*

Proof. Lemma ; weighted Cauchy–Schwarz gives $\sigma \otimes I \ge t_\sigma W_d$, and the input $\sum_i \sqrt{\lambda_i^{-1}/\mathrm{Tr}\,\sigma^{-1}}\,|ii\rangle$ attains. Also $J(T \otimes \mathrm{id}_s) = J(T) \otimes W_s$ after reordering, and $\mathrm{Tr}_{\mathrm{out}}\, W_s = I_s$. Check DC-40 ($d = 3$, $s = 2$).

par *Non-claim.* The witness is independent of κ and of every early label. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Energy-constrained cut and residual floors without a fictitious dimension

Let M bosonic modes have number operator N and supplied energy ΔN, $\Delta > 0$, chosen independently of κ. Write $\varepsilon_E(\Phi, \Psi) = \sup \frac{1}{2}\|(\Phi \otimes \mathrm{id})(\rho) - (\Psi \otimes \mathrm{id})(\rho)\|_1$ over inputs with $\mathrm{Tr}\,(\Delta N \rho_A) \le E$ [Shirokov2018,WinterECD]. Put $P_n = 1_{\{N \le n\}}$ and $\mathscr{C}_n(X) = P_n X P_n + \mathrm{Tr}\,[(I - P_n)X]\mathrm{proj}\,0$.

Theorem (Exact cutoff-and-reset error; draft t279u_2). *For every* $n \ge 0$, $E \ge 0$ *and* $M \ge 1$,

$$\varepsilon_E(\mathscr{C}_n, \mathrm{id}) = F(q) := \frac{q + \sqrt{4q - 3q^2}}{2}, \qquad q = \min\{1, \frac{E}{\Delta(n+1)}\}.$$

A two-dimensional reference suffices. For $n = 0$ *this is the exact energy-constrained distance of vacuum replacement. Since* $F(q) \le \sqrt{q} + q/2$*, this closes the upper bound of Theorem (c).*

Proof. Reduce to pure referenced inputs $\sqrt{1 - q'}\,|u\rangle + \sqrt{q'}\,|v\rangle$, with u supported in P_n and v in $I - P_n$. Markov's inequality gives $q' \le q$. The output difference is $q'\mathrm{proj}\,v + \sqrt{q'(1-q')}(|u\rangle\langle v| + |v\rangle\langle u|) - q'\mathrm{proj}\,0 \otimes v_R$. The 2×2 block has trace norm $\sqrt{4q' - 3q'^2}$, and the last term has trace q', so the error is at most $F(q') \le F(q)$, since F is increasing. The input $\sqrt{1-q}|0\rangle|0\rangle + \sqrt{q}|n+1\rangle|1\rangle$ has energy $\Delta(n+1)q$, and its reset term is orthogonal to both u and v, so the triangle inequality is an equality. The energy-constrained SDP on a truncated Fock space gives $F(q)$ for $n = 0, 1$ and $q = 0.1, 0.5, 1$ (DC-41).

par *Non-claim.* An exact value for one channel. It is not a universal energy-constrained error for every replacement or residual channel. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corollary (Continuous-variable residual bracket). *For every channel $\mathcal{N}$, $\varepsilon_E(\mathcal{N},\mathcal{NC}_n) \le F(q)$, and optimized recovery errors of $\mathcal{N}$ and $\mathcal{NC}_n$ differ by at most $F(q)$. For $\mathcal{S}_\tau(X) = P_n X P_n + \mathrm{Tr}\,[(I-P_n)X]\tau$ with τ supported outside P_n,*

$$\varepsilon_E^*(\mathcal{S}_\tau) \le F(q), \qquad \varepsilon_E^*(\mathcal{S}_\tau) \ge 1 - \frac{1}{1+j^2} \text{ if } E \ge \Delta\frac{j^2}{1+j^2}(n+\frac{j+1}{2}),\ j \in \mathbb{N},$$

whatever the entropy of τ. Without an energy constraint and with an infinite lost sector, the error is 1*, the limit of the finite values* $1 - 1/(1+j^2)$ *of Theorem (b).*

Proof. Postprocessing contraction and the triangle inequality. A decoder that resets the outside sector to the vacuum turns $\mathcal{S}_\tau$ into $\mathcal{C}_n$. For the lower bound, test with $\sqrt{w}|0\rangle|0\rangle + \sqrt{1-w}|\Phi_j\rangle$ on the number vectors $n+1,\dots,n+j$, with $w = 1/(1+j^2)$. The preserved branch contributes at most w^2 to the overlap, and the lost branch at most $(1-w)^2/j^2$. The finite case $k=1$, $j=2$ gives 0.8 (DC-41).

par *Non-claim.* A two-sided bracket, not an equality for every residual state. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Hard support versus mean energy; draft t279u_1, corrected). *Let P be a supplied finite-rank projection of rank $D \ge 2$, with $\overline{E}_P = D^{-1}\mathrm{Tr}\,(P\Delta N)$.*

• *If all inputs are supported on $P\mathcal{H}$ (a hard promise), complete access through a flagged m-dimensional quantum cut has exact error* $1 - \min\{m,D\}/D$*, which lies below the replacement floor by* $(mD-1)/D^2$ *for $m \le D$. This is Lemma on $P\mathcal{H}$.*

• *Under a mean-energy promise only, with $E \ge \overline{E}_P$, every decoded flagged m-cut has $\varepsilon_E \ge [1-m/D]_+$ and every decoded replacement has $\varepsilon_E \ge 1 - D^{-2}$. These are test floors, not optima. For $P = P_n$ they are Theorem (d), with $D = \frac{n+M}{M}$ and $\overline{E}_{P_n} = \Delta Mn/(M+1)$. A cutoff n on each mode separately gives dimension* $(n+1)^M$.

• *A mean-energy promise alone supplies no finite dimension. Mixtures of the vacuum with* $\mathrm{proj}\,n$ *are admissible for arbitrarily large n, and a vacuum mixture with weights $\propto (n+1)^{-3}$ has finite mean number and infinite support. Substituting $D = e^S$, or any noninteger ``effective dimension'', into (a) is not justified. At $E = 0$ the only input is the vacuum, so there is no positive energy-independent floor.*

Proof. (a) Lemma and the attainer Λ_m of Theorem (c) on $P\mathcal{H}$. (b) The Bell state on $P\mathcal{H}$ has marginal P/D; apply the Schmidt-rank and replacement overlaps. The counting identity $\sum_{r\le n} r\frac{r+M-1}{M-1} = M\frac{n+M}{M+1}$ is checked in DC-41. (c) Direct.

par *Non-claim.* The energy promise is external. No noise channel or temperature is derived, and the plan's $1 - m/d_{\mathrm{eff}}$ is replaced by (a) and (b). Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Coding: an additive information-defect ledger

For an isometry $V : \mathbb{C}^K \to A\otimes B\otimes C$ with encoded Bell state ψ_{RABC}, put $\Xi(V) = \frac{1}{2}(I(R:A)_\psi + I(R:B)_\psi)$.

Theorem (Additive information-defect action; draft t279u_2).

• $\ln K \le \ln\dim C + \Xi$*, so* $\ln\dim(ABC) - \ln K \ge \ln\dim A + \ln\dim B - \Xi$.

• *Under $V \mapsto V\otimes V'$ with regrouped factors,* $\ln K$, $\ln\dim C$, Ξ *and* $\ln\dim(ABC) - \ln K$ *are additive.*

• *If erasures of A and B are recoverable with errors $\varepsilon_A, \varepsilon_B$, then $\Xi \le (\varepsilon_A+\varepsilon_B)\ln K + \frac{1}{2}(g(\varepsilon_A)+g(\varepsilon_B))$, with g as in*

Theorem (c). The blockwise allowances add for independent allocations. Exact correction gives $\Xi = 0$*, and the weight-t overhead stays at least* $4t\ln q$ *per block.*

• *(No dimension-free cost.) For* $\rho_0 = \mathrm{proj}\,0$ *and* $\rho_\varepsilon = (1-\varepsilon)\mathrm{proj}\,0 + \frac{\varepsilon}{D-1}(I - \mathrm{proj}\,0)$*, the trace distance is* ε *while* $S(\rho_\varepsilon) - S(\rho_0) = h(\varepsilon) + \varepsilon\ln(D-1)$*. So an additive* $O(\varepsilon\ln(1/\varepsilon))$ *cost cannot come from entropy continuity, and the term* $(\varepsilon_A + \varepsilon_B)\ln K$ *in (c) is the correct allowance. The decoupling statement of version 2,* $\frac{1}{2}\|\psi_{RX} - I/K \otimes \psi_X\|_1 \le 2\sqrt{\varepsilon_X}$*, concerns a different quantity, a trace distance.*

• *For one mode with hard cutoff* n_i *per site,* $\ln K \le \sum_{i\in C}\ln(n_i+1) + \Xi$*; for* M_i *modes with total cutoff* n_i*, replace* $n_i + 1$ *by* $\frac{n_i + M_i}{M_i}$.

Proof. (a) Purity and subadditivity give $2S(R) \le 2S(C) + I(R:A) + I(R:B)$. (b) Entropies of product states add. (c) This is the proof of Theorem (c), using the cited continuity bound [Winter2016]. (d) Eigenvalues. (e) Counting. Checks DC-42.

par *Non-claim.* The ledger is declared. No physical error model or holographic code is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Correlated gauge balance). *Under* $\rho_{LG} \mapsto (\mathrm{id}_L \otimes \mathscr{F}_G)(\rho_{LG})$*, with any retained reference, the L (and RL) marginals are unchanged and* $\Delta S(LG) = \Delta S(G) - \Delta I(L:G)$*. For product inputs the mutual-information terms vanish. Logical recovery need not imply gauge recovery, and* $\Delta S(G)$ *alone specifies no heat.*

Proof. Subtract $S(LG) = S(L) + S(G) - I(L:G)$ at the two times; replacement on G keeps RL. Numerical check DC-42.

par *Non-claim.* No thermodynamic bill is assigned. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Partial access: erased observations and damping mixtures

Theorem (Observation-erasure interpolation; draft t279u_2). *A d-dimensional carrier is encrypted by a uniform Weyl key, and the carrier survives. A deterministic transcript partitions the* d^2 *keys into L nonempty classes. It is delivered with probability p independently of the key; otherwise an erasure flag is delivered. No other key-dependent interaction exists. Then* $\varepsilon^* = 1 - (1 + p(L-1))/d^2 = (1-p)a_d + p(1 - L/d^2)$*. With at most b transcript bits,* $L \le \min\{2^b, d^2\}$*, and for* $L < d^2$ *even perfect delivery leaves a positive error.*

Proof. For any decoder, the decoded Bell fidelity is at most the optimal key-guessing probability, $\sum_z \max_g \Pr(g,z) = pL/d^2 + (1-p)/d^2$, as in Theorem . Guessing a maximizing key and inverting it attains this value. SDP check DC-42.

par *Non-claim.* The plan's ``convex combination between zero error and the no-access value'' holds only for full-key transcripts ($L = d^2$). The delivery probability is an availability parameter, not extra syndrome bits. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Amplitude-damping mixtures). *For* $T_{p,\gamma} = (1-p)\mathrm{id} + pA_\gamma$*,* $\frac{1}{2}\|T_{p,\gamma} - \mathrm{id}\|_\diamond = p\gamma$*. The certificate is the two-sided one* $(Y \pm J \ge 0)$ *of Proposition , scaled by p. It is not the rank-one sandwich, for which* $t^* = 0$ *(Proposition).*

Proof. Homogeneity of the norm (DC-42).

par *Non-claim.* An installed distance, not an optimized recovery error. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corrections to version-2 statements (by addition)

Version 2 is frozen. The following corrections are recorded here and in the status table; the version-2 text is not

edited.

Proposition (Sharpness in Theorem (a)). *The bound $c\varepsilon/(h_0(h_0-\varepsilon))$ of Theorem (a) is valid, but its equality example is not admissible: the pair of constant profiles h_0 and $h_0-\varepsilon$ violates the hypothesis $h_a, h_b \geq h_0$. Under that hypothesis the sharp bound is $c\varepsilon/(h_0(h_0+\varepsilon))$ (Theorem (c)), which needs no condition $\varepsilon < h_0$. The other clauses of Theorem (a) are unaffected.*

Proof. Theorem (c). In DC-35, with $h_0 = 0.5$ and $\varepsilon = 0.3$, the sharp bound is 0.75 against 3.0, and adversarial piecewise-constant pairs stay below 0.75.

par *Non-claim.* A correction of wording and constant only. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Direction of the entropy comparison in Theorem (b)). *For full-rank τ on a j-dimensional lost sector, the valid chain is*

$$\varepsilon(\mathscr{S}_\tau) = 1 - \frac{1}{1+\operatorname{Tr}\tau^{-1}} \geq 1 - \frac{1}{1+j^2} \geq 1 - \frac{1}{1+e^{2S(\tau)}}.$$

The second inequality printed in Theorem (b), $1-1/(1+e^{2S(\tau)}) \geq 1-1/(1+j^2)$, has its direction reversed; it fails for every non-uniform τ. The conclusion $\varepsilon(\mathscr{S}_\tau) \geq 1-1/(1+j^2)$ stays true, by $\operatorname{Tr}\tau^{-1} \geq j^2$, and so does the envelope (b').

Proof. Cauchy–Schwarz gives $\operatorname{Tr}\tau^{-1} \geq j^2$, and $S(\tau) \leq \ln j$ gives $e^{2S} \leq j^2$, with equality iff τ is uniform. In DC-43 the reversed inequality fails on all 21 sampled spectra, none of which is uniform.

par *Non-claim.* A correction of direction only; no version-2 conclusion that was used elsewhere changes. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

What is deliberately not done (version 3)

• No derivation of $a(t)$ from Einstein equations or from a Big-Bang initial condition.

• No physical noise channel at T_r, and no thermal state.

• No identification of early radiation entropy with late horizon entropy.

• No filling of any coefficient slot κ. No quantum coefficient is admitted.

• No discharge of O.1, XV-B, XVI-N or Gate 0.

• No revision of any parent sheet. Citations identify source records only; no proof transfer. The early-universe interface is external labelled data. The causal and fibre certificates separate it from the late-time logical floors, and the Big-Bang narrative stays motivational background only.

Emergence-page sentences (not registered)

The following sentences are recorded for use if the sheet is later registered. They are not registered here. Big02-A-v3 records dark-capacity and early-universe thermodynamic separations on prescribed hosts. None of its theorems is spacetime emergence, a Page curve, or a derived temperature or entropy. These four packaging theorems unify existing floors and ceilings on prescribed hosts. None of them is spacetime emergence, a Page curve, or a derived temperature.

Exact (L, m) values from a Ky Fan branch bound (version 4)

This section implements targets P1 and P4 of the derived version-4 agenda (Appendix). The notation is that of Definition . For $Z \geq 0$ on $\mathbb{C}^d$ and $1 \leq m \leq d$, write $\|Z\|_{(m)}$ for the sum of the m largest eigenvalues of Z, the Ky Fan m -norm [KyFan1949].

Theorem (Ky Fan branch bound). *Let $A_a : \mathbb{C}^d \to \mathbb{C}^m$ and $B_b : \mathbb{C}^m \to \mathbb{C}^d$ be finite families with $\sum_a A_a^\dagger A_a \leq I_d$ and $\sum_b B_b^\dagger B_b \leq I_m$. Then for every $Z \geq 0$ on $\mathbb{C}^d$,*

$$\sum_{a,b} |\mathrm{Tr}\,(Z B_b A_a)|^2 \le \|Z\|_{(m)}^2 .$$

Equality holds when B_1 is an isometry onto a spectral subspace of the m largest eigenvalues of Z and $A_1 = B_1^\dagger$.

Proof. Let $z_1 \ge \dots \ge z_d \ge 0$ be the eigenvalues of Z, put $z_{d+1} = 0$, and let Π_j be the spectral projector onto the first j eigenvectors. Then $Z = \sum_{j=1}^{d} (z_j - z_{j+1})\Pi_j$, a sum with nonnegative coefficients. Write $K_{ab} = B_b A_a$ and $t_{j,ab} = \mathrm{Tr}\,(\Pi_j K_{ab})$.

First, $\sum_{a,b} |t_{j,ab}|^2 \le \min(j,m)^2$. For any finite family L_k of operators on a space of dimension n with $\sum_k L_k^\dagger L_k \le I_n$, the vector $\omega_n = \sum_i |ii\rangle$ gives $\sum_k |\mathrm{Tr}\, L_k|^2 = \sum_k |\langle \omega_n | (L_k \otimes I) | \omega_n \rangle|^2 \le \|\omega_n\|^2 \sum_k \mathrm{Tr}\, L_k^\dagger L_k \le n \cdot n$, by the Cauchy–Schwarz inequality. Apply this on $\Pi_j \mathbb{C}^d$ (so $n = j$) to $L_{ab} = \Pi_j B_b A_a \Pi_j$, whose sum $\sum L^\dagger L \le \Pi_j$; and on $\mathbb{C}^m$ (so $n = m$) to $L_{ab} = A_a \Pi_j B_b$, which has the same traces and satisfies $\sum_{a,b} L_{ab}^\dagger L_{ab} = \sum_b B_b^\dagger \Pi_j (\sum_a A_a^\dagger A_a) \Pi_j B_b \le I_m$.

Second, the Minkowski inequality in ℓ^2 over the pairs (a,b) gives

$$\Big(\sum_{a,b} |\mathrm{Tr}\,(Z K_{ab})|^2\Big)^{1/2} \le \sum_j (z_j - z_{j+1}) \Big(\sum_{a,b} |t_{j,ab}|^2\Big)^{1/2} \le \sum_j (z_j - z_{j+1}) \min(j,m) = \sum_{i \le m} z_i .$$

The last step is summation by parts. For the equality case, K_{11} is the spectral projector, so $\mathrm{Tr}\,(Z K_{11}) = \|Z\|_{(m)}$. Check DC-44: 240 random factorized families give no violation (largest ratio 0.726), and per-branch seesaws attain the bound.

Status: proved. For $Z = I_d$ the bound is the dimension bound m^2 used in Theorem . For a density operator $Z = \rho$ it bounds the entanglement fidelity of ρ through any channel that factors through m dimensions by $(\sum_{i \le m} \lambda_i(\rho))^2$. par

Non-claim. A matrix inequality for channels between prescribed finite systems. It derives no Page curve, no temperature and no compression rate for a physical source. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Variational formula for the hybrid cut). *For $d \ge 2$, $1 \le m \le d$ and $L \ge 1$,*

$$d^2 F_{\max}(d,L,m) = \max \Big\{ \sum_{x=1}^{L} \|\sqrt{E_x}\|_{(m)}^2 \;:\; E_x \ge 0,\ \sum_{x=1}^{L} E_x = I_d \Big\}.$$

The maximum is attained, and every POVM is realized by an unseeded cut with this value. Hence $e_{\mathrm{seed}} = 1 - F_{\max}$ depends on the instrument only through the spectra of its effects.

Proof. Upper bound. In branch x write $A_{x,a} = \widetilde{A}_{x,a} \sqrt{E_x}$, with $\widetilde{A}_{x,a} = A_{x,a} (E_x^{1/2})^+$. This is valid because $A_{x,a}^\dagger A_{x,a} \le E_x$ forces $\ker E_x \subseteq \ker A_{x,a}$, and $\sum_a \widetilde{A}_{x,a}^\dagger \widetilde{A}_{x,a}$ is the support projector of E_x. Then $\mathrm{Tr}\,(B_{x,b} A_{x,a}) = \mathrm{Tr}\,(\sqrt{E_x} B_{x,b} \widetilde{A}_{x,a})$, and Theorem with $Z = \sqrt{E_x}$ bounds the branch contribution to $d^2 F_e$ (as in the proof of Theorem) by $\|\sqrt{E_x}\|_{(m)}^2$.

Attainment. Let P_x be a rank-m spectral projector of E_x that contains the m largest eigenvalues, and let $V_x : \mathbb{C}^m \to \mathrm{ran}\, P_x$ be unitary. Take the Kraus operators $V_x^\dagger \sqrt{E_x} P_x$ and $|e_1\rangle\langle j| \sqrt{E_x} (I - P_x)$ for $j = 1, \dots, d$, and the decoder V_x. Since P_x commutes with E_x, the effect is E_x. The first operator contributes $\mathrm{Tr}\,(P_x \sqrt{E_x}) = \|\sqrt{E_x}\|_{(m)}$, and the others contribute $\langle j | \sqrt{E_x} (I - P_x) V_x | e_1 \rangle = 0$. The maximum exists by compactness and continuity. Check DC-44 (attainer exact).

Status: proved. It recovers Theorem (a), (c) and (d). Indeed $\|\sqrt{E}\|_{(m)}^2 \le m \|E\|_{(m)} \le m \mathrm{Tr}\, E$, with equality iff $E = wP$ for a rank-m projector P; and $\|\sqrt{E}\|_{(m)}^2 \le m^2$. par

Non-claim. An exact optimization over declared cuts. No universal closed formula for $L \ge 3$ is claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Two labels: complete closed form). *Let $L = 2$, and put $r = \min\{m, d-m\}$ and $k = \max\{0, 2m-d\}$. Then*

$$d^2F_{\max}(d,2,m) = 2r^2 + 2\sqrt{2}\,rk + k^2, \qquad e_{\text{seed}}(d,2,m) = 1 - \frac{2r^2 + 2\sqrt{2}\,rk + k^2}{d^2}.$$

For $2m \le d$ this is $2m^2$ (Proposition). In particular $F_{\max}(3,2,2) = (3+2\sqrt{2})/9$ and

$$e_{\text{seed}}(3,2,2) = \frac{6-2\sqrt{2}}{9} \approx 0.352397,$$

so the upper end of the version-3 bracket is the exact value. The optimum is attained by $E_1 = \operatorname{diag}(1_r, \frac{1}{2}1_k, 0_r)$ and $E_2 = I - E_1$ with the construction of Theorem .

Proof. $E_2 = I - E_1$ commutes with E_1, so both are diagonal in one basis, with eigenvalues x_i and $1 - x_i$. By Ky Fan's maximum principle, $\|\sqrt{E_1}\|_{(m)} = \max_{|S|=m} \sum_{i\in S}\sqrt{x_i}$, and likewise for E_2. Fix m-sets S_1, S_2 with overlap $k' = |S_1 \cap S_2|$; then $k' \ge \max\{0, 2m-d\}$. The value increases if $x_i = 1$ on $S_1 \setminus S_2$ and $x_i = 0$ on $S_2 \setminus S_1$. On the overlap put $u = \sum \sqrt{x_i}$ and $v = \sum \sqrt{1-x_i}$. By Cauchy–Schwarz, $u^2 + v^2 \le k'\sum_i (x_i + 1 - x_i) = k'^2$, so $u + v \le \sqrt{2}\,k'$. With $a = m - k'$ the value is

$$(a+u)^2 + (a+v)^2 = 2a^2 + 2a(u+v) + u^2 + v^2 \le 2(m-k')^2 + 2\sqrt{2}(m-k')k' + k'^2 =: f(k'),$$

with equality at $x_i = \frac{1}{2}$ on the overlap. The derivative $f'(k') = (2\sqrt{2}-4)m + (6-4\sqrt{2})k'$ is affine and negative at $k' = 0$ and at $k' = m$, so f decreases on $[0,m]$ and is maximal at the least admissible overlap $k' = \max\{0, 2m-d\}$. Substituting gives the formula. Theorem gives attainment. Check DC-45 (exact symbolic algebra; diagonal optimization for $d \le 5$; full instrument seesaws for $(3,2,2)$, $(4,2,3)$ and $(5,2,3)$ reproduce 0.647603, 0.728553 and 0.586274) and DC-46.

Status: proved. This closes B2-O20 for $(3,2,2)$ and for every two-label case. par *Non-claim.* The value concerns a declared finite cut with two classical labels. It does not open a Page curve or identify a temperature, and every value has κ-derivative 0. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corollary (Sharpened two-label resource condition). *In Theorem with $L = 2$, the condition $1 - e \le \min\{m/d, 2m^2/d^2\}$ sharpens to $1 - e \le (2r^2 + 2\sqrt{2}rk + k^2)/d^2$. For $m = 2$ and $e = 0.35$, the version-3 condition allows $d \le 3$, but $d = 3$ is excluded, because $F_{\max}(3,2,2) = 0.6476 < 0.65$.*

Proof. Theorem and $1 - e \le F_e$. Check DC-53.

par *Non-claim.* A conditional resource statement; the capacity promise is supplied, not derived. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Three or more labels; partial). *For $L \ge 3$ the value is the maximum in Theorem over possibly noncommuting POVMs. It equals m/d iff $I_d = \sum_x w_x P_x$ with rank-m projectors, a weighted tight fusion frame. The construction literature for such frames is recorded in [Casazza2011]* (source record; not a premise). *Numerical optimization over POVMs (DC-46, evidence only) reaches m/d for $(5,4,2)$ and $(6,4,4)$. It stays below m/d for $(5,3,2)$ (0.395959), $(5,3,3)$ (0.597128), $(4,3,3)$ (0.743686) and $(7,3,3)$ (0.424448).*

Proof. Theorem and Theorem (d). The numbers are local optima of a nonconvex search and are not certificates.

Status: partial. The closed form for $L \ge 3$ and the decomposition criterion stay open (B2-O21, B2-O24). par *Non-claim.* No closed formula or existence criterion for $L \ge 3$ is claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Energy-constrained replacement floor (version 4)

This section implements target P2 (Appendix). The setting is that of Section : M bosonic modes, supplied energy ΔN with $\Delta > 0$, the energy-constrained distance ε_E, and $F(q) = (q + \sqrt{4q - 3q^2})/2$ from Theorem . Write $\nu = E/\Delta$

and $\varepsilon_E^*(\mathcal{N}) = \inf_{\mathcal{D}} \varepsilon_E(\mathcal{DN}, \mathrm{id})$. For a replacement channel, $\mathcal{DR} = \mathcal{R}_\sigma$ with $\sigma = \mathcal{D}(\text{output})$, so $\varepsilon_E^*(\mathcal{R}) = \inf_\sigma \varepsilon_E(\mathcal{R}_\sigma, \mathrm{id})$. This is the energy-constrained value left open in Theorem (f).

Lemma (Pure inputs and a scalar dual test). *Restricting the supremum in $\varepsilon_E(\mathcal{R}_\sigma, \mathrm{id})$ to pure inputs ψ_{AR}, with any reference R, loses nothing. For such ψ,*

$$\tfrac{1}{2}\|\psi\psi^\dagger - \sigma \otimes \psi_R\|_1 = \max_{\|v\| = 1} (|\mathrm{ip}\, v\psi|^2 - \langle v|\sigma \otimes \psi_R|v\rangle).$$

Let $\alpha > 0$ and $\beta \geq 0$. If $\langle v|(\alpha + I_A \otimes Y_v + \beta N \otimes I_R)^{-1}|v\rangle \leq 1$ for every unit v of finite energy, where $Y_v = \mathrm{Tr}_A[(\sigma \otimes I)vv^\dagger]$, then $\tfrac{1}{2}\|\psi\psi^\dagger - \sigma \otimes \psi_R\|_1 \leq \alpha + \beta\langle\psi|N \otimes I|\psi\rangle$ for every finite-energy pure input.

Proof. A mixed input with a purification on a larger reference has the same energy, and appending an identity on the extra reference does not lower the distance. The difference $\psi\psi^\dagger - \sigma \otimes \psi_R$ is a rank-one positive operator minus a positive operator and has trace 0, so it has exactly one nonnegative eigenvalue, which equals half the trace norm. For fixed v, $\langle v|\sigma \otimes \psi_R|v\rangle = \langle\psi|I_A \otimes Y_v|\psi\rangle$, so the bracket minus $\beta\langle\psi|N \otimes I|\psi\rangle$ is $\langle\psi|(vv^\dagger - K_v)|\psi\rangle$ with $K_v = I \otimes Y_v + \beta N \otimes I \geq 0$. Finally, $\lambda_{\max}(vv^\dagger - K_v) \leq \alpha$ iff $vv^\dagger \leq \alpha + K_v$ iff $\langle v|(\alpha + K_v)^{-1}|v\rangle \leq 1$.

par *Non-claim.* A variational identity for declared channels only. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Exact low-energy regimes).

• *(Any $M \geq 1$.) For $0 \leq v \leq \tfrac{1}{3}$, $\varepsilon_E^*(\mathcal{R}) = F(v)$, attained by the vacuum output. For the cutoff-and-reset channel $\mathcal{C}_n$ with $q = E/(\Delta(n+1)) \leq \tfrac{1}{3}$, $\varepsilon_E^*(\mathcal{C}_n) = \varepsilon_E(\mathcal{C}_n, \mathrm{id}) = F(q)$: the identity decoder is optimal.*

• *(One mode.) For $\tfrac{1}{3} \leq v \leq v_2 := 2 - \tfrac{\sqrt{22}}{3} \approx 0.436528$, $\varepsilon_E^*(\mathcal{R}) = 3v(1-v)$, attained by the output $\sigma_v = (2 - 3v)\mathrm{proj}\,0 + (3v - 1)\mathrm{proj}\,1$. For $v > \tfrac{1}{3}$ the vacuum output is strictly worse: $F(v) > 3v(1-v)$ on $(\tfrac{1}{3}, 1)$.*

• *(One mode, beyond.) For $v \geq v_2$,*

$$\max\{3\bar{v}(1-\bar{v}),\ 1 - \frac{1}{K_v^2}\} \leq \varepsilon_E^*(\mathcal{R}) \leq \min\{1,\ 3v_2^2 + (3 - 6v_2)v\},$$

with $\bar{v} = \min\{v, \tfrac{1}{2}\}$ and $K_v = \lfloor 2v \rfloor + 1$.

• *(Any M, lower bounds.) $\varepsilon_E^*(\mathcal{R}) \geq \max\{F(\min\{v, \tfrac{1}{3}\}), 3\bar{v}(1-\bar{v})\}$ for every v, and $\varepsilon_E^*(\mathcal{R}) \leq F(\min\{v, 1\})$.*

Proof. Lower bounds. Use the test input $\psi_q = \sqrt{1-q}|0\rangle|0\rangle + \sqrt{q}|1\rangle|1\rangle$ (one quantum in one mode, energy $q\Delta$, so $q \leq v$ is admissible), and any output σ. For every unit p, the distance is at least $|\mathrm{ip}\, p\psi_q|^2 - \langle p|\sigma \otimes \psi_R|p\rangle$ (Lemma). (i) Let $s = \sqrt{q(1-q)}$ and $F = F(q)$, and take $p \propto s|00\rangle + F|11\rangle$, the positive eigenvector of $\psi_q\psi_q^\dagger - \mathrm{proj}\,0 \otimes \psi_R$. For the vacuum output, the value is F. For a general σ, the subtracted term is $(s^2(1-q)\sigma_{00} + F^2 q\sigma_{11})/(s^2 + F^2) \leq s^2(1-q)/(s^2 + F^2)$, because $\sigma_{00} + \sigma_{11} \leq 1$ and $s^2(1-q) \geq F^2 q$. The last inequality is equivalent to $F \leq 1 - q$, that is, to $(3q - 1)(q - 1) \geq 0$, that is, to $q \leq \tfrac{1}{3}$. Hence the distance is at least $F(q)$ for $q = \min\{v, \tfrac{1}{3}\}$. (ii) Take $p = \sqrt{q}|00\rangle + \sqrt{1-q}|11\rangle$. Then $|\mathrm{ip}\, p\psi_q|^2 = 4q(1-q)$ and $\langle p|\sigma \otimes \psi_R|p\rangle = q(1-q)(\sigma_{00} + \sigma_{11}) \leq q(1-q)$, so the distance is at least $3q(1-q)$. (iii) The Bell state on levels $0, \ldots, K-1$ of one mode has energy $\Delta(K-1)/2$, and a decoded replacement has Bell overlap at most K^{-2} (Theorem (d)). For $\mathcal{C}_n$, use $\sqrt{1-q}|0\rangle|0\rangle + \sqrt{q}|n+1\rangle|1\rangle$, where $|n+1\rangle$ has $n+1$ quanta in one mode. Then $\mathcal{C}_n \otimes \mathrm{id}$ maps it to $\mathrm{proj}\,0 \otimes \psi_R$, since $P_n|n+1\rangle = 0$ and $\mathrm{Tr}[(I - P_n)|0\rangle\langle n+1|] = 0$. Every decoded output is therefore $\sigma \otimes \psi_R$, and (i) applies with $|1\rangle$ replaced by $|n+1\rangle$.

Upper bounds. (a) The vacuum output is $\mathcal{C}_0$, and for $\mathcal{C}_n$ the identity decoder gives $F(q)$; both by Theorem . (b) Apply Lemma with $\alpha = 3v^2$, $\beta = 3 - 6v$ and σ_v, so $s_0 = 2 - 3v$ and $s_1 = 3v - 1$. Write $v = \sum_k |k\rangle \otimes |r_k\rangle$ in the number

basis. Then $\langle v|(\alpha + K_v)^{-1}|v\rangle = \sum_k \langle r_k|(\alpha + \beta k + Y)^{-1}|r_k\rangle$ with $Y = s_0|r_0\rangle\langle r_0| + s_1|r_1\rangle\langle r_1|$. Use $Y \geq s_0|r_0\rangle\langle r_0|$, $Y \geq s_1|r_1\rangle\langle r_1|$ and $Y \geq 0$, together with operator monotonicity of inversion. With $u = \|r_0\|^2$ and $w = \|r_1\|^2$, this sum is at most

$$\Phi(u, w) = \frac{u}{\alpha + s_0 u} + \frac{w}{\alpha + \beta + s_1 w} + \frac{1 - u - w}{\alpha + 2\beta}.$$

Φ is concave on $\{u, w \geq 0, u + w \leq 1\}$. At $(u, w) = (v, 1 - v)$ we have $\alpha + s_0 v = 2v$ and $\alpha + \beta + s_1(1 - v) = 2(1 - v)$, so $\Phi = \frac{1}{2} + \frac{1}{2} = 1$, and both partial derivatives equal $\frac{3}{4} - (\alpha + 2\beta)^{-1}$. This is ≥ 0 iff $9v^2 - 36v + 14 \geq 0$, which for $v \leq \frac{1}{2}$ means $v \leq v_2$. The Karush–Kuhn–Tucker conditions then give $\max \Phi = 1$. Hence the distance is at most $\alpha + \beta v = 3v(1 - v)$. Strictness: $F(v) - 3v(1 - v) = \frac{1}{2}[g - (5v - 6v^2)]$ with $g = \sqrt{4v - 3v^2}$, and $g^2 - (5v - 6v^2)^2 = -4v(3v - 1)^2(v - 1) > 0$ for $v \in (\frac{1}{3}, 1)$. (c) The certificate of (b) with its parameters frozen at v_2 satisfies $\Phi \leq 1$ independently of E. So the distance is at most $\alpha_2 + \beta_2 E(\psi)/\Delta$ for every input, which gives the upper bound. (d) The lower bounds (i)–(ii) use one mode, and the upper bound is the vacuum output. Checks DC-47 and DC-48: a truncated joint SDP over outputs and dual variables reproduces $F(v)$ for $v \leq \frac{1}{3}$ and $3v(1 - v)$ on $[\frac{1}{3}, v_2]$, with the optimal output $\mathrm{diag}\,(2 - 3v, 3v - 1)$. The dual inequality holds in a truncated SDP and on random two-vector configurations (maximum 0.99954). The polynomial identities are verified in SymPy. For $\mathscr{C}_n$, the optimized SDP equals $F(q)$ at $q = 0.1, 0.2, 0.3$.

Status: (a) and (b) proved; (c) and (d) brackets. At $v = 0.5, 0.7$ and 1 the truncated SDP gives 0.761206, 0.824663 and 0.891380. The certified brackets are $[0.75, 0.762086]$, $[0.75, 0.838252]$ and $[0.888889, 0.952502]$. The SDP values are evidence only (B2-O23). par *Non-claim.* The energy promise, Δ and the mode count are declared data. No temperature, thermal state, noise channel or physical cutoff is derived, and no dimension e^S is substituted. The optimal output σ_v is a decoder choice, not a bath. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Scaling of the approximate coding cost (version 4)

This section implements target P3. Theorem (d) showed only that entropy continuity cannot give a dimension-free $O(\varepsilon \ln(1/\varepsilon))$ cost. Here a code shows that the $\ln K$ term is necessary for the entropy forms of the cost, and that the dimension form remains open. Notation: h and g as in Theorem , and Ξ as in Theorem .

Theorem (Flag code: the coefficient $\varepsilon_A + \varepsilon_B$ of $\ln K$ is tight). *Let $K \geq 2$ and $a, b \geq 0$ with $a + b \leq 1$. On $\mathscr{H}_C \otimes \mathscr{H}_A \otimes \mathscr{H}_B$, each factor $\mathbb{C}^K \oplus \mathbb{C}|\varnothing\rangle$, put*

$$V|i\rangle = \sqrt{1 - a - b}\,|i\rangle_C|\varnothing\rangle_A|\varnothing\rangle_B + \sqrt{a}\,|\varnothing\rangle_C|i\rangle_A|\varnothing\rangle_B + \sqrt{b}\,|\varnothing\rangle_C|\varnothing\rangle_A|i\rangle_B.$$

Then V is an isometry, and:

- *erasures of A and of B are recoverable with errors $\varepsilon_A = a(1 - K^{-2})$ and $\varepsilon_B = b(1 - K^{-2})$;*
- *$I(R : A) = 2a \ln K$ and $I(R : B) = 2b \ln K$, so $\Xi = (a + b)\ln K = (\varepsilon_A + \varepsilon_B)\ln K/(1 - K^{-2})$;*
- *$\ln K - S(C) = (a + b)\ln K - h(a + b)$. Consequently, over all codes with erasure errors at most $\varepsilon_A, \varepsilon_B$,*

$$(\varepsilon_A + \varepsilon_B)\ln K \leq \sup \Xi \leq (\varepsilon_A + \varepsilon_B)\ln K + \tfrac{1}{2}(g(\varepsilon_A) + g(\varepsilon_B)),$$

and the same upper bound holds for $\sup(\ln K - S(C))$, which is at least $(\varepsilon_A + \varepsilon_B)\ln K - h((\varepsilon_A + \varepsilon_B)/(1 - K^{-2}))$ when $a + b \leq \frac{1}{2}$. No bound on Ξ or on $\ln K - S(C)$ that is independent of K holds at fixed errors. The replacement for the refuted dimension-free cost is $(\varepsilon_A + \varepsilon_B)\ln K + O(\varepsilon \ln(1/\varepsilon))$, and it is tight to first order in $\ln K$.

Proof. The three branches carry orthogonal flags, so V is isometric. For an erasure of A, the decoder on BC outputs i from C when $C \neq \varnothing$ and from B when $B \neq \varnothing$, and outputs I/K when both are $\varnothing$. Each surviving branch returns Φ_K exactly, so the output is $(1 - a)\Phi_K + a\, I/K \otimes I/K$, at trace distance $a(1 - K^{-2})$ from Φ_K; similarly for B.

The marginals are $\rho_A = a\,I/K \oplus (1-a)\mathrm{proj}\,\varnothing$ and $\rho_{RA} = a\Phi_K \oplus (1-a)I/K \otimes \mathrm{proj}\,\varnothing$, so $I(R:A) = \ln K + (a\ln K + h(a)) - (h(a) + (1-a)\ln K) = 2a\ln K$. Also $\rho_C = (1-a-b)I/K \oplus (a+b)\mathrm{proj}\,\varnothing$. The upper bounds are Theorem (a),(c) and the proof of , where $2\ln K \le 2S(C) + I(R:A) + I(R:B)$. Check DC-49 ($K = 2, \ldots, 5$; mutual informations, entropies and decoder errors agree with the formulas to 10^{-9}).

Status: proved. Theorem (d) said the allowance $(\varepsilon_A + \varepsilon_B)\ln K$ is ``the correct allowance''. That is now justified for Ξ and for $\ln K - S(C)$, not for $\ln\dim\mathscr{H}_C$ (Proposition). par *Non-claim.* A declared code on finite registers. No physical error model, holographic code or threshold is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (The dimension form; partial). *Let* $G_{\dim} = \ln K - \ln\dim\mathscr{H}_C$.

• $G_{\dim} \le (\varepsilon_A + \varepsilon_B)\ln K + \frac{1}{2}(g(\varepsilon_A) + g(\varepsilon_B))$, *by* .

• *The flag code has* $G_{\dim} = -\ln(1 + K^{-1}) < 0$.

• *The allocation code stores* $c < K$ *basis vectors in* C *(so* $\dim\mathscr{H}_C = c + 1$*) and the rest in A. It has* $\varepsilon_B = 0$, $\varepsilon_A \le F(x)$ *with* $x = (K-c)/K$*, and* $G_{\dim} = \ln(K/(c+1)) \to -\ln(1-x) \ge x$ *as* $K \to \infty$ *at fixed x. This is a dimension-free gain of order* ε_A^2*. Whether* $G_{\dim}$ *can grow like* $\varepsilon\ln K$ *is open (B2-O22).*

Proof. (b) $\dim\mathscr{H}_C = K + 1$. (c) The decoder returns the index from C when $C \ne \varnothing$, and $|0\rangle$ otherwise. On span $\{\hat{\xi}, \hat{\eta}\}$, with $\hat{\xi}, \hat{\eta}$ the normalized kept and lost parts of Φ_K, the error operator is the block $\left(\begin{smallmatrix} 0 & s \\ s & x \end{smallmatrix}\right)$ with $s = \sqrt{x(1-x)}$, plus an orthogonal term of trace $-x$. Its positive part is $F(x)$, as in Theorem . Check DC-49 ($K = 6, 10, 20$).

Status: (a)–(c) proved; the growth question is open. par *Non-claim.* No dimension-free bound on $G_{\dim}$ is claimed, and none is refuted. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Early-universe interface: power-law prefixes and rate-bounded futures (version 4)

This section implements target P5. It keeps the conventions of Section : $B = c/H_0$, $D = c/H_1$, and every statement concerns a *prescribed* metric and *declared* labels.

Definition (Power-law prefix). Fix $\gamma \ge 0$, $c, H_0 > 0$ and t_i with $1 + \gamma H_0 t_i > 0$. Put $a_\gamma(t) = (1 + \gamma H_0 t)^{1/\gamma}$ for $\gamma > 0$ and $a_0(t) = e^{H_0 t}$, on $[t_i, 0]$. Then $H = H_0/(1 + \gamma H_0 t)$ and $\frac{\mathrm{d}}{\mathrm{d}t}(1/H) = \gamma$. The exponent is a declared number, and the name $\gamma = \frac{3}{2}(1+w)$ is a bookkeeping convention: $\gamma = 2$ ($w = \frac{1}{3}$) is the radiation-type prefix of Definition , and $\gamma = 0$ is a de Sitter prefix. Comparison futures are defined as in Definition , with a_γ in place of a_{r}. On the prefix, with $z = a_\gamma(t)$, we have $\mathrm{d}t = z^{\gamma-1}\mathrm{d}z/H_0$ and

$$D_\gamma(t) := c\int_t^0 \frac{\mathrm{d}s}{a_\gamma} = \begin{cases} B(1 - z^{\gamma-1})/(\gamma - 1), & \gamma \ne 1, \\ B\ln(1/z), & \gamma = 1. \end{cases}$$

Definition (Label dictionary for a declared exponent $w > 0$). Supply $U, \alpha, V_c > 0$ and an energy-scale slot $\varepsilon_s > 0$, all independently of the geometry, and keep $\kappa > 0$ external. Put $n_w = (1+w)/w$, $u = Ua^{-3(1+w)}$, $V = V_c a^3$ and

$$T_w^{(\kappa)} = \frac{1}{k_B}\left(\frac{u\,\kappa^3 c^3 \varepsilon_s^{n_w - 4}}{\alpha}\right)^{1/n_w}, \qquad S_w^{(\kappa)} = \frac{(1+w)\,uV}{T_w^{(\kappa)}}.$$

For $w = \frac{1}{3}$ this is Definition , and ε_s drops out. The labels are defined only for $w > 0$.

Analogy (heuristic, not claimed) (Fluid reading of the exponent). With an Einstein equation and a perfect fluid with $p = w\rho c^2$ *supplied*, $\frac{\mathrm{d}}{\mathrm{d}t}(1/H) = \frac{3}{2}(1+w)$ and $u \propto a^{-3(1+w)}$, and S_w has the Gibbs–Duhem form $(\rho c^2 + p)V/T$ at

zero chemical potential. None of this is used below.

Theorem (General-*w* label certificate). *At fixed* $(g_a|_{[t_i,0]}, c, k_B, U, \alpha, V_c, \varepsilon_s, w)$*:*

• $T_w \propto \kappa^{3w/(1+w)} a^{-3w}$ *and* $S_w \propto \kappa^{-3w/(1+w)}$*, with* S_w *independent of t. So* $\partial_\kappa \ln T_w = \frac{3w}{(1+w)\kappa} > 0$ *and* $\partial_\kappa \ln S_w = -\frac{3w}{(1+w)\kappa} < 0$*. Each label ranges over* $(0,\infty)$*, and every data-only estimator of either label has infinite worst-case error.*

• $T_w S_w = (1+w) U V_c a^{-3w}$*, so* $U = a^{3w} T_w S_w / ((1+w) V_c)$*.*

• *The Jacobian of* $(\ln T_w, \ln S_w)$ *with respect to* $(\ln U, \ln \kappa)$ *is* $-3w/(1+w) \neq 0$*; for* $w = \frac{1}{3}$ *it is* $-\frac{3}{4}$*. So the pair* (T_w, S_w) *at one time determines* (U, κ)*, whereas* T_w *alone is invariant under* $(U,\kappa) \mapsto (r^{-3}U, r\kappa)$*.*

Proof. Substitution gives $\ln T_w = \frac{w}{1+w}(\ln U + 3\ln\kappa) - 3w \ln a + \text{const}$ and $\ln S_w = \frac{1}{1+w}\ln U - \frac{3w}{1+w}\ln\kappa + \text{const}$, and the determinant is $-\frac{3w^2+3w}{(1+w)^2}$. The rest is as in Theorem . Check DC-50 (SymPy).

par *Non-claim.* Calibration is conditional on declared data. No thermal state or equation of state is derived, and the fluid reading is heuristic. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Flip four-volume on a power-law prefix). *In the setting of Theorem , on the prefix* a_γ *with* $\gamma \neq 1$*, put* $\beta_\gamma = B/(\gamma - 1)$*,* $C_\pm = R_\pm + \beta_\gamma$ *and* $\Delta C^j = C_+^j - C_-^j$*. Then*

$$\mathscr{V}_4 = \frac{4\pi c}{3H_0}\Big[\frac{\Delta C^3}{\gamma+3} z^{\gamma+3} - \frac{3\beta_\gamma \Delta C^2}{2\gamma+2} z^{2\gamma+2} + \frac{3\beta_\gamma^2 \Delta C}{3\gamma+1} z^{3\gamma+1}\Big]_{z_a}^{z_b}.$$

For $\gamma = 1$*,* $\mathscr{V}_4 = \frac{4\pi c}{3H_0}\int_{z_a}^{z_b} z^3 [(B\ln\frac{1}{z} + R_+)^3 - (B\ln\frac{1}{z} + R_-)^3]\mathrm{d}z$*, which is elementary. For* $\gamma = 2$ *the formula reduces to Theorem (there* $C_\pm = B + R_\pm$*, since* $\beta_2 = B$*), and for* $\gamma = 0$ *it is the de Sitter prefix. The minimax statement of Theorem holds unchanged.*

Proof. $D_\gamma + R_\pm = C_\pm - \beta_\gamma z^{\gamma-1}$. Expanding the cubic difference and integrating $z^3 \cdot z^{\gamma-1}\mathrm{d}z/H_0$ termwise gives the formula; all exponents are positive for $\gamma \geq 0$. Check DC-51: formula against quadrature for $\gamma \in \{0, \frac{1}{2}, 1, 2, 3\}$ ($c = 1$, $H_0 = 2$, $R_\pm = 1, \frac{3}{4}$, $t_a = -\frac{1}{16}$); SymPy reduction at $\gamma = 2$; the version-3 instance 0.278293.

par *Non-claim.* A geometric measure on a prescribed host. It is not an entropy or an information count. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Rate-bounded futures and deceleration). *Use the radiation-type host of Definition , and let* $0 < H_1 < H_0$*,* $\rho = D/B > 1$ *and* $\gamma_{\max} \geq 2$*. Let* $\mathscr{F}^{\gamma_{\max}}(H_1)$ *be the futures in* $\mathscr{F}(H_1)$ *with* $0 \leq \frac{\mathrm{d}}{\mathrm{d}t}(1/H) \leq \gamma_{\max}$ *on* $(0,\infty)$*, and* $\mathscr{F}_\tau^{\gamma_{\max}}(H_1)$ *those that also settle by* τ*. Put*

$$\overline{R} = B\frac{\gamma_{\max}\rho^{1-1/\gamma_{\max}} - 1}{\gamma_{\max} - 1}, \qquad t_* = \frac{1/H_1 - 1/H_0}{\gamma_{\max}}.$$

• $\{R_0[a] : a \in \mathscr{F}^{\gamma_{\max}}(H_1)\} = (B, \overline{R})$ *and* $B < \overline{R} < D$*; for* $\gamma_{\max} = 2$*,* $\overline{R} = 2\sqrt{BD} - B$*. As* $\gamma_{\max} \to \infty$*,* $\overline{R} \to D$*, the range of Theorem (b).*

• *For* $\tau > t_*$*,* $\{R_0[a] : a \in \mathscr{F}_\tau^{\gamma_{\max}}(H_1)\} = (B + (\overline{R} - B)e^{-H_0(\tau - t_*)}, \overline{R})$*, and for* $\tau < t_*$ *the class is empty.*

• *The exact minimax error of an early-data-only estimate of* $R_e(t)$*,* $t \leq 0$*, over* $\mathscr{F}^{\gamma_{\max}}(H_1)$ *is* $z(\overline{R} - B)/2$*.*

• *If* $\ddot{a} \leq 0$ *on* $(0,\infty)$*, then* $a(t) \leq 1 + H_0 t$ *and* $R_0 = \infty$*: no decelerating future has an event horizon, whatever its labels.*

Proof. Write $u = 1/H$, so $R_0 = c\int_0^\infty e^{-b}$ with $b(t) = \int_0^t \mathrm{d}s/u$. The constraints are linear in u: $u(0) = 1/H_0$, the radiation

germ ($u'(0) = 2$ and $u^{(k)}(0) = 0$ for $k \geq 2$, from smooth matching at 0), $0 \leq u' \leq \gamma_{\max}$, and $u = 1/H_1$ eventually. They give the pointwise bounds $1/H_0 \leq u(t) \leq \min\{1/H_0 + \gamma_{\max} t, 1/H_1\}$. Since R_0 decreases in b and b decreases in u, R_0 is monotone in u. The upper profile gives $c\int_0^{t_*}(1+\gamma_{\max}H_0 t)^{-1/\gamma_{\max}}\mathrm{d}t + (c/H_1)\rho^{-1/\gamma_{\max}} = \overline{R}$, and the lower gives B. Both inequalities are strict. Near 0, u' is close to 2, so $u > 1/H_0$ on some $(0,\delta)$, and for $\gamma_{\max} > 2$ also $u < 1/H_0 + \gamma_{\max} t$ there. For $\gamma_{\max} = 2$ the upper profile has a corner at t_*, so a smooth u lies strictly below it on an open set. The extremal profiles are limits of admissible smooth profiles (smooth the corners with a flat germ cutoff as in Theorem (b)). The admissible set is convex and R_0 is continuous along segments, so the open interval is filled. $\overline{R} < D$ because $(\gamma\rho^{1-1/\gamma} - 1)/(\gamma - 1) < \rho$ for $\rho > 1$ (strict concavity of $\rho \mapsto \rho^{1-1/\gamma}$). (b) Settling by τ forces $u(t) \geq \max\{1/H_0, 1/H_1 - \gamma_{\max}(\tau - t)\}$. The corresponding profile gives $B(1 - e^{-H_0(\tau - t_*)}) + e^{-H_0(\tau - t_*)}\overline{R}$, and the upper profile settles at $t_* < \tau$. (c) As in Theorem (b). (d) a is nonincreasing, so $a \leq a(0) = H_0$, and $\int_0^\infty \mathrm{d}t/(1 + H_0 t) = \infty$. Check DC-52: extremal profiles reproduce $\overline{R}$ and the deadline infimum to 10^{-6}; four admissible profiles per row lie strictly inside; SymPy limits; logarithmic divergence of the partial integrals.

Status: proved. The comparison with the version-3 range (B, D) shows that the earlier interval is the $\gamma_{\max} \to \infty$ limit of a quantitative family.

Analogy (heuristic, not claimed) (Energy-condition reading). Under a *supplied* Einstein equation with a single effective fluid, $\frac{\mathrm{d}}{\mathrm{d}t}(1/H) = \frac{3}{2}(1 + w_{\mathrm{eff}})$. Then $0 \leq \frac{\mathrm{d}}{\mathrm{d}t}(1/H)$ is the null-energy condition, and $\gamma_{\max} = 3$ is the bound $w_{\mathrm{eff}} \leq 1$ that one associates with the dominant energy condition. Here both are geometric conditions on a prescribed H.

par *Non-claim.* $\gamma_{\max}$ and the deadline are declared. No energy condition is derived, and no matter model is selected. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Re-check of version-3 statements (version 4)

This section implements target P6. The displayed constants and formulas of Sections and were recomputed independently (DC-53, plus the cross-checks inside DC-45 to DC-52):

• the radiation Jacobian $-\frac{3}{4}$ and C_{r} (Theorem);

• the flip four-volume polynomial and the instance $19049\pi/215040 \approx 0.278293$ (Theorem);

• the range endpoints of Theorem (b);

• the cutoff-and-reset values $F(q)$ (Theorem);

• the replacement gap $(md - 1)/d^2$, the erasure interpolation and the Λ_m coefficient $(m - 1)/(d - 1)$ (Theorem);

• epoch composition (Proposition);

• the bracket of Proposition . **No erratum was found.** The version-3 text stands unchanged. The following are *sharpenings*, recorded by addition and not as corrections:

• Proposition : the upper end $(6 - 2\sqrt{2})/9$ is the exact value of $e_{\mathrm{seed}}(3,2,2)$ (Theorem); the version-3 sentence ``remains open'' is superseded by addition.

• Theorem (f) and B2-O10: the energy-constrained replacement value is now exact for $v \leq \frac{1}{3}$, and for one mode up to v_2 (Theorem); beyond that it is bracketed.

• Theorem (d): ``the correct allowance'' is now proved to first order for Ξ and for $\ln K - S(C)$ (Theorem); for $\ln\dim\mathscr{H}_C$ it is an upper allowance whose sharpness is open (Proposition).

• Theorem : for $L = 2$ the resource condition is sharpened (Corollary).

• Theorem (b): the range (B, D) is the $\gamma_{\max} \to \infty$ limit of Theorem (a). par *Non-claim.* A re-check is a recomputation by the same author with new scripts. It is not an independent review. Ledger unchanged: no

quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

What is deliberately not done (version 4)

• No closed form for $F_{\max}(d, L, m)$ with $L \geq 3$, and no existence criterion for weighted rank-m decompositions of I_d.

• No exact energy-constrained replacement value for one mode beyond ν_2, or for $M \geq 2$ modes beyond $\nu = \frac{1}{3}$. The SDP values are evidence only.

• No dimension-free bound on $\ln K - \ln \dim \mathscr{H}_C$, and none is refuted.

• No derivation of $a(t)$, of an equation of state or of an energy condition. The exponent γ and the bound $\gamma_{\max}$ are declared.

• No thermal state, physical noise channel or entropy identification. No coefficient slot κ or ε_s is filled, and no quantum coefficient is admitted.

• No discharge of O.1, XV-B, XVI-N or Gate 0. No revision of any parent sheet. Citations identify source records only; no proof transfer.

Emergence-page sentence, version 4 (not registered)

The following sentence is recorded for use if the sheet is later registered. It is not registered here. Big02-A-v4 closes the two-label seeded optimum and two low-energy replacement regimes, and quantifies the coding-cost scaling and the rate-bounded early-junction ranges on prescribed hosts. None of its theorems is spacetime emergence, a Page curve, or a derived temperature or entropy.

General (L, m): decomposition criteria and a finite conic value (version 5)

This section implements item 1 of the plan t283u (Appendix) and the hybrid-cut items of the user draft t285u (Appendix). The notation is that of Definition and Theorem . We say that (d, L, m) *decomposes* if $I_d = \sum_{x=1}^{L} w_x P_x$ with $w_x \geq 0$ and rank-m orthogonal projectors P_x. By Theorem (d), when $Lm > d$ this is exactly the condition for $F_{\max}(d, L, m) = m/d$.

Unconditional rank conditions and the case L = 3

Proposition (Rank lemma, complement and Naimark moves). *Let $1 \leq m < d$, and let $I_d = \sum_{x=1}^{L'} w_x P_x$ with all $w_x > 0$ and rank-m projectors P_x.*

• *Every $w_x \leq 1$. For each y, either $w_y < 1$, and then $(L' - 1)m \geq d$; or $w_y = 1$, and then $\sum_{x \neq y} w_x P_x = I - P_y$ is a decomposition of type $(d - m, L' - 1, m)$ on $\operatorname{ran}(I - P_y)$.*

• *(N1) $(L' - 1)m \geq d$ or $L'm = d$. In particular, if $(L - 1)m < d < Lm$, then (d, L, m) does not decompose.*

• *(Complement.) $\sum_x w_x = d/m =: W$, and if $W > 1$, then $I_d = \sum_x \frac{w_x}{W-1}(I - P_x)$. Hence (d, L, m) decomposes iff $(d, L, d - m)$ does, and (N2) $(L' - 1)(d - m) \geq d$ or $L'(d - m) = d$.*

• *(Naimark move.) If all $w_x < 1$, there are rank-m projectors P'_x on $\mathbb{C}^{L'm-d}$ with $I = \sum_x (1 - w_x)P'_x$.*

Proof. (a) $w_y P_y \leq I$ gives $w_y \leq 1$. If $(L' - 1)m < d$, the kernels of the P_x, $x \neq y$, meet in a nonzero subspace. For a unit vector v there, $w_y P_y v = v$, so $w_y = 1$. If $w_y = 1$, then $\sum_{x \neq y} w_x P_x = I - P_y$, and positivity puts every $\operatorname{ran} P_x$, $x \neq y$, inside $\operatorname{ran}(I - P_y)$. (b) If $(L' - 1)m < d$, (a) forces all weights to be 1, and the trace gives $L'm = d$. (c) Take traces. The complement identity is $\sum_x w_x(I - P_x) = (W - 1)I$, and the rank of $I - P_x$ is $d - m$. Apply (b) to the complement. (d) Let $V_x : \mathbb{C}^m \to \mathbb{C}^d$ be isometries onto $\operatorname{ran} P_x$. Then $T = (\sqrt{w_x} V_x^\dagger)_x : \mathbb{C}^d \to \mathbb{C}^{L'm}$ is an isometry, and the diagonal blocks of $TT^\dagger$ are $w_x I_m$. Let $U = (U_x)_x$ be an isometry onto $\operatorname{ran}(I - TT^\dagger)$, of dimension $L'm - d$. Then $\sum_x U_x^\dagger U_x = I$

and $U_x U_x^\dagger = (1 - w_x) I_m$, so $P_x' = U_x^\dagger U_x/(1 - w_x)$ is a rank-m projector. Check DC-67.

Status: proved. Part (b) is part (a) of the draft's Corollary cor:projector-obstructions, and part (c) is its part (b). Moves (c) and (d) are the spatial and Naimark dualities of the fusion-frame literature [Casazza2011,BLR2015] (source record; not a premise). par *Non-claim.* A statement about weighted projector sums on a declared finite space. No physical cut is identified. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Three labels: complete classification). $(d, 3, m)$ *decomposes if and only if* $d/m \in \{1, \frac{3}{2}, 2, 3\}$. *The constructions are:* $m = d$ *(one projector);* $d = 2m$ *(the trine* $\{|\varphi_k\rangle\langle\varphi_k|\}_{k=0,1,2}$, $\varphi_k = (\cos\frac{k\pi}{3}, \sin\frac{k\pi}{3})$, *tensored with* I_m, *weights* $\frac{2}{3}$*);* $d = 3m$ *(orthogonal blocks, weights* 1*);* $m = \frac{2d}{3}$ *(complements of three orthogonal rank-*$\frac{d}{3}$ *projectors, weights* $\frac{1}{2}$*). Consequently:*

• *if* $3m > d$, *then* $F_{\max}(d, 3, m) = m/d$ *exactly when* $d/m \in \{1, \frac{3}{2}, 2\}$, *and otherwise* $F_{\max}(d, 3, m) < m/d$*;*

• *for* $(3, 3, 2)$, $P_x = I_3 - \operatorname{proj} x$ *and* $w_x = \frac{1}{2}$, *so* $e_{\text{seed}}(3, 3, 2) = \frac{1}{3}$*;*

• $(5, 3, 2)$, $(5, 3, 3)$, $(4, 3, 3)$ *and* $(7, 3, 3)$ *do not decompose, so the version-4 values of Proposition lie strictly below* m/d.

Proof. Necessity. With at most three nonzero weights, Proposition (b),(c) with $L' \le 3$ give: $2m \ge d$ or $3m = d$ (or $2m = d$, or $m = d$); and $m \le d/2$ or $3(d - m) = d$ (or the corresponding $L' \le 2$ cases). Intersecting gives $d/m \in \{1, \frac{3}{2}, 2, 3\}$. Sufficiency is the list of constructions, each verified by direct summation (the trine satisfies $\sum_k |\varphi_k\rangle\langle\varphi_k| = \frac{3}{2} I_2$). For (a), when $3m \le d$, $F_{\max} = 3m^2/d^2$ (Theorem (c)), which equals m/d only at $d = 3m$; when $3m > d$, equality with m/d is the decomposition (Theorem (d)), and the maximum is attained (Theorem). Checks DC-55 (Horn linear program agrees, $d \le 7$) and DC-67.

Status: proved. This closes B2-O21 for $L = 3$ without any external spectral theorem, and it gives parts (c),(d) of the draft's Corollary cor:projector-obstructions unconditionally. par *Non-claim.* An exactness criterion for a declared three-label cut. No closed form of $F_{\max}(d, 3, m)$ below m/d is claimed here. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

General L: a finite linear criterion and a finite conic value (conditional on a cited theorem)

Definition (Horn index sets; draft t285u). For $I = \{i_1 < \dots < i_r\} \subseteq \{1, \dots, d\}$ put $\lambda(I) = (i_r - r, \dots, i_1 - 1)$. Let $\mathscr{H}_{d,L}$ be the finite set of tuples $(r; I_0, I_1, \dots, I_L)$, $1 \le r < d$, $|I_x| = r$, for which the Littlewood–Richardson coefficient $[s_{\lambda(I_0)}]\, s_{\lambda(I_1)} \cdots s_{\lambda(I_L)}$ is positive. Let $\mathscr{P}_{d,L}$ be the polytope of arrays (λ_{xj}) with $1 \ge \lambda_{x1} \ge \dots \ge \lambda_{xd} \ge 0$, $\sum_{x,j} \lambda_{xj} = d$ and $\sum_x \sum_{j \in I_x} \lambda_{xj} \ge r$ for every tuple in $\mathscr{H}_{d,L}$.

The external input is the multi-summand Horn–Klyachko theorem [KnutsonTao2001,Fulton2000], in the form recorded as Theorem 4.1 of [BLR2015]: nonincreasing real lists $\lambda^0, \dots, \lambda^L$ are the spectra of Hermitian $H_0 = \sum_{x \ge 1} H_x$ iff their traces agree and $\sum_{j \in I_0} \lambda_j^0 \le \sum_{x \ge 1} \sum_{j \in I_x} \lambda_j^x$ for every tuple in $\mathscr{H}_{d,L}$. It is cited, not reproved. With $H_0 = I_d$ it says that $\mathscr{P}_{d,L}$ is exactly the set of spectra of L-outcome POVMs on $\mathbb{C}^d$.

Theorem (Finite linear criterion; draft t285u, adopted conditionally). *Assume the cited theorem. For* $1 \le m \le d$, (d, L, m) *decomposes iff the rational linear system*

$$0 \le w_x \le 1, \qquad m \sum_x w_x = d, \qquad \sum_{x=1}^{L} |I_x \cap \{1, \dots, m\}|\, w_x \ge r \quad ((r; I_0, \dots, I_L) \in \mathscr{H}_{d,L})$$

is feasible. A feasible system has a rational solution, and an infeasible one has a finite rational Farkas certificate.

Proof. $w_x P_x$ has spectrum $(w_x^{(m)}, 0^{(d-m)})$, and I_d has spectrum $(1^{(d)})$. Substituting into the cited inequalities gives the

system. Conversely, the cited theorem supplies Hermitian H_x with these spectra summing to I_d, and each is w_x times a rank-m projector (any rank-m projector if $w_x = 0$). Linear-programming duality over $\mathbb{Q}$ gives the last sentence.

Status: proved, conditional on the cited Horn–Klyachko theorem. Check DC-55 generates $\mathscr{H}_{d,L}$ by exact integer arithmetic (Schur polynomials in r variables, Jacobi–Trudi, alternant extraction), reproduces the Weyl inequalities for $d = 2$, and solves the system for $d \le 6$, $L \in \{3,4,5\}$, and for $(7,3,m)$. In all 48 cases the system is feasible exactly when the unconditional rank conditions (N1) and (N2) of Proposition hold for some $L' \le L$. Whether (N1) and (N2) suffice for every (d,L,m) is open (B2-O25). par *Non-claim.* A finite decision procedure; no closed criterion in (d,L,m) beyond $L = 3$ is claimed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Finite conic value for every L; draft t285u, adopted conditionally). *Assume the cited theorem. Let $Q_{d,L,m}$ be the maximum of*

$$\sum_{x=1}^{L}\Big(\sum_{i=1}^{m}\lambda_{xi} + 2\sum_{1\le i<j\le m} z_{xij}\Big) \quad \text{over} \lambda \in \mathscr{P}_{d,L},\ z_{xij} \ge 0,\ \begin{pmatrix} \lambda_{xi} & z_{xij} \\ z_{xij} & \lambda_{xj} \end{pmatrix} \ge 0.$$

Then $F_{\max}(d,L,m) = Q_{d,L,m}/d^2$ and $e_{\text{seed}}(d,L,m) = 1 - Q_{d,L,m}/d^2$. Each 2×2 constraint is a rotated second-order cone, so this is a finite second-order-cone program with rational data. If $Lm > d$, then $Q_{d,L,m} = md$ iff the system of Theorem is feasible; otherwise $Q_{d,L,m} < md$, a strict gap by compactness.

Proof. For fixed $\lambda \ge 0$ the maximal z_{xij} is $\sqrt{\lambda_{xi}\lambda_{xj}}$, so the branch objective is $(\sum_{i\le m}\sqrt{\lambda_{xi}})^2 = \|\sqrt{E_x}\|_{(m)}^2$ for a POVM element with spectrum λ_x. Theorem and the cited description of POVM spectra give the value. The equality criterion is Theorem (d) together with Theorem .

Status: proved, conditional on the cited theorem. Check DC-56 solves the program for eleven cases. It reproduces the two-label closed form of Theorem for $(3,2,2)$, $(4,2,3)$ and $(5,2,3)$, and the version-4 search values 0.395959, 0.597128 and 0.743686 for $(5,3,2)$, $(5,3,3)$ and $(4,3,3)$ to $2\cdot 10^{-6}$. An independent POVM search attains each conic value from below. A linearization certificate is computed at each optimum: since the objective is concave, $Q \le f(\lambda^*) + \max_{\lambda\in\mathscr{P}} \nabla f(\lambda^*)\cdot(\lambda-\lambda^*)$, and the linear program for the last term returns at most 10^{-5}. Numerically, $Q_{5,3,2} = 5 + 2\sqrt{6}$, $Q_{4,3,3} = 7 + 2\sqrt{6}$, $Q_{5,3,3} = 8 + 4\sqrt{3}$ and $Q_{5,4,4} = 13 + 4\sqrt{3}$ to 10^{-5}. These identifications are evidence only, not theorems (B2-O24). par *Non-claim.* The program is finite but not a closed formula in (d,L,m). The upper bound uses the necessity half of the cited theorem, and the numbers are solver output. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Design certificate; draft t285u). *Write the program of Theorem as* $\max c^{\text{top}}x$ *subject to $\mathscr{B}x = b$ and $h - \mathscr{C}x \in \mathscr{K}$, with $\mathscr{K}$ a product of nonnegative rays and real 2×2 positive-semidefinite cones. Every (y,Z) with $\mathscr{B}^*y + \mathscr{C}^*Z = c$ and $Z \in \mathscr{K}^*$ gives $Q_{d,L,m} \le b^{\text{top}}y + \operatorname{ip} Zh$. After facial reduction, a dual certificate attaining $Q_{d,L,m}$ exists.*

Proof. For feasible x, $c^{\text{top}}x = b^{\text{top}}y + \operatorname{ip} Z\mathscr{C}x \le b^{\text{top}}y + \operatorname{ip} Zh$. The primal is nonempty and compact. On its minimal face it has a relatively strictly feasible point, and finite-dimensional conic duality gives a zero gap and dual attainment.

Status: proved (standard conic duality). It is a certificate for the design problem, and is distinct from the channel witnesses below. par *Non-claim.* No certificate is claimed for any untwirled or unseeded object. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Remark (The all-L channel witnesses are already in version 3). The draft's Theorem thm:allLdual (isotropic channel J_t, witnesses Y_t, Z_t, H_t, both Choi signs, value $1-t$) is Theorem (b), with $t = F_{\max}$. Since $F_{\max} = Q_{d,L,m}/d^2$, the plan's request for ``exact dual witnesses for every L'' is met by that theorem together with Theorem . Check

DC-54 verifies the identities exactly for $(d,t) = (3,\frac{2}{3})$ and $(2,\frac{1}{2})$, and numerically at the conic values of $(5,3,2)$ and $(4,3,3)$. Disposition: duplicate.

Proposition (Joint observers; draft t285u, corrected). *For a state σ on $\mathbb{C}^d$, put $t_\sigma = (\mathrm{Tr}\,\sigma^{-1})^{-1}$, with $t_\sigma = 0$ if σ is singular. Then*

$$\tfrac{1}{2}\|\mathscr{R}_\sigma - \mathrm{id}\|_\diamond = 1 - t_\sigma,$$

with the witness $Z_\sigma = (1-t_\sigma)W_d$, $Z_\sigma - (W_d - \sigma\otimes I) = \sigma\otimes I - t_\sigma W_d \geq 0$, and the input $\sum_i \sqrt{p_i}|ii\rangle$, $p_i \propto \sigma_i^{-1}$, in an eigenbasis of σ. If no receiver is causally reachable from the source, the complete joint receiver channel is a replacement channel (Theorem (a)), and its witness must use the joint output σ. Product witnesses built from the marginals of a correlated σ do not certify it. For $\sigma = \mathrm{diag}\,(\frac{2}{5},\frac{1}{10},\frac{1}{10},\frac{2}{5})$ on two qubits, both marginals are $I_2/2$, but $t_\sigma = \frac{1}{25}$ and the error is $\frac{24}{25}$, not the product value $1-\frac{1}{16} = \frac{15}{16}$.

Proof. The value is Proposition (a) and Theorem ; we record the witness. $\sigma\otimes I \geq tW_d$ iff $t\langle\Omega_d|(\sigma^{-1}\otimes I)|\Omega_d\rangle \leq 1$, that is, $t\mathrm{Tr}\,\sigma^{-1} \leq 1$. So Z_σ is feasible in the dual of [Watrous2009], with $\mathrm{Tr}_{\mathrm{out}}\, Z_\sigma = (1-t_\sigma)I$. For the input, the error is at least $1 - \langle\psi|\sigma\otimes\psi_R|\psi\rangle = 1 - \sum_i p_i^2\sigma_i$, which is $1-t_\sigma$ at $p_i \propto \sigma_i^{-1}$. Check DC-57 (SDP: 24/25 to 10^{-6}; six random σ with $d = 2,3$ agree with $1-t_\sigma$).

Status: proved; the value formula duplicates Proposition (a), and the joint-channel statement duplicates Theorem (a). What is new is the correlated example. The plan's sentence ``the same dual witness tensors across observers'' is corrected: it holds for an untouched identity factor and for independent payloads with independent replacement outputs, not for correlated joint outputs. par *Non-claim.* No observer model or receiver network is constructed; the causal premise is the declared one of Theorem . Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

All-energy replacement error and cutoff recovery (version 5)

This section implements item 2 of the plan t283u and the energy items of the draft t285u. The setting and notation are those of Section : M supplied modes, total number operator N with occupation basis $N|i\rangle = e_i|i\rangle$, multiplicity $g_M(n) = \frac{n+M-1}{M-1}$ of $e_i = n$, $\nu = E/\Delta$, and $\varepsilon_E^*(\mathscr{R}) = \inf_\sigma \varepsilon_E(\mathscr{R}_\sigma, \mathrm{id})$, written $\mathfrak{r}_M(\nu)$ below. The energy-constrained distance is the one of [WinterECD,Shirokov2018].

Lemma (Diagonal reduction and the secular equation; draft t285u). *An optimal output may be taken diagonal, $\sigma = \sum_i s_i \mathrm{proj}\, i$. For such σ the supremum may be restricted to the inputs $\psi_p = \sum_i \sqrt{p_i}|i\rangle_A|i\rangle_R$ with $\sum_i p_i = 1$ and $\sum_i e_i p_i \leq \nu$. The error of ψ_p is the unique root $\lambda_s(p) \geq 0$ of $\sum_i p_i/(\lambda + s_i p_i) = 1$. For $t > 0$, $\lambda_s(p) \leq t$ iff $G_t(s,p) := \sum_i p_i/(t + s_i p_i) \leq 1$.*

Proof. The phase unitaries $e^{i\theta\cdot N_{\mathrm{modes}}}$ preserve the energy constraint and commute with id. Conjugating σ by them and the input by their inverses leaves the error unchanged, and the error is convex in σ. So the phase average, which is diagonal in the occupation basis, is no worse. For diagonal σ, the error of a pure input is a function f of its marginal ρ_A. This function is phase invariant, and it is concave: a flagged purification of $\sum_j q_j\rho_j$, followed by dephasing of the flag, gives the direct sum of the separate experiments, and the trace norm contracts. So a diagonal marginal is no worse, and its purification can be taken as ψ_p. On span $\{|ii\rangle\}$ the difference $\psi_p - \sigma\otimes\psi_R$ is $|\sqrt{p}\rangle\langle\sqrt{p}| - \mathrm{diag}\,(s_i p_i)$, and on the other $|ij\rangle$ it is $-s_i p_j \leq 0$. By Lemma the error is the positive eigenvalue, which solves the rank-one secular equation. Finite truncation and continuity cover the countable basis.

Status: proved. par *Non-claim.* A variational identity for declared channels and a declared energy promise. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Lemma (Scalar dual inequality; draft t285u). *For $t,s > 0$ and $a,x \geq 0$: $\frac{x}{t+sx} \leq ax + \frac{(1-\sqrt{at})_+^2}{s}$, and the supremum*

over x of the left side minus ax equals the last term.

Proof. If $at < 1$, the maximizer satisfies $t + sx = \sqrt{t/a}$, and substitution gives $(\sqrt{t/a} - t)(\sqrt{a/t} - a)/s = (1 - \sqrt{at})^2/s$. If $at \geq 1$, the maximum is at $x = 0$.

Theorem (Exact all-energy formula; draft t285u, adopted). *For $v > 0$ put $A_M(u,v) = \sum_{n \geq 0} g_M(n)(1 - \sqrt{u + vn})_+$, a finite sum for $u, v > 0$. Then*

$$\mathfrak{r}_M(v) = \min_{\text{substack } u,v > 0}$$

the minimum is attained, and $0 < \mathfrak{r}_M(v) < 1$. For every feasible (u,v), with $A = A_M(u,v)$, $t = (u + vv)/(1 - A^2)$ and $s_i = (1 - \sqrt{u + ve_i})_+/A$, the output $\sigma = \sum_i s_i \mathrm{proj}\, i$ has error at most t. For fixed t, $\mathfrak{r}_M(v) \leq t$ iff $\inf_{u,v \geq 0} \{(u + vv)/t + A_M(u,v)^2\} \leq 1$, a convex condition.

Proof. Upper bound. Apply Lemma with $a_i = (u + ve_i)/t$ to each term of $G_t(s,p)$. Levels with $s_i = 0$ have $u + ve_i \geq 1$, so their remainder vanishes. Summing, and using $\sum p_i = 1$ and $\sum e_i p_i \leq v$, gives $G_t(s,p) \leq (u + vv)/t + \sum_i (1 - \sqrt{u + ve_i})_+^2/s_i = (u + vv)/t + A^2 = 1$. Lemma gives error at most t.

Truncated equality. Restrict the inputs to $e_i \leq K$. The truncated value c_K is at most $\mathfrak{r}_M(v)$, and an optimal output may be supported on these levels, since output weight elsewhere only lowers the subtracted term. G_t is convex in s and concave in p on finite compact convex sets, so Sion's theorem [Sion1958] exchanges $\min_s$ and $\max_p$. The concave dual of the p-maximization has multipliers $\mu \in \mathbb{R}$ and $\beta \geq 0$; Lemma forces $\mu + \beta e_i \geq 0$. Then $\min_s \sum_i a_i^2/s_i = (\sum_i a_i)^2$ over the simplex, and the substitution $u = t\mu$, $v = t\beta$ gives the truncated formula with $A_{M,K}$ and $v \geq 0$.

Limit. The output $1_{\{N \leq J\}}/D$ with $J > v$ has error at most $t_0 < 1$ uniformly. So the truncated minimizers satisfy $u + vv \leq t_0$ and $A_{M,K} < 1$. If v were small, many levels with $u + vn < 1$ would each contribute at least $1 - \sqrt{t_0}$ to A, so v is bounded below. Then $1 - A^2 \geq vv/t_0$ is bounded below, u is bounded below (the $n = 0$ term is $1 - \sqrt{u}$), and all active levels satisfy $e_i < 1/v$. For large K the truncated and full functions agree on this compact set. A limit point (u^*, v^*) is feasible, and its upper certificate equals $\lim c_K \leq \mathfrak{r}_M(v)$. This proves the formula and attainment. Each $(1 - \sqrt{u + vn})_+$ is convex, which gives the convex test.

Status: proved. This closes B2-O23 and the replacement part of B2-O10 as an exact two-parameter variational value. Check DC-58 evaluates the formula and the matching primal of Theorem for $M = 1$ at ten energies and $M = 2$ at eight. The primal–dual gap is at most $2 \cdot 10^{-8}$. The values agree with $F(v)$ and $3v(1 - v)$ on the version-4 intervals and with the version-4 truncated SDP values $0.761206, 0.824663, 0.891380$ at $v = 0.5, 0.7, 1$, and they lie inside the version-4 brackets. Fresh truncated SDPs ($M = 1$, and $M = 2$ with total occupation at most 3) agree to $2 \cdot 10^{-5}$. par *Non-claim.* The value is a variational formula, not a single elementary expression for all v. The energy promise, Δ and M are declared; no bath or temperature is derived, and the optimal output is a decoder choice. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Saturating input; draft t285u, adopted). *Let (u,v) attain the minimum in Theorem , $A = A_M(u,v)$ and $t = \mathfrak{r}_M(v)$. There are probabilities with $p_i = tA/\sqrt{u + ve_i}$ if $u + ve_i < 1$, $0 \leq p_i \leq tA$ if $u + ve_i = 1$, and $p_i = 0$ if $u + ve_i > 1$, such that $\sum_i p_i = 1$ and $\sum_i e_i p_i = v$. The input ψ_p has error t against every output.*

Proof. The quotient is a convex-composite function of (u,v). On active levels $\partial_u A = -1/(2\sqrt{u + ve_i})$, and on boundary levels the one-sided derivatives lie between $-\frac{1}{2}$ and 0. Stationarity in the interior variables u, v gives the normalization and energy identities with these p_i. For them, $t + s_i p_i = t/\sqrt{u + ve_i}$ on active levels, so the scalar inequality is an equality on every occupied level, $G_t(s,p) = 1$, and $\lambda_s(p) = t$. Moreover s minimizes $G_t(\cdot, p)$ on the

simplex: the partial derivatives are $-A^2$ on active levels and $-p_i^2/t^2 \geq -A^2$ on boundary levels. So ψ_p is a lower test against every output.

Status: proved. Generically the minimizer has one boundary shell. Check DC-58 reconstructs p, including the boundary weights from the normalization, and verifies normalization, energy and the secular equation. Rows: $\mathfrak{r}_1(0.5) = 0.761206$, $\mathfrak{r}_1(1) = 0.891380$, $\mathfrak{r}_1(2) = 0.960379$; $\mathfrak{r}_2(0.6) = 0.872727$ (numerically 48/55), $\mathfrak{r}_2(1) = 0.939578$. par *Non-claim.* No numerical value is promoted to an exact constant beyond the identities proved. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Degeneracy-extended vacuum interval; draft t285u, adopted). *Put* $q_g = g^2/(g^2 + g + 1)$, *so* $q_1 = \frac{1}{3}$, $q_2 = \frac{4}{7}$, $q_3 = \frac{9}{13}$.

• $\mathfrak{r}_M(\nu) = F(\nu)$ *for* $0 \leq \nu \leq q_M$, *attained by the vacuum output.*

• *For* $\mathscr{C}_n$, *put* $g = g_M(n+1)$ *and* $q = E/(\Delta(n+1))$. *If* $q \leq q_g$, *then* $\varepsilon_E^*(\mathscr{C}_n) = \varepsilon_E(\mathscr{C}_n, \mathrm{id}) = F(q)$.

• *For* $M = 1$, $g = 1$ *and this is Theorem (a). For* $M \geq 2$ *the intervals are strictly larger.*

Proof. Use the g orthonormal states of the first rejected shell, and the input $\sqrt{1-q}|0\rangle|0\rangle + \sqrt{q/g}\sum_{i=1}^{g}|i\rangle|i\rangle$, whose energy is admissible. For $\mathscr{C}_n$ every decoder sees the vacuum on it. Put $s = \sqrt{q(1-q)}$ and test with the unit vector proportional to $s|00\rangle + F(q)|\Phi_g\rangle$, $|\Phi_g\rangle = g^{-1/2}\sum_i |ii\rangle$. For a general output, the subtracted term is proportional to $s^2(1-q)\sigma_{00} + F(q)^2(q/g^2)\sum_i \sigma_{ii}$, which is at most its vacuum value iff $s^2(1-q) \geq F(q)^2 q/g^2$, that is, $F(q) \leq g(1-q)$. Using $F^2 - qF - q(1-q) = 0$, this is $q \leq q_g$. The lower bound $F(q)$ follows, and the vacuum output (or the identity decoder) attains it by Theorem .

Status: proved. Check DC-59: the threshold identity in SymPy; the equivalence $F(q) \leq g(1-q) \Leftrightarrow q \leq q_g$ on a grid for $g \leq 4$; 900 random outputs give witness slack $\geq -10^{-10}$; and an output that moves weight to the shell defeats the witness beyond q_g. DC-58 confirms $\mathfrak{r}_2 = F$ at $\nu = 0.57$ and $\mathfrak{r}_2 < F$ at $\nu = 0.58$. par *Non-claim.* As for Theorem . Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Certified recovery limit for $\mathscr{C}_n$; draft t285u, adopted). *Fix* n, M *and* $\nu > 0$. *For* $K \geq n$ *let* $c_{n,K}(\nu)$ *be the optimized recovery error over inputs supported on* $P_K = 1_{\{N \leq K\}}$ *with mean occupation at most* ν. *Then* $c_{n,K}$ *is nondecreasing in* K *and*

$$c_{n,K}(\nu) \leq \varepsilon_E^*(\mathscr{C}_n) \leq c_{n,K}(\nu) + F(\min\{1, \nu/(K+1)\}),$$

so $\varepsilon_E^*(\mathscr{C}_n) = \lim_K c_{n,K}(\nu)$. *Each* $c_{n,K}$ *is a finite SDP: minimize* $\alpha + \beta\nu$ *over decoder Choi operators,* $\beta \geq 0$ *and* $Y \geq 0$ *with* $Y \geq J(\mathrm{id} - \mathscr{D}\mathscr{C}_n)$ *and* $\mathrm{Tr}_{\mathrm{out}}\, Y \leq \alpha I + \beta N|_{P_K}$ *[Watrous2009]. For* $n = 0$ *the exact gap to the installed value is* $F(\min\{1,\nu\}) - \mathfrak{r}_M(\nu)$. *If* $E \geq \Delta(n+1)$, *then* $\varepsilon_E(\mathscr{C}_n, \mathrm{id}) = 1 > \mathfrak{r}_M(\nu) \geq \varepsilon_E^*(\mathscr{C}_n)$.

Proof. Lower bound: restrict any decoder to P_K inputs and postprocess by $\mathscr{C}_K$. Since $\mathscr{C}_K$ fixes the target on P_K and contracts trace distance, this does not increase the error. The same argument gives monotonicity. Upper bound: for a decoder $\mathscr{D}$ that is optimal for the truncated problem, $\mathscr{D}\mathscr{C}_n = \mathscr{D}\mathscr{C}_n\mathscr{C}_K$ for $K \geq n$. So $(\mathscr{D}\mathscr{C}_n - \mathrm{id}) = (\mathscr{D}\mathscr{C}_n - \mathrm{id})\mathscr{C}_K + (\mathscr{C}_K - \mathrm{id})$. Since $\mathscr{C}_K$ does not increase energy, the first term is at most $c_{n,K}$, and the second is at most $F(\min\{1, \nu/(K+1)\})$ by Theorem . Preparing an optimal replacement output is an allowed decoder, which gives the last statement.

Status: proved. This is an exact certified-limit description, not an elementary formula for $n \geq 1$ (B2-O26). The allowance decays only like $\sqrt{\nu/K}$. Check DC-60: $c_{n,K}$ for $n \in \{0,1\}$, $K \in \{2,3,4,5\}$, $\nu \in \{0.5, 1, 2\}$ is nondecreasing; $c_{1,5}(0.5) = F(\frac{1}{4}) = 0.575694$ within the interval of Theorem ; at $\nu = 1$, $c_{1,5} = 0.772373$ lies below

the installed $F(\frac{1}{2}) = 0.809017$, but the certified interval is too wide to decide whether optimized decoding improves on it; and the $n = 0$ gaps are 0 at $v = 0.2$ and positive at $v = 0.5, 1, 2$. par *Non-claim.* No physical oscillator noise model, and no unrestricted no-seed register problem, is closed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Remark (Hard support is the promise that supplies a dimension). The draft's Proposition prop:hardpromise is Proposition (a),(c). Its one addition is that for $P = P_n$, $D = \frac{n+M}{M}$, a budget $E \geq \Delta n$ admits every state supported on P_n, so that the hard-support contract of Proposition (a) is not further restricted by the mean-energy promise. Disposition: duplicate, with this addition. The plan's ``effective dimension d_{eff}'' is this contract; a mean-energy promise alone supplies none.

Coding dimension: a sharper Singleton bound and the error metric (version 5)

This section implements item 3 of the plan t283u and the coding items of the draft t285u. Notation: h and g as in Theorem , and $H_K(x) = h(x) + x\ln(K^2 - 1)$, the entropy of an isotropic state on $\mathbb{C}^K \otimes \mathbb{C}^K$ with Bell error x.

Theorem (Bell-twirled approximate Singleton bound; draft t285u, adopted). *In the setting of Theorem (c), with Bell-form errors $\varepsilon_A, \varepsilon_B \leq 1 - K^{-2}$ (implied by half-diamond errors),*

$$\ln K \leq \ln \dim \mathscr{H}_C + \tfrac{1}{2}(H_K(\varepsilon_A) + H_K(\varepsilon_B)),$$

equivalently $(1 - \varepsilon_A - \varepsilon_B)\ln K \leq \ln \dim \mathscr{H}_C + \frac{1}{2}(h(\varepsilon_A) + h(\varepsilon_B)) + \frac{1}{2}(\varepsilon_A + \varepsilon_B)\ln(1 - K^{-2})$. *This sharpens , since* $H_K(\varepsilon) \leq 2\varepsilon \ln K + g(\varepsilon)$.

Proof. Let ω_{RL} be the decoded Bell state after erasure of A, so $\langle \Phi_K | \omega | \Phi_K \rangle \geq 1 - \varepsilon_A$. Its R-marginal is I/K, and $\overline{U} \otimes U$ twirling keeps it. $I(R:L) = S(R) - S(R|L)$ is convex on states with fixed R-marginal, since $S(R|L)$ is concave, and each conjugation preserves it. So the twirled isotropic state, with Bell error $\delta \leq \varepsilon_A$, has $I(R:L) \leq I(R:L)_\omega$. Its entropy is $H_K(\delta) \leq H_K(\varepsilon_A)$, because H_K increases on $[0, 1 - K^{-2}]$. Hence $I(R:L)_\omega \geq 2\ln K - H_K(\varepsilon_A)$. Data processing and purity give $I(R:A) = 2\ln K - I(R:BC) \leq H_K(\varepsilon_A)$, and likewise for B. Finally $2\ln K = I(R:B) + I(R:AC) \leq I(R:B) + I(R:A) + 2S(C)$, since $I(R:C|A) \leq 2S(C)$. The comparison $H_K(\varepsilon) \leq 2\varepsilon \ln K + g(\varepsilon)$ follows from $\ln(K^2 - 1) < 2\ln K$ and $h \leq g$.

Status: proved. No continuity bound is used. Check DC-66: $H_K(\varepsilon) \leq 2\varepsilon \ln K + g(\varepsilon)$ on grids for $K \in \{2, 3, 5, 10, 100\}$; the isotropic entropy and the monotonicity of H_K; flag codes satisfy $I(R:A) = 2a\ln K \leq H_K(\varepsilon_A)$; and the allocation codes satisfy the new bound. par *Non-claim.* The bound concerns declared factorizations. No physical error model or threshold is derived. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corollary (Weight form; draft t285u, adopted). *For n supplied q-ary sites, if every r-site erasure has half-diamond (or Bell-form) recovery error at most $\varepsilon < \frac{1}{2}$, then* $(1 - 2\varepsilon)\ln K \leq (n - 2r)\ln q + h(\varepsilon) + \varepsilon \ln(1 - K^{-2})$. *With* 2*t-site erasures,* 2*r becomes* 4*t. Approximate recovery from all weight-t errors must not be converted into the* 2*t-erasure hypothesis by the exact Knill–Laflamme product argument alone. At zero error that argument gives distance* $2t + 1$ *(Theorem (a)); at nonzero error the* 2*t-erasure recovery is a separate hypothesis.*

Proof. Theorem with two disjoint erased sets and the rest in C.

Status: proved. This is the plan's ``approximate Singleton under diamond distance''; the plan's version is already Theorem (c), and the form kept here is the sharper one. par *Non-claim.* As for Theorem . Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (The allocation code is not small in diamond distance; draft t285u, adopted). *Let $K = c + r$ and encode $V|i\rangle = |i\rangle_C |\varnothing\rangle_A$ for $i \leq c$ and $V|i\rangle = |\varnothing\rangle_C |i\rangle_A$ for $i > c$, with B trivial. After erasure of A, the optimized*

half-diamond recovery error is exactly $1 - \frac{1}{1+r^2} \geq \frac{1}{2}$*, while the optimized Bell-form error is at most* $F(r/K)$*, which tends to* 0 *as* $r/K \to 0$.

Proof. The remaining channel keeps the c-block, removes its coherence with the r-block, and maps the r-block to the flag. The decoder that keeps the c-block and resets the flag to I_r/r has error $1 - 1/(1+r^2)$, by the harmonic direct-sum certificate (Theorem : one identity block and one replacement block with $\mathrm{Tr}\,(I_r/r)^{-1} = r^2$). For the lower bound, use the input $\sqrt{w}|\Phi_c\rangle + \sqrt{1-w}|\Phi_r\rangle$ with $w = 1/(1+r^2)$. Every decoder has target overlap at most $w^2 + (1-w)^2/r^2 = 1/(1+r^2)$. The Bell-form bound is Proposition (c).

Status: proved. Check DC-61: joint decoder SDPs give the diamond value $1 - 1/(1+r^2)$ for $(c, r) \in \{(1,1),(2,1),(1,2),(2,2),(1,3),(3,3)\}$ to 10^{-5}, and the explicit decoder attains it for $(1,2)$ and $(2,3)$. The optimal Bell-form errors lie below $F(r/K)$ in all six cases (for example 0.5 against 0.809017 at $c = r = 1$), so the version-4 decoder is not Bell-optimal; its bound stands. par *Non-claim.* This is a clarification of the error metric, not an erratum. Proposition (c) is stated, and remains valid, for Bell-form errors, which are the hypotheses of Theorem (c). It does not supply a small-diamond-error example. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Remark (The dimension form stays open). Theorems and sharpen the upper bound and separate the two metrics. They do not show that $\ln K - \ln \dim \mathscr{H}_C$ can grow like $\varepsilon \ln K$ at fixed small error, in either metric, and they do not exclude it. The flag code has $\dim \mathscr{H}_C = K + 1$, so its entropy calculation gives no dimension gain. The plan's request for a matching lower bound is therefore not met: B2-O22 stays open, now with the upper allowance $\frac{1}{2}(H_K(\varepsilon_A) + H_K(\varepsilon_B))$ and with the metric made explicit.

Several epochs, tail moduli, a two-epoch benchmark and label invariants (version 5)

This section implements item 4 of the plan t283u and the geometric items of the draft t285u. Conventions are those of Sections and , with $R_j = c\,a(t_j)\int_{t_j}^{\infty} \mathrm{d}s/a(s)$.

Proposition (Several epochs: simultaneous minimax and power-law coefficients; draft t285u, adopted in part). *Let the scale factor be prescribed and continuous on* $[t_0, t_N]$*, with junctions* $t_0 < \ldots < t_N$*. The transfer law* $R_i = C_i + \frac{a(t_i)}{a(t_N)} R_N$, $C_i = c\,a(t_i)\int_{t_i}^{t_N} \mathrm{d}s/a$*, is Theorem (d), equivalently the product of the matrices*

$$\begin{pmatrix} a(t_i)/a(t_{i+1}) & c\,a(t_i)\int_{t_i}^{t_{i+1}} a^{-1} \\ 0 & 1 \end{pmatrix}.$$

• *If* R_N *ranges over a set with infimum u and supremum v, then for every norm on* $\mathbb{R}^{N+1}$ *the exact minimax error of a simultaneous estimate of* $(R_0, \ldots, R_N)$ *from the prefix is* $\frac{v-u}{2}\|(a(t_i)/a(t_N))_{i=0}^{N}\|$*, attained by the midpoint.*

• *On a power-law epoch* $a(t) = a_i(1 + \gamma H_i(t - t_i))^{1/\gamma}$ *with* $z = a(t_{i+1})/a_i$*, the matrix entries are* $1/z$ *and* $\frac{c}{H_i}\frac{z^{\gamma-1}-1}{\gamma-1}$ *(*$\frac{c}{H_i}\ln z$ *for* $\gamma = 1$*;* $\frac{c}{H_i}(1 - z^{-1})$ *for the de Sitter case* $\gamma = 0$*).*

Proof. (a) The target set is a segment with direction $\lambda = (a(t_i)/a(t_N))_i$ and length $(v-u)\|\lambda\|$. The triangle inequality at its two ends gives the lower bound, and the midpoint attains it; open ends are handled by suprema. (b) Substitute $y = 1 + \gamma H_i(s - t_i)$ in $a_i \int a^{-1} \mathrm{d}s$.

Status: proved; the transfer law and the scalar minimax duplicate Theorem (d). The plan's phrase that each junction radius ``is independent of all earlier data'' is corrected: each radius is not a function of its own past, but the radii are tied by the transfer law. Check DC-62 (three junctions mixing radiation-type, de Sitter and $\gamma = \frac{1}{2}$ epochs; coefficient formula against quadrature for $\gamma \in \{0, \frac{1}{2}, 1, 2, 3\}$ to 10^{-9}; midpoint optimality in the ℓ^∞, ℓ^2 and ℓ^1 norms). par *Non-claim.* The prefix and futures are prescribed; no dynamics selects them. Ledger unchanged: no

quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Window-plus-tail modulus; draft t285u, corrected). *Normalize* $a(0) = \widehat{a}(0) = 1$, *put* $b = \ln a$, $\widehat{b} = \ln \widehat{a}$, *and suppose* $b, \widehat{b} \ge ht - B$ *on* $t \ge 0$ *with* $h > 0$. *For every* $T > 0$,

$$|R_0[a] - R_0[\widehat{a}]| \le \frac{ce^B(1 - e^{-hT})}{h}\|b - \widehat{b}\|_{C^0[0,\,T]} + \frac{ce^B e^{-hT}}{h},$$

and the last term is absent if the scale factors agree after T. The tail constant 1 *is sharp. Any norm that dominates the* $C^0[0, T]$ *norm, such as a* $C^{k,\alpha}[0, T]$ *norm or a Gevrey norm* $\sup_j L^j \|f^{(j)}\|_{\infty,\,[0,\,T]}/(j!)^s$, *may replace it.*

Proof. On $[0, T]$, $|e^{-b} - e^{-\widehat{b}}| \le e^{B - ht}|b - \widehat{b}|$ by the mean-value theorem. The two tail integrals are positive and each is at most $ce^B e^{-hT}/h$, so their difference is at most $ce^B e^{-hT}/h$. For sharpness, take equal windows, $b = ht - B$ after T, and $\widehat{b} \to \infty$ after T.

Status: proved. The draft printed the tail term with a factor 2; that bound is valid but not sharp, and the sharp constant is 1 (correction). The draft's parts (c) ($|\ln R_0 - \ln \widehat{R}_0| \le T\|H - \widehat{H}\|$ when the profiles agree after T) and (d) (local regularity gives no modulus) are Theorem (a),(b); disposition duplicate. Check DC-68: 60 random admissible pairs satisfy the bound (largest ratio to it below 1), and in the sharpness example with cutoff slope $k = 10^3$ the tail difference is $k/(h + k)$ times the tail term, to 10^{-6}. par *Non-claim.* The growth constants h, B and the tails are hypotheses, not derived. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (A radiation–de Sitter benchmark; draft t285u, adopted). *Let* $c = 1$ *and prescribe* $a(t) = \sqrt{1 + 4t}$ *on* $[-\frac{3}{16}, 0]$ *and* $a(t) = e^t$ *on* $[0, \ln 2]$. *After* $T = \ln 2$ *take* $a_K(t) = 2e^{K(t - T)}$ *with* $K \in \{\frac{1}{2}, 1\}$. *The scale factors are continuous and piecewise smooth; they are not claimed to solve any field equation. Then* $R_0(K) = \frac{1}{2} + \frac{1}{2K}$, *so* $R_0(\frac{1}{2}) = \frac{3}{2}$ *and* $R_0(1) = 1$, *and the status-flip four-volume of Theorem over* $[-\frac{3}{16}, \ln 2]$ *is*

$$\mathscr{V}_4 = \frac{9293\pi}{26880} + \frac{35\pi}{9} = \frac{341479\pi}{80640} \approx 13.303422.$$

It contains no early label and no κ.

Proof. $a(T) = 2$, and splitting the conformal integral gives $R_0(K)$. On the radiation interval ($\gamma = 2$, $H_0 = 2$, $B = \frac{1}{2}$), with $z = \sqrt{1 + 4t} \in [\frac{1}{2}, 1]$, the comoving radii are $2 - \frac{z}{2}$ and $\frac{3}{2} - \frac{z}{2}$, and $a^3 \mathrm{d}t = \frac{1}{2}z^4 \mathrm{d}z$; this gives $9293\pi/26880$, which is also Theorem with $R_+ = \frac{3}{2}$, $R_- = 1$. On $[0, \ln 2]$ the proper radii are $1 + \frac{e^t}{2}$ and 1, and $\frac{4\pi}{3}\int_0^{\ln 2}[(1 + \frac{e^t}{2})^3 - 1]\mathrm{d}t = \frac{35\pi}{9}$.

Status: proved. Check DC-63 (exact SymPy integrals, the version-4 formula, and quadrature of the conformal radii to 10^{-8}). This is the plan's ``quantitative dark volume on a radiation-to-de Sitter transition''. The draft's further sentence on smoothed junctions is not adopted: its convergence claim needs common window and tail bounds that the draft does not specify, and Theorem gives such control only for R_0. par *Non-claim.* A geometric measure on a prescribed piecewise host. It is not an entropy, and it does not verify any statement of nuADaM01-A. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Coefficient-free combinations of labels; draft t285u, adopted). *Let positive labels have the declared form* $\ell_j = c_j(\mathfrak{a})\kappa^{a_j}$, *where* $\mathfrak{a}$ *is the complete coefficient-free datum.*

• *A monomial* $\prod_j \ell_j^{b_j}$ *is independent of* κ *iff* $\sum_j a_j b_j = 0$.

• *A function of the labels is invariant along the* κ*-orbits iff it factors through the projection of* $(\ln \ell_j)_j$ *along* $\mathrm{span}\{(a_j)_j\}$. *On the declared orbit its value is a function of* $\mathfrak{a}$.

• *For the labels of Definition , with* $p = 3w/(1 + w)$, $T_w \propto \kappa^p$ *and* $S_w \propto \kappa^{-p}$. *So* $T_w S_w$, $T_w(t)/T_w(t')$ *and* $S_w(t)/S_w(t')$ *are* κ*-free, whereas* $T_w/S_w \propto \kappa^{2p}$ *is not.*

• *The datum cannot be dropped:* $T_w S_w = (1 + w)UV_c a^{-3w}$ *is* κ*-free but depends on* U.

Proof. $\ln \ell = \ln c(\mathfrak{a}) + a \ln \kappa$, so the orbits are translations along $(a_j)_j$, which gives (a) and (b). Theorem gives (c) and (d).

Status: proved. The plan's sentence ``ratios of early labels are κ-free'' is corrected: the ratio T_w/S_w is not, and κ-free combinations are functions of the admitted datum, not of geometry alone. Check DC-64 (SymPy). par *Non-claim.* No label is calibrated, and κ is not filled. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

The spectator monoid and the new certificates (version 5)

This section implements item 5 of the plan t283u and the monoid item of the draft t285u. The monoid is $\mathscr{M}_{\mathrm{sp}}$ of Definition (the draft's SCM), acting through its spectator lift. For energy-constrained accounts we add one clause: appended spectators carry zero Hamiltonian, start independent of the payload, and never become a message-bearing path or an energy resource.

Corollary (Typed compatibility; draft t285u, merged). *Under the spectator lift:*

• *the optimized recovery error, including its energy-constrained version* ε_E^*, *is invariant;*

• d, L, m, *the systems of Theorems and , the value* $Q_{d,L,m}$ *and the witnesses of Theorem (b) are unchanged;*

• M, Δ, E, *the value* $\mathfrak{r}_M(v)$ *of Theorem and the intervals of Theorem are unchanged;*

• *the log-dimension padding cost rises by* $n \ln 2$, *while the active erasure guarantees and the inequalities of Theorem are unchanged;*

• *junction radii, fibres and flip volumes are unchanged;*

• *an early tightly-coupled phase may be added only as metadata unless its channel and access contract are separately built; it is not a fifth independent category (Proposition).*

Proof. Theorem (a)–(c). Preparing a fixed spectator has discard as a left inverse, so decoders transfer both ways. A zero-Hamiltonian, disconnected spectator leaves the admissible active inputs unchanged. The remaining data are fixed by definition, and (f) is Proposition .

Status: proved; a merge of Theorem with the version-5 certificates. The plan's ``five categories'' is corrected to the four categories with an epoch tag (Proposition ). The plan's ``no reorganization reduces a certified floor'' holds for the spectator lift only: a host-preserving declaration change can add a bypass and lower the error (Proposition). Output postprocessing preserves lower floors by inclusion of decoder classes, but it is access-degrading and can raise the error. Check DC-65 (amplitude damping with $\gamma = 0.3$: the optimized recovery SDP gives 0.242131 with and without an appended spectator, below the identity-decoder value γ of Proposition ; the appended Hamiltonian is zero). par *Non-claim.* No conversion rate between categories is supplied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Re-check of version-4 statements (version 5)

The constants of Sections – were recomputed (DC-69, and the cross-checks inside DC-55 to DC-66): $\nu_2 = 0.436528$, $\alpha_2 = 0.571672$, $\beta_2 = 0.380832$; $e_{\mathrm{seed}}(3, 2, 2) = 0.352397$; the DC-46 values (reproduced by the conic program of Theorem); the allocation-code value $F(0.1) = 0.354138$ and gain 0.051293; and the version-4 energy brackets, which contain the exact values of Theorem .

No erratum was found. The version-4 text stands unchanged. The following are sharpenings and clarifications, recorded by addition:

• Proposition : the decomposition criterion is now complete for $L = 3$ (Theorem) and finite for every L (Theorem

); the values below m/d are strict.

• Theorem (c),(d): the brackets are superseded by the exact value of Theorem , and the vacuum interval for $M \geq 2$ extends to q_M (Theorem).

• Proposition (c): its errors are Bell-form errors, the hypotheses of Theorem (c). In half-diamond distance the same code has error $1 - 1/(1 + r^2) \geq \frac{1}{2}$ (Theorem). This is a clarification, not an erratum.

• Proposition (a): the allowance sharpens to $\frac{1}{2}(H_K(\varepsilon_A) + H_K(\varepsilon_B))$ (Theorem). par *Non-claim.* A re-check is a recomputation by the same author with new scripts. It is not an independent review. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

What is deliberately not done (version 5)

• No closed formula for $F_{\max}(d, L, m)$ below m/d for $L \geq 3$; the finite programs of Theorems and rest on a cited theorem, and the identified surds are numerical (B2-O24). Whether the rank conditions (N1), (N2) suffice for every L is open (B2-O25).

• No elementary all-energy formula; the exact value is a two-parameter minimization. No elementary value of $\lim_K c_{n,K}$ for $n \geq 1$ (B2-O26).

• No matching lower bound for the dimension form of the coding cost, in either metric (B2-O22).

• No smoothed-junction volume, no derivation of $a(t)$, of an equation of state or of an energy condition.

• No thermal state, physical noise channel, Page curve, finite Hilbert space or entropy identification. No coefficient slot κ or ε_s is filled, and no quantum coefficient is admitted.

• No discharge of O.1, XV-B, XVI-N or Gate 0. No revision of any parent sheet. Citations identify source records only; no proof transfer.

Emergence-page sentence, version 5 (not registered)

The following sentence, from the plan t283u, is recorded for use if the sheet is later registered. It is not registered here. Big02-A version 4 (and its expansions) records dark-capacity certificates, including exact (L, m) and energy-constrained floors, on prescribed hosts. None of them is spacetime emergence, a Page curve, or a derived temperature.

Closed (L, m) values below m/d: meeting subspaces (version 6)

This section implements item 3 of the plan t287u (Appendix). The notation is that of Definition and Theorem . Most of item 3 is already proved in versions 3–5: the variational formula (Theorem), the projector-sum criterion (Theorem (d)), the twirled channel witnesses in both sign conventions (Theorem (b), Lemma , Remark), the three-label classification (Theorem) and joint observers (Corollary , Proposition). What is new here is an elementary certificate for the *value* of $F_{\max}$ when I_d is not a weighted projector sum. It gives closed forms for every L in two bands of d and for every d when $L = 3$. None of these uses the Horn–Klyachko theorem cited in Theorems and .

For $E \geq 0$ on $\mathbb{C}^d$ with eigenvalues $\lambda_1(E) \geq \ldots \geq \lambda_d(E)$ and $1 \leq j \leq m \leq d$, put

$$\beta_{m,j}(E) = \sum_{i=m-j+1}^{m} \lambda_i(E),$$

the sum of the j smallest among the m largest eigenvalues. A *top-m spectral subspace* of E is the span of orthonormal eigenvectors for $\lambda_1(E), \ldots, \lambda_m(E)$. A *sub-POVM* is a family $E_1, \ldots, E_L \geq 0$ with $\sum_x E_x \leq I_d$.

Lemma (Meeting subspaces). *Let $(E_x)_{x=1}^{L}$ be a sub-POVM on $\mathbb{C}^d$, let V_x be a top-m spectral subspace of E_x, and let $M \subseteq \mathbb{C}^d$ be a subspace with $\dim(M \cap V_x) \geq j_x$, where $1 \leq j_x \leq m$. Then*

$$\sum_{x=1}^{L} \beta_{m,j_x}(E_x) \le \dim M.$$

Proof. Choose $W_x \subseteq M \cap V_x$ with $\dim W_x = j_x$. The space V_x is invariant under E_x, and the restriction has eigenvalues $\lambda_1(E_x), \dots, \lambda_m(E_x)$. By the Ky Fan minimum principle, $\mathrm{Tr}\,(\Pi_{W_x} E_x) \ge \beta_{m,j_x}(E_x)$. Since $\Pi_M - \Pi_{W_x} \ge 0$ and $E_x \ge 0$, $\mathrm{Tr}\,(\Pi_M E_x) \ge \mathrm{Tr}\,(\Pi_{W_x} E_x)$. Summing over x gives $\sum_x \beta_{m,j_x}(E_x) \le \mathrm{Tr}\,(\Pi_M \sum_x E_x) \le \mathrm{Tr}\,\Pi_M = \dim M$.

Status: proved. par *Non-claim.* A matrix inequality for declared effects; it identifies no physical register. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Lemma (Extension from general position). *Let $\mathcal{G}$ be a dense subset of $\mathrm{Gr}(m,d)^L$. Suppose that $\sum_x \beta_{m,j_x}(E_x) \le D$ for every sub-POVM that has top-m spectral subspaces $(V_x) \in \mathcal{G}$. Then the inequality holds for every sub-POVM, and in particular for every POVM.*

Proof. Let (E_x) be a sub-POVM with top-m spectral subspaces V_x, and let $0 < \delta < 1$. For unitaries U_x put $E_x' = (1-\delta) U_x E_x U_x^\dagger$. Since $\|E_x\| \le 1$, $\|U_x E_x U_x^\dagger - E_x\| \le 2\|U_x - I\|$, so $\sum_x E_x' \le (1-\delta)(1 + 2\sum_x \|U_x - I\|) I \le I$ whenever $\sum_x \|U_x - I\| \le \delta/(2(1-\delta))$. The orbit map $U \mapsto UV_x$ of the transitive action of $\mathrm{U}(d)$ on $\mathrm{Gr}(m,d)$ is open. Since $\mathcal{G}$ is dense, there are such U_x with $(U_x V_x) \in \mathcal{G}$. The space $U_x V_x$ is a top-m spectral subspace of E_x', and $\beta_{m,j}(E_x') = (1-\delta)\beta_{m,j}(E_x)$. Hence $(1-\delta)\sum_x \beta_{m,j_x}(E_x) \le D$. Let $\delta \to 0$.

Status: proved. Each general-position set used below is the set where a finite family of matrices, whose entries are entries of basis matrices of the V_x, has full column rank. Such a set is Zariski open in the irreducible variety $\mathrm{Gr}(m,d)^L$, and so it is dense as soon as it is nonempty. Nonemptiness is shown by an explicit example in each case. par *Non-claim.* A perturbation argument on declared effects. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Valid meeting inequalities). *Let $L \ge 2$, $1 \le m < d < Lm$, and let (E_x) be any sub-POVM on $\mathbb{C}^d$.*

- *(Common intersection.) If $c = Lm - (L-1)d \ge 1$, then $\sum_x \beta_{m,c}(E_x) \le c$.*
- *(Kernel; band.) If $(L-1)m \le d$ and $k = Lm - d$, then $\sum_x \beta_{m,k}(E_x) \le (L-1)k$.*
- *(s-wise intersections.) Let $2 \le s \le L-1$, $r = sm - (s-1)d \ge 1$ and $j = \frac{L-1}{s-1} r \le m$. Let $\mathcal{G}_s$ be the set of (V_x) such that, for every x, the intersections $W_S = \bigcap_{y \in S} V_y$ over the s-sets $S \ni x$ are linearly independent. If $\mathcal{G}_s \ne \varnothing$, then $\sum_x \beta_{m,j}(E_x) \le \frac{L}{s} r$. The set $\mathcal{G}_2$ is nonempty for $L = 3$ whenever $3m \le 2d$, and $\mathcal{G}_3$ is nonempty for $(d, L, m) = (7, 4, 5)$.*

Proof. Every intersection of s of the m-dimensional spaces V_x has dimension at least $sm - (s-1)d$. (I1) The common intersection M of all V_x has $\dim M \ge c$. Choose c dimensions of it and apply Lemma with $j_x = c$. No general position is needed. (I2) Let $\mathcal{G}$ be the set where, for every x, the spaces V_y with $y \ne x$ are linearly independent; this is a full-column-rank condition on the $d \times (L-1)m$ matrix of their bases. On $\mathcal{G}$, let $K = \{(w_x) \in \mathrm{bigoplus}_x V_x : \sum_x w_x = 0\}$, which has $\dim K \ge Lm - d = k$. A vector of K with $w_x = 0$ is a dependency among the V_y, $y \ne x$, so it vanishes; hence each projection $\pi_x : K \to V_x$ is injective. Take $K' \subseteq K$ with $\dim K' = k$, put $U_x = \pi_x(K')$ (so $\dim U_x = k$) and $M = U_1 + \dots + U_{L-1}$. Then $\dim M \le (L-1)k$, and $U_L \subseteq M$ because $w_L = -\sum_{x<L} w_x$. Lemma with $j_x = k$ gives the inequality on $\mathcal{G}$. An example in $\mathcal{G}$: in $\mathbb{C}^{L(m-k)} \oplus (H \otimes \mathbb{C}^k)$, with $H \subset \mathbb{C}^L$ the sum-zero hyperplane and φ_x the projection of e_x to H, take $V_x = (\mathrm{block}x\mathrm{of}\mathbb{C}^{L(m-k)}) \oplus (\varphi_x \otimes \mathbb{C}^k)$. Any $L-1$ of the φ_x are independent. Lemma removes the restriction to $\mathcal{G}$. (I3) For each x, independence of the W_S $(S \ni x)$ is the injectivity of the linear map that sends tuples of vectors $(a_{S,y})_{S \ni x,\, y \in S}$ of $\mathbb{C}^m$ to the consistency differences $B_y a_{S,y} - B_{y'} a_{S,y'}$ and to the sum $\sum_S B_x a_{S,x}$, where B_y are basis matrices. This is a full-column-rank condition, so $\mathcal{G}_s$ is

Zariski open. On $\mathcal{G}_s$, choose $U_S \subseteq W_S$ with $\dim U_S = r$ and put $M = \sum_{|S|=s} U_S$. Then $\dim M \le \frac{L}{s} r$, and $M \cap V_x \supseteq \mathrm{bigoplus}_{S \ni x} U_S$ has dimension $\frac{L-1}{s-1} r = j$. Lemma applies, and Lemma extends it. Examples: for $L = 3$, $s = 2$, with $p = 2m - d \ge 1$ and $q = 2d - 3m \ge 0$, take in $(H \otimes \mathbb{C}^q) \oplus (\mathbb{C}^3 \otimes \mathbb{C}^p)$ the spaces $V_x = (\varphi_x \otimes \mathbb{C}^q) \oplus (\mathrm{span}\{e_y : y \ne x\} \otimes \mathbb{C}^p)$. Then $V_x \cap V_y = e_z \otimes \mathbb{C}^p$ for the third index z, and the two such spaces containing a given x are independent. For $(7, 4, 5)$ take in $H \oplus \mathbb{C}^4$ (with $H \subset \mathbb{C}^4$) the spaces $V_x = (\varphi_x^{\perp} \cap H) \oplus \mathrm{span}\{e_y : y \ne x\}$. Any three φ_x span H, so the triple intersections are $W_S = \mathrm{span}\{e_w\}$ with $w \notin S$, and for each x the three of them containing x are independent. Check DC-70 verifies the ranks of these examples in exact arithmetic.

Status: proved. (I1) is the case $s = L$ of (I3), where no general position is needed. Whether $\mathcal{G}_s \ne \varnothing$ for every admissible (d, L, m, s) is not decided here. par *Non-claim.* Linear algebra on declared effects; no channel is implemented by these inequalities. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Meeting certificate for the value). *Let $1 \le j \le m < d$ and $D < jd/m$. If every POVM with L outcomes on $\mathbb{C}^d$ satisfies $\sum_x \beta_{m,j}(E_x) \le D$, then*

$$d^2 F_{\max}(d, L, m) \le \Phi_{d,m}(j, D) := \left(\sqrt{(m-j)(d-D)} + \sqrt{jD}\right)^2 < md.$$

Proof. By Theorem , $d^2 F_{\max}$ is the maximum of $\sum_x \left(\sum_{i \le m} \sqrt{\lambda_i(E_x)}\right)^2$. Fix a POVM, and put $A_x = \sum_{i \le m-j} \lambda_i(E_x)$ and $s_x = \beta_{m,j}(E_x)$. By Cauchy–Schwarz on the two groups of indices, $\sum_{i \le m} \sqrt{\lambda_i} \le \sqrt{(m-j)A_x} + \sqrt{j s_x}$. The function $g(A, s) = \left(\sqrt{(m-j)A} + \sqrt{js}\right)^2 = (m-j)A + js + 2\sqrt{(m-j)j}\sqrt{As}$ is concave, nondecreasing and positively homogeneous of degree one. The constraints are $\sum_x (A_x + s_x) \le \mathrm{Tr}\, I_d = d$ and $\sum_x s_x \le D$. They are symmetric in x, so by concavity $\sum_x g(A_x, s_x) \le L g(\overline{A}, \overline{s})$ with $\overline{A} + \overline{s} \le d/L$ and $\overline{s} \le D/L$. Raising $\overline{A}$ does not decrease g, so we may take $\overline{A} + \overline{s} = d/L$. Write $\overline{s} = (1 - \alpha) d/L$. Then $Lg = d\, h(\alpha)$ with $h(\alpha) = \left(\sqrt{(m-j)\alpha} + \sqrt{j(1-\alpha)}\right)^2$, a concave function with maximum $h(\alpha^*) = m$ at $\alpha^* = (m-j)/m$. The constraint reads $\alpha \ge 1 - D/d$, and $1 - D/d > 1 - j/m = \alpha^*$. So h is nonincreasing on the feasible range, and $d\, h(1 - D/d) = \Phi_{d,m}(j, D)$. The strict inequality $\Phi < md$ holds because h is strictly concave for $0 < j < m$, and for $j = m$ because then $\Phi = mD$ and $D < d$.

Status: proved. The certificate is the pair $(M, (j, D))$ of Proposition , together with the Cauchy–Schwarz weights. It is a dual object for the design problem of Theorem , which is distinct from the channel witnesses of Theorem (b). par *Non-claim.* An upper bound on a declared optimization. It does not open a Page curve or identify a temperature, and it has κ-derivative 0. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Direct-sum constructions). *Let $\mathbb{C}^d = \mathrm{bigoplus}_b \mathbb{C}^{d_b}$ and, on each summand, let $I_{d_b} = \sum_x w_{x,b} P_{x,b}$ with rank-m_b projectors and $\sum_b m_b = m$. Then $E_x = \mathrm{bigoplus}_b w_{x,b} P_{x,b}$ is a POVM, and*

$$d^2 F_{\max}(d, L, m) \ge \sum_{x=1}^{L} \left(\sum_b m_b \sqrt{w_{x,b}}\right)^2.$$

Admissible summands, each tensored with I_r if needed: (i) the s-subset piece on $\mathbb{C}^{\binom{L}{s}}$, $P_x = \sum_{S \ni x} |e_S\rangle\langle e_S|$ of rank $\frac{L-1}{s-1}$, with $w_x = 1/s$ (orthogonal blocks for $s = 1$, complements of blocks for $s = L - 1$, and $\frac{1}{L} I_1$ for $s = L$); (ii) the simplex piece on $H \cong \mathbb{C}^{L-1}$, P_x the projector onto φ_x, with $w_x = (L-1)/L$; (iii) for $L \ge 3$, its complement $I_H - P_x$ of rank $L - 2$, with $w_x = (L-1)/(L(L-2))$.

Proof. Summation gives $\sum_x E_x = I$. The effect E_x has eigenvalue $w_{x,b}$ with multiplicity m_b, so the sum of its m largest square-root eigenvalues is at least $\sum_b m_b \sqrt{w_{x,b}}$. Apply Theorem . For (i), each s-set contains s indices, so $\sum_x P_x = sI$. For (ii), $\sum_x \varphi_x \varphi_x^{\dagger}$, with $\varphi_x = e_x - \frac{1}{L} 1$, is the projector $I - \frac{1}{L} 1 1^{\dagger}$ onto H, and $\|\varphi_x\|^2 = (L-1)/L$. For (iii),

$\sum_x (I_H - P_x) = (L - \frac{L}{L-1})I_H$.

Status: proved. par *Non-claim.* Explicit finite constructions; no physical encoder is implied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Band value for every *L*). *Let $L \geq 2$ and $(L-1)m \leq d \leq Lm$, and put $k = Lm - d \in \{0, \dots, m\}$. Then*

$$d^2 F_{\max}(d, L, m) = L(m - k + k\sqrt{1 - \tfrac{1}{L}})^2, \qquad md - d^2 F_{\max} = L(1 - \sqrt{1 - \tfrac{1}{L}})^2 k(m-k).$$

So $F_{\max} = m/d$ iff $k \in \{0, m\}$, and $e_{\text{seed}} = 1 - d^{-2} L(m - k + k\sqrt{1 - 1/L})^2$. For $L = 2$ this is Theorem .

Proof. For $k = 0$ and $k = m$ the value is Lm^2 or md by Theorem (c),(d) (orthogonal blocks; $m \mid d$). For $0 < k < m$ we have $(L-1)m < d$, so $D = (L-1)k < kd/m$. Proposition (I2) and Theorem with $j = k$ give $\Phi = (\sqrt{(m-k)L(m-k)} + \sqrt{(L-1)k^2})^2$, which is the stated value, since $d - (L-1)k = L(m-k)$. For the lower bound, Proposition with orthogonal blocks of rank $m - k$ and the simplex piece tensored with I_k gives effects with eigenvalues 1 (multiplicity $m - k$) and $(L-1)/L$ (multiplicity k), and the same value. The difference formula is algebra: with $u = 1 - \sqrt{1 - 1/L}$, $md - L(m - ku)^2 = km(2Lu - 1 - Lu^2) + Lu^2 k(m-k)$, and $2Lu - 1 - Lu^2 = L(1 - (1-u)^2) - 1 = 0$. For $L = 2$, $r = m - k$ and the value is $2r^2 + 2\sqrt{2}rk + k^2$. Check DC-71.

Status: proved, unconditional. It gives, for example, $d^2 F_{\max}(5, 3, 2) = 5 + 2\sqrt{6}$, $d^2 F_{\max}(7, 3, 3) = 11 + 4\sqrt{6}$, $d^2 F_{\max}(8, 3, 3) = 14 + 4\sqrt{6}$ and $d^2 F_{\max}(7, 4, 2) = 7 + 4\sqrt{3}$. par *Non-claim.* A closed value for declared cuts. It derives no temperature, Page curve or compression rate, and every value has κ-derivative 0. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Co-band value for every *L*). *Let $L \geq 2$, $m \leq d$, and $c = Lm - (L-1)d \geq 0$, and put $n = d - m$, so that $m = c + (L-1)n$ and $d = c + Ln$. Then*

$$d^2 F_{\max}(d, L, m) = (c + n\sqrt{L(L-1)})^2, \qquad md - d^2 F_{\max} = cn(2L - 1 - 2\sqrt{L(L-1)}).$$

So $F_{\max} = m/d$ iff $c = 0$ or $n = 0$.

Proof. If $n = 0$, then $F_{\max} = 1$. If $c = 0$, complements of L orthogonal blocks of rank n give $I = \sum_x \frac{1}{L-1}(I - P_x)$, and Theorem (d) gives md. Let $c, n \geq 1$. Then $D = c < cd/m$, and Proposition (I1) with Theorem ($j = D = c$) gives $(\sqrt{(m-c)(d-c)} + c)^2$, with $(m-c)(d-c) = L(L-1)n^2$. For the lower bound, Proposition with $\frac{1}{L}I_c$ and the complement piece tensored with I_n gives eigenvalues $\frac{1}{L}$ (multiplicity c) and $\frac{1}{L-1}$ (multiplicity $(L-1)n$), and the value $L(c/\sqrt{L} + n\sqrt{L-1})^2$. The difference is expansion. Check DC-72.

Status: proved, unconditional. Examples: $d^2 F_{\max}(4, 3, 3) = 7 + 2\sqrt{6}$, $d^2 F_{\max}(5, 4, 4) = 13 + 4\sqrt{3}$, $d^2 F_{\max}(6, 5, 5) = (1 + 2\sqrt{5})^2$. par *Non-claim.* As for Theorem . Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Three labels: complete closed form). *For $1 \leq m \leq d$,*

$$d^2 F_{\max}(d, 3, m) = \begin{cases} 3m^2, & d \geq 3m, \\ 3(m - k + k\sqrt{2/3})^2, \ k = 3m - d, & 2m \leq d \leq 3m, \\ 2(q + \sqrt{3}\,p)^2, \ p = 2m - d, \ q = 2d - 3m, & \frac{3}{2}m \leq d \leq 2m, \\ (c + \sqrt{6}\,(d-m))^2, \ c = 3m - 2d, & m \leq d \leq \frac{3}{2}m. \end{cases}$$

The pieces agree at $d = 3m$, $2m$ and $\frac{3}{2}m$. The value equals md exactly when $d/m \in \{1, \frac{3}{2}, 2, 3\}$, in agreement with Theorem .

Proof. The first case is Theorem (c), the second is Theorem , and the fourth is Theorem . In the third case, if $p = 0$

or $q = 0$ the value is md ($d = 2m$ or $d = \frac{3}{2}m$; Theorem). Otherwise $3m < 2d < 4m$, and Proposition (I3) with $s = 2$, $r = p$, $j = 2p \le m$ and $D = 3p < 2pd/m$ applies. Theorem gives $(\sqrt{(m-2p)(d-3p)} + \sqrt{6p^2})^2 = (\sqrt{2q^2} + \sqrt{6}p)^2 = 2(q + \sqrt{3}p)^2$, using $m - 2p = q$ and $d - 3p = 2q$. For the lower bound, Proposition with the trine (simplex piece, $L = 3$) tensored with I_q and the complement piece tensored with I_p gives eigenvalues $\frac{2}{3}$ (multiplicity q) and $\frac{1}{2}$ (multiplicity $2p$), and the value $3(q\sqrt{2/3} + 2p/\sqrt{2})^2$. At $d = 2m$ both adjacent formulas give $2m^2$, and at $d = \frac{3}{2}m$ both give $\frac{3}{2}m^2$. Check DC-73: the formula, the constructions and a POVM search agree for every (d, m) with $d \le 7$.

Status: proved, unconditional. This closes B2-O24 for $L = 3$. par *Non-claim.* A closed value for declared three-label cuts. No closed formula is claimed for $L \ge 4$ outside the two bands. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corollary (The version-5 surds are theorems). $Q_{5,3,2} = 5 + 2\sqrt{6}$, $Q_{4,3,3} = 7 + 2\sqrt{6}$, $Q_{5,3,3} = 8 + 4\sqrt{3}$ *and* $Q_{5,4,4} = 13 + 4\sqrt{3}$, *where* $Q_{d,L,m} = d^2 F_{\max}(d, L, m)$. *These were numerical identifications in Theorem . Also* $Q_{7,3,3} = 11 + 4\sqrt{6}$, *so the version-4 search value* 0.424448 *of Proposition is exact to six digits. For* $L = 4$, $Q_{7,4,5} = 18 + 12\sqrt{2}$, *from Proposition (I3) with* $s = 3$ *(*$j = 3$, $D = 4$*) and the construction ``complement of the simplex* $\oplus$ *the* 3*-subset piece''.*

Proof. Theorems , and . For $(7, 4, 5)$: $D = 4 < jd/m = \frac{21}{5}$, so $\Phi = (\sqrt{2 \cdot 3} + \sqrt{12})^2 = 18 + 12\sqrt{2}$. The construction has eigenvalues $\frac{3}{8}$ (multiplicity 2) and $\frac{1}{3}$ (multiplicity 3), and gives $4(2\sqrt{3/8} + 3/\sqrt{3})^2 = 18 + 12\sqrt{2}$. Exact identities: DC-73 and DC-74.

Status: proved, unconditional. The values do not rest on the cited Horn–Klyachko theorem, and they agree with the version-5 conic values to $2 \cdot 10^{-6}$ (DC-73). par *Non-claim.* Values of declared cuts only. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corollary (Channel witnesses at the closed values; unseeded attainers).

• *For each value* $t = F_{\max}$ *of Theorems –, the isotropic channel* J_t *and the witnesses* Y_t, Z_t, H_t *of Theorem (b) are explicit in closed form. They satisfy Lemma for* $T - \mathrm{id}$ *and for* $\mathrm{id} - T$*, and in the output-first and input-first orders, so* $e_{\mathrm{seed}} = 1 - t$ *is certified from both sides. For an untouched factor of dimension* s*, tensoring with* W_s *certifies the same error (Corollary).*

• *For the unseeded attainers of Theorem at* $(5, 3, 2)$, $(3, 2, 2)$ *and* $(4, 3, 3)$, Ω *is not in the range of the Choi operator, so Lemma (d) gives* $t^* = 0 < F_e$ *and the rank-one witness does not saturate. The half-diamond errors of these attainers are* 0.666667, 0.5 *and* 0.333333, *against the seeded values* $1 - F_e = 0.604041$, 0.352397 *and* 0.256314 *(DC-75, numerical).*

Proof. (a) Theorem (b) and Lemma hold for every $t \in [0, 1]$. The order change conjugates by the swap, which fixes W_d and I. DC-75 verifies the identities in exact arithmetic at $t = (5 + 2\sqrt{6})/25$, $(7 + 2\sqrt{6})/16$, $(8 + 4\sqrt{3})/25$ and $(3 + 2\sqrt{2})/9$, and the tensored witnesses numerically at $(5, 3, 2)$ with $s = 2$. (b) Lemma (d) computes t^*, and a semidefinite program computes the error. These are numbers, not certificates.

Status: (a) proved; (b) numerical. Part (b) corrects the plan's outline, which applies the sandwich lemma to the Ky Fan maximizer itself: exactness holds after the twirl, not for the untwirled attainer. par *Non-claim.* The side channel opens no Page curve and identifies no temperature. No certificate is claimed for an untwirled object. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Same-temperature separation for early labels (version 6)

This section implements item 6 of the plan t287u. Proposition already proves the zero-Hamiltonian form: one early label is compatible with late errors 0, $1-1/d$ and $1-1/d^2$. Here the system Hamiltonian is any supplied finite H_S on $\mathbb{C}^d$, chosen independently of κ. We decide which of the three channels are thermal operations at a common inverse temperature, and we test the plan's claim that the late error does not depend on the early label. Thermal operations are as in Big01-A v5 thm:Bthermal (source record; not a premise): $X \mapsto \mathrm{Tr}_B[U(X\otimes\gamma_{\beta,B})U^\dagger]$ with a finite bath Hamiltonian H_B, $\gamma_{\beta,B}=e^{-\beta\mathrm{H}_B}/Z$, and U commuting with $\mathrm{H}_S\otimes I + I\otimes\mathrm{H}_B$. Write $\gamma_\beta = e^{-\beta\mathrm{H}_S}/Z_\beta$.

Proposition (Which of the three channels are thermal operations). *Fix $\beta>0$ and H_S.*

• *Every thermal operation at β fixes γ_β.*

• *The identity is a thermal operation at β for every H_S.*

• *For an orthonormal basis $\mathscr{B}$, complete dephasing $\Delta_{\mathscr{B}}$ is a thermal operation at β iff H_S is diagonal in $\mathscr{B}$.*

• *A replacement channel $\mathscr{R}_\sigma$ is a thermal operation at β iff $\sigma=\gamma_\beta$. In particular $\mathscr{R}_{I/d}$ is a thermal operation at some $\beta>0$ iff H_S is a multiple of I, and then at every β.*

• *The errors are $\varepsilon(\mathrm{id})=0$, $\varepsilon(\Delta_{\mathscr{B}})=1-1/d$ and $\varepsilon(\mathscr{R}_{\gamma_\beta})=1-1/(Z_\beta Z_{-\beta})$, and the recovery errors are $\varepsilon^*=0$, $1-1/d$ and $1-1/d^2$, for every H_S and β.*

Proof. (a) U commutes with $e^{-\beta(\mathrm{H}_S\otimes I + I\otimes\mathrm{H}_B)}$, so $U(\gamma_\beta\otimes\gamma_{\beta,B})U^\dagger=\gamma_\beta\otimes\gamma_{\beta,B}$, and the partial trace returns γ_β. (b) $U=I$. (c) If H_S is diagonal in $\mathscr{B}$, the construction of Big01-A v5 thm:Bdephase(a) (source record; not a premise) applies: a d-dimensional bath with $\mathrm{H}_B=0$ and $U=\sum_a Z^a\otimes|a\rangle\langle a|$, with Z the clock matrix in $\mathscr{B}$, commutes with $\mathrm{H}_S\otimes I$ and gives $\Delta_{\mathscr{B}}$. Conversely, by (a), $\Delta_{\mathscr{B}}(\gamma_\beta)=\gamma_\beta$, so γ_β is diagonal in $\mathscr{B}$, and hence so is $\mathrm{H}_S=-\beta^{-1}\ln(Z_\beta\gamma_\beta)$. (d) Sufficiency is Big01-A v5 thm:Bthermal(a) (source record; not a premise): a copy of the system as the bath, with the swap. Necessity: $\mathscr{R}_\sigma(\gamma_\beta)=\sigma$, so (a) gives $\sigma=\gamma_\beta$. Finally, $\gamma_\beta=I/d$ iff H_S is scalar. (e) Proposition (a),(b), and for ε^* Theorems (e) and , as in the proof of Proposition . With $\sigma=\gamma_\beta$, $\mathrm{Tr}\,\sigma^{-1}=Z_\beta Z_{-\beta}$. Check DC-76 verifies the commutations, the channels and the errors by SDP for $d=2,3$ and $\beta\in\{0.3,1,3\}$, with scalar and non-scalar H_S.

Status: proved. Parts (a), (c) (necessity) and (d) (necessity) are new. The rest is Big01-A v5 thm:Bthermal, Big01-A v5 thm:Bdephase and Proposition . par *Non-claim.* Thermal operations here are finite ancillary models with supplied Hamiltonians. No bath located in the early universe is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (One early label, three late errors, for any system Hamiltonian). *Fix a comparison future (Definition), the early datum $\mathfrak{E}$, a time $t\le 0$ and a supplied H_S with an eigenbasis $\mathscr{B}$. Declare a late interface to be a thermal operation at the label temperature, $\beta_\kappa=1/(k_B T_{\mathrm{r}}^{(\kappa)}(t))$, after ideal transport from a visible early source.*

• *For every $\kappa>0$, the channels id, $\Delta_{\mathscr{B}}$ and $\mathscr{R}_{\gamma_{\beta_\kappa}}$ are thermal operations at β_κ, with recovery errors 0, $1-1/d$ and $1-1/d^2$. These errors do not depend on κ, so each is a function of admitted data (Lemma (a)).*

• *The undecoded errors 0 and $1-1/d$ do not depend on κ. The undecoded error of the third channel is $e(\kappa)=1-1/(Z_{\beta_\kappa}Z_{-\beta_\kappa})$. If H_S is scalar, $e\equiv 1-1/d^2$. Otherwise e is strictly decreasing in κ, with range $(1-1/d^2,1)$ over $\kappa\in(0,\infty)$. Then e is not a function of admitted data, and the exact minimax error of an admitted-data rule for e is $\frac{1}{2d^2}$; the midpoint rule attains it as a supremum, which is not attained.*

• *If the interface is required to be $\mathscr{R}_{I/d}$, it is a thermal operation at β_κ for some κ iff H_S is scalar, and then for every κ. The same holds with the late label $T_{\mathrm{late}}^{(\kappa)}=\kappa K/(2\pi k_B)$ of Theorem at a fixed rate K in place of $T_{\mathrm{r}}^{(\kappa)}(t)$.*

Proof. (a) and (c) are Proposition . A function constant on the κ-family is constant on that fibre of the admitted data, so Lemma (a) applies. (b) With eigenvalues E_i of H_S, $Z_\beta Z_{-\beta} = \sum_{i,l} e^{-\beta(E_i - E_l)} = d + 2\sum_{i<l} \cosh(\beta(E_i - E_l))$. This is strictly increasing in $\beta > 0$ unless all E_i coincide. It tends to d^2 as $\beta \to 0$ and to ∞ as $\beta \to \infty$. By Theorem (a), $\kappa \mapsto T_{\mathrm{r}}^{(\kappa)}(t)$ is an increasing bijection onto $(0,\infty)$, so $\kappa \mapsto \beta_\kappa$ is a decreasing bijection onto $(0,\infty)$. The admitted data are constant on the family $\kappa \in (0,\infty)$, so Lemma (b) gives worst-case error at least $\frac{1}{2}|e(\kappa_1) - e(\kappa_2)|$ for every pair, and the supremum of these is $\frac{1}{2d^2}$. The rule $r = 1 - \frac{1}{2d^2}$ has $|r - e(\kappa)| < \frac{1}{2d^2}$ for every κ, because the range is open. For $T_{\mathrm{late}}^{(\kappa)}$, the map $\kappa \mapsto T_{\mathrm{late}}^{(\kappa)}$ is also an increasing bijection onto $(0,\infty)$ at fixed K. Check DC-77 sweeps $\kappa \in [10^{-1}, 10^{6}]$ at $d = 3$.

Status: proved. Part (a) extends Proposition and nuADaM01-A v3 thm:negid(a) (source record; not a premise) from zero Hamiltonians to every supplied H_S. Part (b) corrects the plan's sentence ``the fibre lemma shows the late error is independent of the early label". This holds for recovery errors, and for the undecoded errors of the identity and of dephasing. It fails for the undecoded Gibbs-replacement error when H_S is not scalar and the bath is placed at the label temperature. par *Non-claim.* No physical noise channel at T_{r} is constructed; placing a bath at a label temperature is a declaration. The early entropy label is not identified with the late horizon entropy, and κ is not supplied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Re-check of version-5 statements (version 6)

The version-5 check script (DC-54 to DC-69) and the version-4, -3 and -2 scripts were rerun, and their outputs are byte-identical to the delivered JSON files. The numerical identifications of Theorem are now exact theorems (Corollary). The conic values of DC-56 and the search values of DC-46 agree with the closed forms to $2 \cdot 10^{-6}$ in every row that the closed forms cover (10 of 11 and 6 of 8 rows). The uncovered rows lie in the middle range of B2-O27, where the recorded value is m/d. Theorem agrees with the three-label classification of Theorem , and Theorem with $L = 2$ reproduces Theorem . The version-5 audit entry ``thm:cert-every is in I-C-Ex-CERT v6, not I-C-CERT v6" was confirmed again (DC-78). Proposition and the B2-O24 entry of version 5 called the $L \geq 3$ closed form open; the open part is now narrower (B2-O24 and B2-O27 below). That is an update by addition, not an erratum. *Errata found: none.* This is a re-check by the same process, not an independent review.

What is deliberately not done (version 6)

• No closed formula for $F_{\max}(d, L, m)$ with $L \geq 4$ and $Lm/(L-1) < d < (L-1)m$, except $(7, 4, 5)$ and the cases where I_d is a weighted projector sum (B2-O27). Whether $\mathscr{G}_s$ is nonempty for every admissible (d, L, m, s) is not decided. Whether the rank conditions (N1), (N2) suffice for every L remains open (B2-O25).

• No certificate for untwirled attainers; the numbers of Corollary (b) are SDP output.

• No thermal state, bath located in the early universe, physical noise channel at a label temperature, Page curve, finite Hilbert space or entropy identification. No coefficient slot κ is filled, and no quantum coefficient is admitted.

• Plan items 2, 4 and 5 (the dual witness for the En08 symmetric-subspace pair, the proper-time bound over the branch sphere for Red06-G, and the dimension count of the gauge-invariant equivariant subspace for V-DST) are not treated in this sheet; they are placed in DSS-CERT.

• No discharge of O.1, XV-B, XVI-N or Gate 0. No revision of any parent sheet. Citations identify source records only; no proof transfer.

Emergence-page sentence, version 6 (not registered)

The following sentence, from the plan t287u, is recorded for use if the five packaging items are later registered. It is not registered here, and this sheet carries only items 3 and 6. These five packaging theorems supply dual witnesses, exact (L, m) criteria, a proper-time modulus, a gauge-invariant dimension count and an early-label separation on prescribed hosts. None of them is spacetime emergence, a Page curve, or a derived temperature.

Split-fibre non-selection of dark capacity (version 7)

This section and Sections – implement items 1–4 of the plan t333u and integrate the user draft t335u (``Version-7 New Findings''), whose section structure they follow; each draft result is re-proved here, and its verdict is in Appendix . Big05-A records that ``Big02-A's operational dark capacity is not a host functional and is not identified with stress-energy'' (Big05-A v3 prop:Big05-dark; the sentence itself is in the version-1 paragraph of its abstract) (source record; not a premise). Here the sentence becomes two internal results, with a qualification: the certificate package varies with declared data at a fixed host, although some of its entries are genuine functionals of the full prescribed metric (Proposition); and every entry is constant on every fixed-account split fibre (Theorem). Dependence on the account, on the complete metric, and on the source attribution at fixed metric and account are three different questions. Every proof below is local. Big05-A, Big04-A-coeff, Big03-A and DS-OB-1-fibre are cited only as parallel records.

Definition (Hosts with a declared source, accounts and fixed-account split fibres). A *host datum* is $h = (g, \Lambda, \chi)$: complete prescribed geometric data (a time-oriented metric g on M, here DSS_H, a comparison future g_K of , a flat FLRW future of Section or a prefix host of Definition , together with the declared observers, clocks and references), a declared constant Λ and a declared coupling $\chi \neq 0$. Its *total source* is $T_{\mathrm{tot}} = \chi^{-1}(G[g] + \Lambda g)$, which is symmetric and divergence-free by the contracted Bianchi identity [HawkingEllis1973]. A *split* on an open $U \subseteq M$ is a triple $\sigma = (T_{\mathrm{b}}, T_{\mathrm{dm}}, T_\Lambda)$ of symmetric, divergence-free tensor fields with $T_{\mathrm{b}} + T_{\mathrm{dm}} + T_\Lambda = T_{\mathrm{tot}}$ on U; it has *vacuum form* if $T_\Lambda = -\rho_\Lambda g$ with ρ_Λ constant. The names ordinary, dark and vacuum are attribution coordinates, not independently constructed matter models. $\mathrm{Sp}(h; U)$ is the set of splits. A *declared account* $\mathscr{A}$ is everything Definition and Section declare: registers and factorizations, codes, encodings, the actual channels, located circuits with their gate positions and event placements, payload dimensions, cuts, residual states, complete decoder access, and optional promises. An *account* is $x = (h, \mathscr{A}, \sigma)$, and $\mathscr{X}$ is the set of accounts. The *split-forgetful map* is $\pi(x) = (h, \mathscr{A})$, and the *host map* is $\pi_0(x) = h$. A *fixed-account split fibre* is any specified nonempty class $\Sigma \subseteq \mathrm{Sp}(h; U)$, at fixed $(h, \mathscr{A})$, such that changing $\sigma \in \Sigma$ changes none of h, $\mathscr{A}$ or T_{tot}; in particular the split is not used to change a preparation, interaction, instrument, source placement or decoder permission. An application with equations of state, positivity conditions, species-dependent interactions or closure laws must use its own smaller class Σ. For a future unit timelike field u put $\rho_X = T_X(u, u)$. An *attribution target* is a function on Σ, for example a dominance indicator $1[\rho_{\mathrm{b}} < \rho_{\mathrm{dm}} + \rho_\Lambda \text{on} U]$ or $1[\rho_{\mathrm{dm}} > \rho_{\mathrm{b}}]$, or a ratio such as $\rho_{\mathrm{dm}}/\rho_{\mathrm{b}}$ or $\mathscr{R} = \int w(\rho_{\mathrm{dm}} + \rho_\Lambda)/\int w\rho_{\mathrm{b}}$ for a weight $w \geq 0$, $\int w = 1$, where the denominators are positive.

These are the definitions of Big05-A v3 def:Big05-split and def:Big05-dominance (source record; not a premise), restated so that the proofs below are self-contained; no unprinted admissibility condition of Big05-A is assumed.

Lemma (The retained certificate package factors through the split-forgetful map). *Let C be any of the following, regarded as a function on $\mathscr{X}$: (I) the dimension count, the Singleton bound and the no-promise family of Theorem (a),(b),(d)–(f), for example $(n \ln q - \ln K, (n - 2(\delta - 1))\ln q)$; (II) $\varepsilon_{\min}(s) = (1 - 1/d^2)1[s \notin J^-(\gamma)]$ of Theorem (c) and Theorem (b), and every error of Theorem (a),(e),(f); (III) every error and floor of Theorem and Theorem , for example $p(1 - 1/k^2)$, $1 - m/k$, $1 - 1/(1 + j^2)$ and $\varepsilon^*(\mathscr{N})$; (IV) χ_e, R_e, $R_0 = c\int_0^\infty \mathrm{d}t/a \in (0, \infty]$, the dark-event indicator and $\varepsilon_{\min}(p)$ of Theorem (a)–(c), and every quantity of Theorem (a)–(e). Then $C = \widehat{C} \circ \pi$ for a function $\widehat{C}$ of $(g, \mathscr{A})$ alone. In particular C depends neither on σ nor on (Λ, χ). The promise form of Theorem (c) and every bound evaluated from a supplied κ also factor through π (their η and κ are part of $\mathscr{A}$), but they are not coefficient-blind and are excluded from Section .*

Proof. Each quantity is defined by a formula, or by an infimum over decoders and located circuits, whose inputs are declared objects of $\mathscr{A}$ and, for (II) and (IV), the causal relation $J^\pm$ of g and the scale factor of g. Located circuits are admissible iff their wires respect J^+ of g (Definition); the receiver channel is fixed by the circuit (Lemma); χ_e, R_e and R_0 are integrals of $1/a$ for the scale factor of g. No definition refers to a split, to Λ or to χ. Lemma (a) then gives $\widehat{C}$.

par *Non-claim.* The lemma is a statement about the definitions used in this sheet. It does not say that a physical dark sector has no stress-energy. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Fixed-account split-fibre non-selection). *Fix $h = (g, \Lambda, \chi)$, a connected open U, a declared account $\mathscr{A}$ and a fixed-account split fibre Σ. Call retained readings the metric, its curvature and all its jets, the fixed observers, clocks and references, $\mathscr{A}$, the certificates of Lemma , the exact total T_{tot} with all its derivatives, and any further split-independent report. A rule is any map of the retained readings, deterministic or randomized, including adaptive protocols whose choices and report laws are functions of the retained readings.*

• *Every retained reading, and the law of every rule, is constant on Σ.*

• *A target F on Σ is reproduced by some rule iff F is constant on Σ. If F takes the values $F_0 \neq F_1$, every rule has worst-case expected absolute error at least $\frac{1}{2}|F_0 - F_1|$.*

• *(Binary targets.) If an attribution predicate takes both values on Σ, the exact minimax error is $\frac{1}{2}$ for real-valued decisions under absolute loss, $\frac{1}{2}$ for randomized Boolean decisions under misclassification loss, and 1 for deterministic Boolean decisions.*

• *(Three-way attribution.) If Σ contains splits at which each of the three coordinates in turn has the uniquely greatest density, every randomized rule naming the dominant coordinate has worst-case misclassification probability at least $\frac{2}{3}$; uniform guessing attains $\frac{2}{3}$, and deterministic rules have worst case 1.*

• *(Ratios.) If an attribution ratio is unbounded on Σ, every rule has infinite worst-case expected absolute error for it. Two distinct splits alone do not give this conclusion.*

• *(Vacuum slot.) If $\sigma = (T_{\mathrm{b}}, T_{\mathrm{dm}}, -\rho_\Lambda g)$ is a split of vacuum form, so is $\sigma^{(\tau)} = (T_{\mathrm{b}}, T_{\mathrm{dm}} + \tau g, -(\rho_\Lambda + \tau) g)$ for every $\tau \in \mathbb{R}$. On the class of vacuum-form splits, ρ_Λ ranges over $\mathbb{R}$, and every rule for it has infinite worst-case error. The absolute vacuum term is not selected.*

• *All of (a)–(f) hold for every value of the slot κ.*

Proof. (a) Lemma , and the definition of Σ for the remaining readings. For an adaptive protocol, induct on the round: the law of the first round is a function of the retained readings, and if the law of the first k rounds is constant on Σ, so is the next, because its kernel is a function of the retained readings and of the transcript. (b) The first claim is Lemma (a). For the bound, the law of the output Y is the same at both splits, and $\mathbb{E}|Y - F_0| + \mathbb{E}|Y - F_1| \geq |F_0 - F_1|$. (c) The lower bounds are (b) with $|F_0 - F_1| = 1$; the constant $\frac{1}{2}$ attains it; a randomized Boolean rule returns 1 with one probability p at both splits, with errors p and $1 - p$; a deterministic Boolean rule is wrong at one of the two splits. (d) A randomized rule names the three coordinates with probabilities $p_{\mathrm{b}} + p_{\mathrm{dm}} + p_\Lambda = 1$, the same at the three splits, so its least success probability is at most $\frac{1}{3}$; uniform guessing has success $\frac{1}{3}$ at each. (e) Apply (b) to pairs with unbounded separation. A bounded ratio (for example under a positive lower bound on ρ_{b} with a bounded total) gives a finite bound. (f) τg is symmetric and parallel, so each new piece is conserved; the sum is unchanged; the last slot keeps vacuum form. Apply (b). (g) Theorem and the list in Lemma : none of the coefficient-blind entries contains κ, and the others carry a fixed supplied κ.

Status: proved. This is the internal form of Big05-A v3 thm:Big05-flagship(a)–(c) and thm:Big05-fibre(e),(f) for the observables of this sheet, and it parallels DS-OB-1-fibre v5 cor:F-ranknotsel and Big03-A v2 prop:R-vac-sep(c),(e) for the vacuum slot; all (source record; not a premise). Checks DC-79 and DC-82. par

Non-claim. The theorem does not forbid declaring a split by convention or by a constitutive law, and it does not say that no split-coupled observation exists; such data are not supplied here. No dark-matter density, clustering mechanism or stress tensor of dark capacity is derived, and absolute vacuum energy stays open. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Explicit nontrivial attribution fibres). *The hypotheses of Theorem (c)–(e) hold in the following*

conservation-only comparisons.

• *(Simplex.) Let* $T_{\mathrm{tot}} = -\rho_* g$ *with* $\rho_* > 0$. *For* $\theta_{\mathrm{b}}, \theta_{\mathrm{dm}}, \theta_\Lambda \geq 0$ *with sum* 1, *the triples* $T_i = -\theta_i \rho_* g$ *are splits of vacuum form with one total, metric, account and certificate package, and* $\rho_i = \theta_i \rho_* \geq 0$ *for every unit timelike u. On the edge* $(1-\theta, \theta, 0)$, $0 < \theta < 1$, $1[\rho_{\mathrm{dm}} > \rho_{\mathrm{b}}] = 1[\theta > \frac{1}{2}]$ *and* $\rho_{\mathrm{dm}}/\rho_{\mathrm{b}} = \theta/(1-\theta)$ *is unbounded; near the three vertices each coordinate is uniquely dominant. On* DSS_H, $T_{\mathrm{tot}} = \chi^{-1}(\Lambda - \Lambda_H)g$ *with* $\Lambda_H = 3H^2/c^2$, *so* $\rho_* = \chi^{-1}(\Lambda_H - \Lambda) > 0$ *whenever* $\chi > 0$ *and* $\Lambda < \Lambda_H$.

• *(Proportional splits.) If* $T_{\mathrm{tot}} \equiv 0$, *the splits* $\sigma_s = (sT_{\mathrm{tot}}, (1-s)T_{\mathrm{tot}}, 0)$, $s \in [0,1]$, *lie in one fibre. If* $\rho_{\mathrm{tot}} > 0$ *on U, the indicator* $1[\rho_{\mathrm{b}} < \rho_{\mathrm{dm}} + \rho_\Lambda]$ *equals* $1[s < \frac{1}{2}]$, *and if* $\int w\rho_{\mathrm{tot}} > 0$, *then* $\mathscr{R}(s) = (1-s)/s$ *on* $(0,1]$, *with range* $[0,\infty)$. *On the comparison future* g_K *with* $\Lambda = 3H_1^2/c^2$, $T_{\mathrm{tot}} = \chi^{-1}\Theta$ *of Theorem (d), with* $\Theta_{tt} = 3(K^2 - H_1^2) \neq 0$ *on* $t \geq 1$ *for* $K \neq H_1$.

Proof. (a) $\nabla g = 0$ gives conservation of every constant multiple of g, and the summands add to $-\rho_* g$; with $g(u,u) = -1$, $\rho_i = \theta_i \rho_*$. On DSS_H, $G = -\Lambda_H g$ (check DC-79). (b) Multiples of the conserved symmetric T_{tot} are conserved and symmetric; $\rho_{\mathrm{b}} = s\rho_{\mathrm{tot}} < (1-s)\rho_{\mathrm{tot}}$ iff $s < \frac{1}{2}$, and $\mathscr{R}$ follows by linearity. The last statement is Theorem (d).

Status: proved. All summands in (a) have vacuum-type tensor form. This is deliberate: conservation and nonnegative energy density alone do not turn attribution names into distinct species. A narrower split class that excludes these directions must exhibit its own nonconstant target before Theorem applies to it. Check DC-79. par

Non-claim. No ordinary-matter sector is shown to admit these tensors, and none is constructed. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

What ``not a host functional'' can correctly mean

Proposition (Account dependence is not the negation of geometric dependence).

• *(Account dependence.) The certificate package is not a function of the host datum h in the unrestricted declared-account class. On* DSS_H*: encoding one logical qubit together with n fixed pure idle qubits changes* $\ln D - \ln K$ *by* $n\ln 2$*; the flagged-loss interfaces of Theorem (c) with* $p = 0$ *and* $p = 1$ *at a visible source give* 0 *and* $1 - 1/k^2$*; payload dimensions* $d_1 \neq d_2$ *at a source with* $u(s) \leq 0$ *give* $1 - 1/d_1^2$ *and* $1 - 1/d_2^2$*; and sources* $s_1 \in J^-(\gamma)$, $s_2 \notin J^-(\gamma)$ *give* 0 *and* $1 - 1/d^2$. *Every rule of h alone has worst-case error at least half of each difference, and the Singleton bound is unbounded in the declared n (Theorem (d)).*

• *(Geometric entries.)* χ_e, R_e *and* R_0 *are functionals of the complete prescribed future metric (not of its common past, Theorem (b)), and* $\varepsilon_{\mathrm{min}}$ *is a functional of the metric, the observer, the source placement and d once those are fixed.*

• *(Different projections.) Constancy on split fibres (Theorem) proves non-identification of every nonconstant split target. It does not show that a certificate is non-geometric, and it does not by itself show non-identification of* κ *(Section): the split, host and coefficient projections forget different coordinates.*

• *(No dictionary.) A dictionary* $T_{\mathrm{dm}} = \Phi(\mathscr{C})$ *that is non-constant in the certificates* $\mathscr{C}$ *changes* T_{dm}, *and hence* T_{b}, *under a change of declared account that changes no host datum; and it is not a consequence of the certificates, which take one value on each split fibre. Any such* Φ *is extra declared data, outside the ledger.*

Proof. (a) The idle qubits multiply D by 2^n at fixed K; Theorem (c) gives $p(1 - 1/k^2)$; Theorem (c) gives the other values; Lemma (b) applied to π_0, whose fibre contains each pair, gives the bounds. (b) The defining integrals and Proposition . (c) The three maps π, π_0 and $\kappa \mapsto$ (data) have different fibres; constancy on the fibres of one map does not imply constancy on those of another. (d) By (a), $\Phi \circ \mathscr{C}$ varies along a fibre of π_0; by Theorem (a) it is constant on split fibres, so the equation $T_{\mathrm{dm}} = \Phi(\mathscr{C})$ restricts accounts and follows from nothing in them.

Status: proved. The internal conclusion is: at fixed complete host and fixed account, dark-capacity certificates do not identify an independently variable source attribution; the certificate package is not determined by geometry alone, although some of its entries are genuine geometric functionals. This qualifies the wording ``dark capacity is not a host functional" of Big05-A v3 prop:Big05-dark(a) (source record; not a premise) (PC7-5), and parallels Big01-A v5 thm:D (source record; not a premise). Check DC-79. par *Non-claim.* Dark capacity remains an access-relative classification of declared registers (Definition), not a substance and not stress-energy. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Future-horizon alignment without a continuation law (version 7)

Fix $H_1 > 0$, identified with the rate H_E of Big05-A v3 thm:Big05-futures (source record; not a premise). Let $\mathscr{F}$ be the smooth flat FLRW futures a with $a = e^{H_1 t}$ on $t \le 0$ and $h = \dot{a}/a > 0$ everywhere, and $\mathscr{F}_{\text{downarrow}} = \{a \in \mathscr{F} : \dot{h} \le 0 \text{on} [0,\infty)\}$. Put $R_0[a] = c\int_0^\infty ds/a \in (0,\infty]$, which is the radius R_* of the cited record with $a_* = 1$. The condition defining $\mathscr{F}_{\text{downarrow}}$ is geometric; it is not imposed as a matter law.

Proposition (The two horizon ranges refer to different future classes).

• $\{R_0[a] : a \in \mathscr{F}\} = (0,\infty]$*: every finite value on the family (Theorem (b)), and* ∞ *on* $h = H_1/(1 + H_1 X)$ *(Theorem (c)). This range carries no energy condition.*

• $\{R_0[a] : a \in \mathscr{F}_{\text{downarrow}}\} = [c/H_1, \infty]$*, with* $R_0 = c/H_1$ *iff* $a = e^{H_1 t}$*. This is the range of Big05-A v3 thm:Big05-futures(e). Hence* $R_0 < c/H_1$ *forces* $\dot{h} > 0$ *somewhere on* $(0,\infty)$*; on the family this happens exactly for* $K > H_1$*. Only after a separately imposed Einstein equation with positive coupling is* $\dot{h} \le 0$ *the null energy condition of the total source (*$G(k,k) = -2\dot{h}$ *for the null field* k *of Theorem (c)).*

• *On the slice* $t = 0$*, a source at comoving radius* r *has* $\varepsilon_{\min} = (1 - 1/d^2)\, 1[r \ge R_0[a]]$*; the left-the-patch certificate is a function of* R_0 *and* d*.*

• *(Common past data.) The splits* $T_i[a] = -\theta_i \rho_* g_a$ *(*$\theta_i \ge 0$*,* $\sum \theta_i = 1$*) are conserved on every member and identical, with all jets, on the common past. A finite experiment with all gates and wires in* $\{t \le 0\}$ *has identical routes and channel maps on every member. So exact past geometry, every present jet, such a past split and every such experiment determine neither* R_0 *nor whether an event horizon exists, in either class.*

• *Enlarging the class of continuations enlarges the range and does not strengthen a theorem inside the smaller class; neither range selects a source split, because* R_0 *and* $\varepsilon_{\min}$ *are constant on split fibres (Lemma). A continuation law or real-analyticity requirement is additional data, not a consequence of the certificate.*

Proof. (a) and the attainment in (b) are the cited parts of Theorems and ; there $h = H_1 + (K - H_1)\chi > 0$ for $K > 0$, and $h_K = (K - H_1)\chi$ is ≤ 0 iff $K \le H_1$. For the lower bound, $\dot{h} \le 0$ and $h(0) = H_1$ give $a \le e^{H_1 t}$, so $R_0 \ge c/H_1$, and equality forces equality of the continuous integrands. (c) Theorem (a) at $t = 0$, where $a = 1$ and $\chi_e(0) = R_0$, with Theorem (b). (d) $\nabla g_a = 0$; the metrics agree on $\{t \le 0\}$, and located circuits there use only J^+ restricted to that region. No Einstein equation is imposed on this total. (e) Lemma .

Status: proved. The difference between the two sheets is the class of futures: Big02-A's range $(0,\infty)$ of Theorem (b) carries no energy condition, while $[c/H_E, \infty]$ of Big05-A v3 thm:Big05-futures assumes $\dot{H} \le 0$ (its part (b) shows that a horizon below c/H_E violates the NEC). With the condition imposed, the two attained sets coincide, so this proposition is compatible with the cited record and does not strengthen it. Big05-A v3 thm:Big05-drift and prop:Big05-predsplit are parallel (source record; not a premise). The optional closure hyp:Big05-closure of Big05-A is not a premise. Check DC-80. par *Non-claim.* No future is selected, no energy condition is imposed on a matter model, and no event horizon is derived from data. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Remark (A monotone power-tail realization). A second realization of (b), from the draft t335u, stays inside $\mathscr{F}_{\text{downarrow}}$ and ends in power laws. With χ the smooth step of (so $\chi(u) + \chi(1-u) = 1$) and $X_\tau(t) = \int_0^t \chi(s/\tau)ds$, put $h_p = H_1/(1 + H_1 X_\tau/p)$, $p > 0$. Then $\dot{h}_p = -h_p^2 \chi(t/\tau)/p \le 0$, every past and present jet agrees with de Sitter, and for $t \ge \tau$, $a_p(t) = a_p(\tau)[(t - \tau/2 + p/H_1)/(\tau/2 + p/H_1)]^p$. So $R_0[a_p] = \infty$ for $p \le 1$; for $p > 1$ it is finite, continuous and strictly decreasing in p, tends to ∞ as $p$downarrow 1 and to c/H_1 as $p \to \infty$. Check DC-80.

Proof. $X_\tau = t - \tau/2$ for $t \ge \tau$ by the symmetry of χ, and integrating h_p gives the tail. h_p increases with p where $X_\tau > 0$; dominated convergence (dominator $1/a_{p_0}$ for $p \ge p_0 > 1$) gives continuity and the limit c/H_1, and monotone convergence gives ∞ as $p$downarrow 1.

Corollary (Prediction errors and finite causal-status ledgers). *In $\mathscr{F}$ and in $\mathscr{F}_{\text{downarrow}}$, with $a_d = 1 - 1/d^2$:*

• *Past-only estimation of R_0 has infinite worst-case error. Under a promise $R_0 \in \mathscr{P}$ with finite* $\inf \mathscr{P} = R_- < R_+ = \sup \mathscr{P}$ *inside the attained set, the exact minimax error is* $\frac{1}{2}(R_+ - R_-)$.

• *For the predicate ``an event horizon exists'', the exact minimax errors are* $\frac{1}{2}$ *(real predictions, absolute loss),* $\frac{1}{2}$ *(randomized Boolean) and* 1 *(deterministic Boolean).*

• *For a source at $t = 0$ and radius r in the interior of the attained set ($r > 0$, resp. $r > c/H_1$), the least access error is* 0 *or a_d at identical past data and past splits, and the exact past-only minimax error is* $\frac{1}{2}a_d$.

• *(Finite ledgers.) Let finitely many past events $p_i = (t_i, \mathbf{x}_i)$, $t_i \le 0$, have certified causal status. Event p_i is visible iff $R_0 > s_i := |\mathbf{x}_i| - (c/H_1)(e^{-H_1 t_i} - 1)$. Each joint-status fibre is $\{R_0 : \max_{\text{vis}} s_i < R_0 \le \min_{\text{dark}} s_j\}$ intersected with the attained set. Every nonempty such fibre has positive length, so no finite ledger identifies R_0; the fibre on which all events are visible is unbounded, and on it the worst-case error is infinite.*

Proof. (a) All past data agree across the class and the attained set is unbounded; for the promise, Lemma (b) at points approaching $R_\pm$, and the midpoint. (b) Members with and without a horizon share the past (Proposition (a),(b)). (c) Choose attained $R < r < R'$; Proposition (c) and Lemma (b). (d) Theorem (a): $\chi_e(t) = (c/H_1)(e^{-H_1 t} - 1) + R_0$ for $t \le 0$. A nonempty set of the form $(L, U']$ with $L < U'$, or $(L, \infty]$, intersected with $(0, \infty]$ or $[c/H_1, \infty]$, has positive length unless it is the single point c/H_1 with $L < c/H_1 = U'$; in that case $R_0 = c/H_1$ is identified only within $\mathscr{F}_{\text{downarrow}}$ and only because the class itself forces it.

Status: proved. Part (d) sharpens the draft's ``an unbounded fibre on which all events are visible'' to every joint-status fibre, with the boundary case stated. Checks DC-80 and DC-83. par *Non-claim.* If R_0 itself is supplied exactly, it is known; the corollary concerns inference from past data and finite status ledgers. The future-complete optimization defining $\varepsilon_{\min}$ is not a finite experiment confined to the past. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Horizon drift is an identity, not a horizon selector). *On a flat FLRW future with finite horizon (Section), $\dot{\chi}_e = -c/a$ and $\dot{R}_e = HR_e - c$. Every solution of $\dot{R} = H\dot{R} - c$ is $R_e + Ca$ for a constant C, and $R = R_e$ iff $R/a \to 0$ at t_{end}. A forecast that factors through split-independent retained data is constant on split fibres; a forecast that uses a separately supplied species law or split-dependent closure is not covered by this statement.*

Proof. Differentiate the convergent tail integral. The difference y of two solutions has $\dot{y} = Hy$, so $y = Ca$. Conversely $\frac{\mathrm{d}}{\mathrm{d}t}(R/a) = -c/a$, and integrating to t_{end} with $R/a \to 0$ gives $R/a = \chi_e$. The last statement is Theorem (a).

Status: proved. Parallel: Big05-A v3 thm:Big05-drift and prop:Big05-predsplit (source record; not a premise). Check DC-85. par *Non-claim.* The identity holds for every prescribed future and selects none. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged;

XV-B is not opened. par

Coefficient-blind adaptive experiments (version 7)

Definition (Operational coefficient blindness). Fix the host, the account and every dictionary parameter other than the slot $\kappa \in \mathscr{K} \subseteq (0,\infty)$. A finite adaptive experiment is *coefficient-blind* if (i) its initial classical and quantum state, including retained memories and references, is independent of κ; (ii) at each history, the kernel selecting the next experiment is independent of κ; and (iii) conditional on the history and the selection, the actual accessible process and the actual instrument, including their action on retained quantum memory, are independent of κ. It may use coefficient-blind certificate values (Lemma , without the promise form) as inputs. It may not use a numerical value of a coefficient-sensitive label, a coefficient-dependent bath preparation or an equivalent absolute calibration. Placement blindness and split independence are separate properties and do not replace (i)–(iii).

Theorem (Adaptive dark-certificate experiments do not calibrate the slot). *For every coefficient-blind experiment:*

• *The transcript law and the final transcript-indexed accessible quantum state are independent of κ, and so is the report law of every coefficient-blind postprocessing, decoder or final tester. This persists under a coefficient-blind stopping rule that terminates almost surely for every κ, and under conditioning on a common transcript event of positive probability.*

• *For every target $L(\kappa)$ and every estimate $\widehat{L}$, $\max_i \mathbb{E}_{\kappa_i}|\widehat{L} - L(\kappa_i)| \geq \frac{1}{2}|L(\kappa_1) - L(\kappa_2)|$. If L is unbounded on $\mathscr{K}$, the worst-case expected error is infinite; if its range on $\mathscr{K}$ has finite infimum ℓ and supremum u, the exact minimax expected error is $\frac{1}{2}(u-\ell)$, also for randomized estimates.*

• *If $\kappa_1, \dots, \kappa_M \in \mathscr{K}$ are pairwise more than 2ε apart, then $\min_j \Pr_{\kappa_j}\{|\widehat{\kappa} - \kappa_j| \leq \varepsilon\} \leq 1/M$. On $\mathscr{K} = (0,\infty)$, under relative loss $|y/\kappa - 1|$ every estimate has worst-case expected loss at least 1, which $y = 0$ attains.*

• *In particular every coefficient-blind estimate of κ, $T^{(\kappa)}$, $S^{(\kappa)}$ or $N^{(\kappa)}$ has infinite worst-case expected error on $(0,\infty)$.*

• *(Compatibility with Theorem .) Each label is injective in κ, so one exact coefficient-sensitive report, for example $T^{(\kappa)}$ together with H, determines $\kappa = 2\pi k_B T^{(\kappa)}/H$. Statements (b)–(d) fail as soon as one such report is admitted, and this sheet admits none.*

• *(Both coordinates.) On accounts with fixed $(h, \mathscr{A})$ and free $(\kappa,\sigma) \in \mathscr{K} \times \Sigma$, every coefficient-blind rule of retained readings has a law that is constant in (κ,σ).*

Proof. (a) For a finite history h let ω_h be the subnormalized accessible memory state (its trace is the probability of h). If x is the next selection and y its report, $\omega_{h,x,y} = p(x \mid h)\,\mathscr{I}_y^{h,x}((\mathscr{N}^{h,x} \otimes \mathrm{id})(\omega_h))$, and every factor is independent of κ; induct from the common initial state. Subnormalized states avoid conditioning on events of probability zero. A stopped law is the countable sum of the common finite-history contributions on which stopping occurs, and almost-sure termination accounts for all its mass; restriction to a common event and normalization preserve equality. (b) Under one common law, $\mathbb{E}|Y - L(\kappa_1)| + \mathbb{E}|Y - L(\kappa_2)| \geq |L(\kappa_1) - L(\kappa_2)|$; take points approaching ℓ and u, and the constant midpoint. (c) The M accuracy events are disjoint under the common law. For relative loss: if $\mathbb{E}|\widehat{\kappa}| < \infty$, then $\mathbb{E}|\widehat{\kappa}/\kappa - 1| \geq 1 - \mathbb{E}|\widehat{\kappa}|/\kappa \to 1$ as $\kappa \to \infty$; otherwise the loss is infinite. (d) Each label maps $(0,\infty)$ onto an unbounded interval (Theorem). (e) Differentiate . (f) Combine (a) with Theorem (a).

Status: proved. This strengthens only the ``no derivation'' direction; the injectivity of Theorem is unchanged. If a coefficient-dependent capacity promise removes members of the coefficient fibre, the theorem applies to the remaining compatible set; a supplied promise is not a blind report. It parallels Big04-A-coeff v5 thm:C-adaptive, prop:C-orbit, thm:C-generic and thm:C-worstcase, Big03-A v2 thm:R-unsupplied, prop:R-vac-sep(d) and prop:R-floors, and DS-OB-1-fibre v5 thm:F-einstein, thm:F-generic and cor:F-ranknotsel; all (source record; not a premise). In prop:C-orbit(d),(f) the orbit is confounding only for a reduced contract that omits α, and an absolute label breaks it; thm:C-generic identifies κ from enough coefficient-sensitive readouts, which is exactly the regime

excluded by Definition . Check DC-81. par *Non-claim.* No value of κ is supplied, and $\kappa = \hbar$ is not admitted. No dictionary parameter such as α is dropped, and no calibrated label is promoted to blind data. No quantum coefficient is admitted. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Proposition (Blind certificate values are not sufficient for process blindness). *Neither split independence, placement blindness nor constancy of the optimized recovery error implies Definition . For a qubit and* $\sigma_\kappa = \mathrm{diag}(\kappa/(1+\kappa), 1/(1+\kappa))$, *every replacement channel* $\mathscr{R}_{\sigma_\kappa}$ *has optimized error* $\frac{3}{4}$, *while its output population* $\kappa/(1+\kappa)$ *determines* κ. *Moreover, equality of every one-round marginal report law does not imply equality of the adaptive law: with a fair bit* Z, $b(\kappa) = 1[\kappa \geq 1]$, $Y_1 = Z$ *and* $Y_2 = Z \oplus b(\kappa)$, *both marginals are fair for every* κ, *while* $Y_1 \oplus Y_2 = b(\kappa)$.

Proof. Every decoding of a replacement is a replacement, and the decoder preparing $I/2$ attains $1 - 1/2^2$ (Theorem (a)). The population p gives $\kappa = p/(1-p)$; the preparation is coefficient-sensitive, so the channel lies outside the blind class. The last claim is direct.

Status: proved. Check DC-81. par *Non-claim.* The example constructs no physical calibration of κ. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Rank and independent parameter supply (version 7)

Proposition (Certificate augmentation does not turn rank into selection). *Fix* $(h, \mathscr{A})$ *and a separately declared ambiguity* $T_\theta = T_{\mathrm{tot}} + \Phi_g(\theta)$, $\Phi_g(\theta) = \sum_{i=1}^{r} \theta_i B_i[g]$, $\theta \in \mathbb{R}^r$, *with fixed symmetric conserved* $B_i[g]$, *where the account and the experiments do not depend on* θ.

• *Adjoining all certificates of Lemma identifies neither* θ *nor, if* $\Phi_g \neq 0$, *the tensor* $\Phi_g(\theta)$, *whatever the rank of* Φ_g; *a component detecting* $\Phi_g(v) \neq 0$ *has infinite worst-case error.*

• *If the value* $\Phi_g(\theta)$ *is separately supplied, its parameter fibre is* $\theta + \ker \Phi_g$. *Full rank makes this inverse unique; it does not supply the value being inverted.*

• *On an Einstein region of a four-dimensional metric, with* $\theta = (c_0, c_G, c_1, c_2)$ *and the basis of DS-OB-1-fibre v5 thm:F-einstein* (source record; not a premise), $\Phi_g(\theta) = (c_0 - \frac{R}{4} c_G) g$; *the rank is one, with kernel spanned by* $(\frac{R}{4}, 1, 0, 0)$, $(0, 0, 1, 0)$, $(0, 0, 0, 1)$, *and no certificate selects the remaining coefficient.*

• *Information about* κ *without a separately imposed bridge to* θ *changes none of (a)–(c); a tensor normalization that contains no* κ *does not calibrate the slot.*

Proof. (a) The retained readings are constant in θ; along v with $\Phi_g(v) \neq 0$ a detecting pairing grows linearly in the coefficient of v; Lemma (c). (b) Linearity: $\Phi_g(\theta') = \Phi_g(\theta)$ iff $\theta' - \theta \in \ker \Phi_g$. (c) On an Einstein region $\mathrm{Ric} = \frac{R}{4} g$ with R constant, so $G = -\frac{R}{4} g$ and the two quadratic conserved tensors vanish (as in the cited proof, re-checked in DC-82); the row $(1, -\frac{R}{4}, 0, 0)$ has rank one and the stated kernel. (d) κ does not occur in Φ_g, and no relation is assumed.

Status: proved (a linear-data separation). It parallels DS-OB-1-fibre v5 cor:F-ranknotsel and thm:F-generic (source record; not a premise); no generic-rank theorem is re-proved. Check DC-82. par *Non-claim.* Whether a physical renormalization prescription admits the whole family is a separate question. No vacuum representative or coefficient value is selected; absolute vacuum energy stays open. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Documentary separations (version 7)

Each paragraph is a documentary separation; source record; not a premise.

Documentary record (En04-A and En04-B). En04-B v4 thm:curvature-sampling gives responding finite

samples of the fixed, nonconstant scalar curvature of settled Host B, with g_B unchanged, and En04-A v3 thm:magic-fibre gives fibres of magic labels on which a leakage indicator takes both values. Its samples come from declared neck modes and an arc carrier on a fixed metric. A finite-channel dark-capacity certificate of this sheet supplies no such sampling interface; by its name or error value it is not a geometric sampling of the Host-B curvature, it moves no metric, and it is not a magic label. The fibre pattern of thm:magic-fibre parallels Lemma . Host A and Host B stay separate. Source record; not a premise.

Documentary record (I-D and Hawk05-B). I-D v5 prop:v5-u6 tabulates tested source prescriptions for the interface moments Γ_1, Γ_2, and U6-sel stays open there; no dark-capacity certificate supplies a prescription, so U6-sel is not moved. Hawk05-B thm:page is the Haar-average entropy history $\overline{S}_k$ of a declared finite release model inside that sheet. The only time-indexed quantity here is the two-valued step of Theorem (f); a recovery floor is not that entropy history, and the dark-capacity floors are not a Page curve. Neither record concerns a McVittie comparison; this sheet has no McVittie host, and its prescribed hosts select no McVittie comparison (Appendix , PC7-10). Source record; not a premise.

Documentary record (OpenAI Research Catalogue, Families 003 and 265). In the catalogue record [OpenAIcat], rem:oai-003 is the quasi-Riemann hypothesis (zero-free half-plane Re $s > 7/8$ for Dirichlet L -functions) and rem:oai-265 is a two-dimensional gapped area law; the catalogue itself notes that a gapped area law is not a Page curve. No family is identified with a dark-capacity realization, and neither is a premise. Source record; not a premise.

Conditional cosmological predictions from causal access: horizon bounds, communication extinction, and falsifiable dynamical closures (version 7)

This section implements the plan t334u. Its results are on three levels, kept apart in Table . Level 1 uses only prescribed flat FLRW hosts and the core of this sheet. Level 2 adds declared promises on the future expansion rate or on register capacity. Level 3 adds an optional, supplied dynamical closure (Assumption). That closure is a set of supplied external hypotheses, not premises; it is outside the ledger, and no result outside Theorems , and Proposition uses it. Cosmological event horizons and their thermodynamic interpretation have their own literature [GH1977] (source record; not a premise); R_e below is the event-horizon radius, not in general the Hubble radius c/H. Each statement is proved here; the cited physics papers are source records, and no proof is transferred from them. No data are fitted, and every number below is illustrative.

Throughout, a *flat FLRW future* is a prescribed host $g = -c^2\mathrm{d}t^2 + a(t)^2\mathrm{d}\mathbf{x}^2$ on $(t_-, t_{\mathrm{end}}) \times \mathbb{R}^3$, with $t_{\mathrm{end}} \le \infty$, $a > 0$ smooth, $H = \dot{a}/a > 0$ and $a(t) \to \infty$ as $t \operatorname{uparrow} t_{\mathrm{end}}$. The comoving observer is $\gamma = \{\mathbf{x} = 0\}$. Put $\chi_e(t) = c\int_t^{t_{\mathrm{end}}} \mathrm{d}s/a(s) \in (0, \infty]$ and $R_e = a\chi_e$, $a_d = 1 - 1/d^2$, and $\varepsilon = -\dot{H}/H^2$.

The prediction interface

Proposition (Interface). *On every flat FLRW future, a source at comoving radius r on the slice t has least located-circuit error*

$$e_{\min}(t, r) = a_d\, 1[r \ge \chi_e(t)] = a_d\, 1[a(t)r \ge R_e(t)].$$

On the boundary $r = \chi_e(t)$ the source is dark. If $\chi_e \equiv \infty$ (no event horizon), $e_{\min} \equiv 0$.

Proof. The proof of Theorem (a) uses only $|\mathrm{d}\mathbf{x}/\mathrm{d}t| \le c/a$ on future causal curves, so it applies verbatim on (t, t_{end}): if $r \ge \chi_e(t)$, then $|\mathbf{x}(t')| > r - \chi_e(t) \ge 0$ for every $t' < t_{\mathrm{end}}$, and if $r < \chi_e(t)$, the radial null ray arrives at a finite time. Theorem (b) converts membership into the two values.

Status: proved. This extends Theorem (a) from the family to every flat FLRW future, including futures with $t_{\mathrm{end}} < \infty$. Check DC-83. par *Non-claim.* The interface is a property of the prescribed metric and the declared payload. It does not predict a future. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Corollary (Past-only prediction; plan Theorem A). *Let $\mathscr{F}$ be a class of futures that agree on $t \le t_0$, and let*

$r > 0$. If $\mathscr{F}$ contains a, b with $\chi_e^a(t_0) \le r < \chi_e^b(t_0)$, then every past-only predictor of $e_{\min}(t_0, r)$ has worst-case error at least $\frac{1}{2}a_d$; the constant $\frac{1}{2}a_d$ attains this, so $\frac{1}{2}a_d$ is the exact minimax error, and every predictor with values in $\{0, a_d\}$ has worst-case error a_d. Instances: the classes $\mathscr{F}$ and $\mathscr{F}_{\text{downarrow}}$ (Corollary (c)) and the open band of Theorem (d).

Proof. Proposition gives the values a_d and 0; Lemma (b) gives the bound, and the midpoint has error $\frac{1}{2}a_d$ at both. A two-valued predictor is wrong by a_d at one of the two futures.

Status: proved. Verdict on plan Theorem A: held, and it is a corollary of Theorem and Lemma ; the stronger exact-minimax form is kept, as in Theorem (e). Check DC-83. par *Non-claim.* No predictor of the future is offered; the corollary is a lower bound. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Horizon bounds from a declared expansion-rate band

Theorem (Horizon bounds; plan Theorem B). *Fix a slice t of a flat FLRW future and put $x = \ln(a(t')/a(t))$ for $t' \ge t$.*

• $R_e(t) = \frac{c}{H(t)}\int_0^\infty \exp(-x + \int_0^x \varepsilon)\mathrm{d}x$, *where ε is regarded as a function of x.*

• *If $\varepsilon_- \le \varepsilon \le \varepsilon_+ < 1$ on $[t, t_{\text{end}})$, then*

$$\frac{c}{H(t)(1-\varepsilon_-)} \le R_e(t) \le \frac{c}{H(t)(1-\varepsilon_+)}.$$

Equality on the left (right) holds iff $\varepsilon \equiv \varepsilon_-$ ($\varepsilon \equiv \varepsilon_+$) on $[t, t_{\text{end}})$. If $\varepsilon_- \ge 0$, the future is complete ($t_{\text{end}} = \infty$), so completeness is a consequence of the band, not an extra hypothesis. If only $\varepsilon \ge \varepsilon_-$ is known, the upper bound is lost: $\sup R_e = \infty$.

• *(Sharpness.) Let $\mathscr{F}_B$ be the class of futures, defined by continuous $\varepsilon(x)$ with values in $[\varepsilon_-, \varepsilon_+]$ and given $H(t)$ and $\varepsilon(0) = \varepsilon_0$. Then $\{R_e(t) : \mathscr{F}_B\}$ is an interval with endpoints $L = c/(H(t)(1-\varepsilon_-))$ and $U = c/(H(t)(1-\varepsilon_+))$; L belongs to it iff $\varepsilon_0 = \varepsilon_-$, and U belongs to it iff $\varepsilon_0 = \varepsilon_+$.*

• *(Certified regions.) For a source at proper distance $D = a(t)r$: if $D < L$ it is visible for every member of $\mathscr{F}_B$; if $D \ge U$ it is dark for every member; if $L < D < U$, both statuses occur, and the exact minimax error of a past-only predictor is $\frac{1}{2}a_d$.*

Proof. (a) Substitute $\mathrm{d}t' = \mathrm{d}x/H$, $a(t)/a(t') = e^{-x}$ and $H(x) = H(t)\exp(-\int_0^x \varepsilon)$, which follows from $\mathrm{d}\ln H/\mathrm{d}x = -\varepsilon$, into $R_e = c\,a(t)\int_t^{t_{\text{end}}} \mathrm{d}t'/a(t')$; $x \to \infty$ at t_{end} by the standing assumption $a \to \infty$. (b) $\varepsilon_- x \le \int_0^x \varepsilon \le \varepsilon_+ x$, and $\int_0^\infty e^{-(1-\varepsilon_\pm)x}\mathrm{d}x = 1/(1-\varepsilon_\pm)$. Equality forces $\int_0^x(\varepsilon - \varepsilon_\pm) = 0$ for all x, so $\varepsilon \equiv \varepsilon_\pm$ by continuity. If $\varepsilon \ge 0$, then $\mathrm{d}t'/\mathrm{d}x = e^{\int_0^x \varepsilon}/H(t) \ge 1/H(t)$, so $t' \to \infty$ as $x \to \infty$. Without an upper bound, $\varepsilon \equiv \varepsilon_1$ with $\varepsilon_1$uparrow 1 gives $R_e = c/(H(1-\varepsilon_1)) \to \infty$. (c) Every continuous $\varepsilon(x)$ in the band defines a member: $H(x)$ as above, $t'(x) = t + \int_0^x \mathrm{d}y/H(y)$ is increasing, and $a = a(t)e^x$. The functional of (a) is continuous along convex combinations of profiles (dominated convergence, with dominating function $e^{-(1-\varepsilon_+)x}$), so its range on the convex class is an interval. The profiles $\varepsilon_n(x) = \varepsilon_0 + (\varepsilon_\pm - \varepsilon_0)\min(1, nx)$ belong to the class and give $R_e \to U$ or L; the endpoints are attained only by constant profiles, which belong to the class iff ε_0 equals the bound. (d) Proposition : dark iff $D \ge R_e$. If $L < D < U$, choose members with $R_e \in (L, D)$ and $R_e \in (D, U)$; then apply Corollary .

Status: proved, conditional on the declared band, which is a Level-2 promise. With $\varepsilon_- = 0$, $\varepsilon_+ = 0.2$ (illustrative only), $L = c/H$ and $U = 1.25\,c/H$: sources with $D < c/H$ are certified visible, sources with $D \ge 1.25\,c/H$ are certified dark, and the band in between is undecided. With $\varepsilon_- = 0$ and no upper bound, the range is Theorem (c): $[c/H, \infty)$ and no horizon. Checks DC-84. par *Non-claim.* The band is a declaration, not a measurement and not a

consequence of the core. No equation of state is supplied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Communication extinction

Theorem (Communication extinction; plan Theorem C). *Let a flat FLRW future have* $\chi_e(t_0) < \infty$.

• $\dot{\chi}_e = -c/a$ *and* $\dot{R}_e = HR_e - c$ *(Proposition), and* $\chi_e(t) \to 0$ *as t*uparrow t_{end}.

• *For a comoving source at r with* $0 < r < \chi_e(t_0)$ *there is exactly one* $t_* \in (t_0, t_{\mathrm{end}})$ *with* $\chi_e(t_*) = r$*; then* $e_{\min}(t, r) = a_d 1[t \geq t_*]$ *for* $t \geq t_0$*. Every comoving source becomes dark, and none returns.*

• *(de Sitter.) If H is constant,* $\chi_e = c/(aH)$, $R_e = c/H$*, and a source at proper distance* $D_0 = a(t_0)r < c/H$ *has* $t_* - t_0 = H^{-1}\ln(c/(HD_0))$.

• *(Power law.) If* $a = (t/t_0)^p$ *with* $p > 1$ *on* $[t_0, \infty)$*, then* $\chi_e = c\, t_0^p t^{1-p}/(p-1)$, $R_e = ct/(p-1)$*, and* $t_* = [c\, t_0^p/((p-1)r)]^{1/(p-1)}$ *for* $r < \chi_e(t_0)$.

• *(Accessible population.) For sources of constant comoving number density n, the number on the slice t that are not dark is* $N_{\mathrm{access}}(t) = \frac{4\pi}{3} n \chi_e(t)^3$*, and* $-\mathrm{d}\ln N_{\mathrm{access}}/\mathrm{d}t = 3c/R_e$.

Proof. (a) Proposition ; the tail of a convergent integral tends to 0. (b) χ_e is continuous and strictly decreasing from $\chi_e(t_0) > r$ to 0; apply Proposition . (c) $\chi_e = c\int_t^\infty e^{-H(s-t_0)}\mathrm{d}s/a(t_0) = c/(aH)$; solve $c/(a(t_*)H) = r$. (d) Integrate s^{-p}; solve $\chi_e(t_*) = r$. (e) Count the ball $|\mathbf{x}| < \chi_e$; then $-3\dot{\chi}_e/\chi_e = 3c/(a\chi_e)$.

Status: proved. Checks DC-85. In the power-law case R_e grows while χ_e shrinks, so a growing horizon does not prevent extinction. par *Non-claim.* The step concerns events of the source at times $t \geq t_*$; light emitted before t_* still arrives. No galaxy count is predicted, and no sources are counted. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Code capacity inside a horizon

Theorem (Capacity promise and critical radius; plan Theorem D). *Suppose a declared register of n sites of dimension q, associated with the horizon patch at time t, satisfies the declared promise* $n\ln q \leq C_\kappa R_e(t)^2$ *with* $C_\kappa = \eta\pi c^3/(G\kappa)$*, for supplied* $\eta, \kappa > 0$.

• *A code of dimension* $K \geq 2$ *that corrects the erasure of every set of r sites has* $\ln K \leq (n-2r)\ln q \leq C_\kappa R_e^2 - 2r\ln q$*, hence* $R_e \geq R_{\mathrm{crit}} = \sqrt{(\ln K + 2r\ln q)/C_\kappa}$*. For codes correcting all errors of weight* t_w, $R_{\mathrm{crit}} = \sqrt{(\ln K + 4t_w \ln q)/C_\kappa}$.

• *The factor* 2 *is sharp:* $[[4,2,2]]$ *(*$r = 1$*) and* $[[5,1,3]]$ *(*$r = 2$*) attain* $\ln K = (n-2r)\ln q$*. The bound with* $n - r$ *in place of* $n - 2r$ *is valid but weaker.*

• *At* $R_e = c/H$ *the promise is the promise* P_η *of Assumption :* $C_\kappa(c/H)^2 = \eta S^{(\kappa)}/k_B$*. For* $R_e \neq c/H$ *it is a new promise.*

• $R_{\mathrm{crit}} \propto \sqrt{\kappa}$*. With* κ *unsupplied,* R_{crit} *is not a function of admitted data, and every rule has infinite worst-case error (Theorem , Lemma (c)).*

Proof. (a) An r-erasure-correcting code has distance at least $r + 1$, so Theorem (b) gives $\ln K \leq (n-2r)\ln q$ (Theorem (a)); $\mathscr{E}_{t_w}$-correction gives distance $2t_w + 1$. Combine with the promise. (b) Theorem (d) with $n = 4$ gives $K = 4 = 2^{4-2}$; the five-qubit code has $K = 2 = 2^{5-4}$ and distance 3. (c) Substitute . (d) Differentiate in κ; the range over $\kappa > 0$ is $(0, \infty)$.

Status: proved, conditional on the supplied promise. Verdict on plan Theorem D: held with $n - 2r$; the pre-check ``likely wrong $(n - r)$'' is rejected, because $n - 2r$ is the quantum Singleton bound and is attained. Check DC-86. par *Non-claim.* No holographic bound on cosmological information is derived; the promise is declared, and κ is not given a value. No quantum coefficient is admitted. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

An optional dynamical closure (Level 3)

Assumption (Supplied closure package X_b; outside the ledger). For a supplied constant $b > 0$, on a flat FLRW future: (X1) a component X has energy density $u_X = 3b^2c^4/(8\pi GR_e^2)$; (X2) matter ($u_m$, pressure 0) and X (pressure $w_X u_X$, which defines w_X) are separately conserved, $\dot{u}_m + 3Hu_m = 0$ and $\dot{u}_X + 3H(1 + w_X)u_X = 0$; (X3) the flat Friedmann equation $H^2 = 8\pi G(u_m + u_X)/(3c^2)$ holds. Put $\Omega_X = 8\pi G u_X/(3c^2H^2)$ and $\Omega_m = 1 - \Omega_X$.

These are supplied external hypotheses. They are not premises of this sheet, they are not consequences of dark capacity, and they add nothing to the ledger. No result outside Theorems , and Proposition (b),(c) uses them. (X1) with R_e the future event horizon is the holographic dark energy model of Li [Li2004], whose equations (9) and (18) are the closed forms in Theorems and ; these forms also appear in Cui and Zhang [CuiZhang2014] (source records; not premises). It is a different closure from Big05-A v3 hyp:Big05-closure (source record; not a premise), and neither is adopted. (X3) is a supplied equation, not derived Einstein dynamics.

Theorem (Equation of state and a closure diagnostic; plan Theorem E). *Assume (X1) and the X-part of (X2); (X3) is not needed. Then*

- $c/(HR_e) = \sqrt{\Omega_X}/b$;
- $w_X = -\frac{1}{3} - \frac{2\sqrt{\Omega_X}}{3b}$;
- *the diagnostic* $\mathscr{B} = -2\sqrt{\Omega_X}/(1 + 3w_X)$ *is identically* b;
- *if reconstructed* (w_X, Ω_X) *give a non-constant* $\mathscr{B}$ *on an interval of epochs, the package* X_b *fails there for every* b.

Proof. (a) From (X1), $\Omega_X = b^2c^2/(H^2R_e^2)$. (b) $\dot{u}_X/u_X = -2\dot{R}_e/R_e = -2H + 2c/R_e = -2H + 2H\sqrt{\Omega_X}/b$ by Proposition and (a); equate with $-3H(1 + w_X)$. (c) $1 + 3w_X = -2\sqrt{\Omega_X}/b$. (d) is the contrapositive of (c).

Status: proved, conditional on the package. Verdict: held, sharpened (the Friedmann equation is not used). Check DC-87. par *Non-claim.* No value of b is supplied and nothing is fitted. A constant $\mathscr{B}$ would not establish the package. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Theorem (Closed evolution, asymptotics and the boundary condition; plan Theorem F). *Assume (X1)–(X3), with* $0 < \Omega_X < 1$ *at* $a = a_0 = 1$, *and write* $' = \mathrm{d}/\mathrm{d}\ln a$.

- $\Omega_X' = \Omega_X(1 - \Omega_X)(1 + 2\sqrt{\Omega_X}/b)$, *and* $\varepsilon = \frac{3}{2} - \frac{1}{2}\Omega_X - \Omega_X^{3/2}/b$.
- $H^2/H_0^2 = \Omega_{m0}a^{-3}/(1 - \Omega_X)$, *so* $H(a)$ *is determined by* (H_0, Ω_{m0}, b); Ω_X *increases strictly to* 1 *as* $\ln a \to \infty$, *with* $1 - \Omega_X \sim C\,a^{-(1 + 2/b)}$; *and* $HR_e \to bc$, $\varepsilon \to 1 - 1/b$.
- *(*$b = 1$*.)* $H \to H_\infty > 0$*: the future is complete and asymptotically de Sitter, with* $R_e \to c/H_\infty$.
- *(*$b > 1$*.)* $\varepsilon \geq 1 - 1/b > 0$*: the future is complete,* $H \propto a^{-(1 - 1/b)}(1 + o(1))$, *so asymptotically* $a \propto t^{b/(b-1)}$ *and* $R_e \simeq (b - 1)ct$.
- *(*$0 < b < 1$*.) The future is incomplete:* $t_{\mathrm{end}} = t_{\mathrm{rip}} < \infty$, $a \to \infty$ *and* $H \to \infty$ *as* tuparrow t_{rip}, *with* $t_{\mathrm{rip}} - t_0 = \int_0^\infty \mathrm{d}x/H(x)$. *In the X-dominated limit* $a \propto (t_{\mathrm{rip}} - t)^{-b/(1-b)}$ *and* $R_e \simeq c(1 - b)(t_{\mathrm{rip}} - t)$. *This happens for every* $b \in (0, 1)$ *and every initial* $\Omega_X \in (0, 1)$.
- *(Boundary condition.) For a solution of (a),(b),* $R := bc/(H\sqrt{\Omega_X})$ *satisfies* $\dot{R} = HR - c$ *and* $R/a \to 0$ *at* t_{end}, *so* $R = R_e$. *The other solutions* $R_e + Ca$, $C \neq 0$, *of Proposition are excluded.*

Proof. (a) By (X2) and (X3), $\varepsilon = \frac{3}{2}(1 + w_X\Omega_X)$; insert Theorem (b). From $\Omega_X = b^2c^2/(H^2R_e^2)$, $\Omega_X'/\Omega_X = 2\varepsilon - 2(1 - c/(HR_e)) = 2\varepsilon - 2 + 2\sqrt{\Omega_X}/b$, which factors as stated. (b) $\Omega_m = 1 - \Omega_X$ and $u_m \propto a^{-3}$. The right

side of (a) is positive on $(0,1)$ and vanishes at 1, so $\Omega_X$uparrow 1; linearizing at 1 gives the rate $1+2/b$. Then $\varepsilon \to 1-1/b$ and $HR_e = bc/\sqrt{\Omega_X} \to bc$. (c) With $\delta = 1-\Omega_X$, $\varepsilon = 2\delta + O(\delta^2)$ is integrable in $\ln a$, so $\ln H$ converges. $\varepsilon \geq 0$ gives completeness as in Theorem (b). (d) ε is decreasing in Ω_X on $[0,1]$, so $\varepsilon \geq 1-1/b$; $\varepsilon - (1-1/b) = O(\delta)$ is integrable, which gives the asymptotics; with $H \simeq p/t$, $p = b/(b-1)$, $R_e \simeq bc/H$. (e) $\varepsilon \to 1-1/b < 0$, so $H \geq C a^{-k}$ eventually, with $k > 0$, and $\int dx/H < \infty$. For constant $\varepsilon = -(1-b)/b$, $H = b/((1-b)(t_{\rm rip} - t))$. (f) $\dot{R}/R = -\dot{H}/H - \frac{1}{2}\dot{\Omega}_X/\Omega_X$; with (a) this gives $H - H\sqrt{\Omega_X}/b = H - c/R$. In every case $aH \to \infty$ ($\varepsilon < 1$ eventually), so $R/a \to 0$, and Proposition gives $R = R_e$.

Status: proved, conditional on the package; the asymptotic forms in (d) and (e) hold in the X-dominated limit, not exactly. Verdict: held; the place where the future becomes incomplete is stated in (e), and (f) shows the boundary condition is satisfied automatically. Checks DC-87 and DC-88. par *Non-claim.* The rip in (e) is a property of the supplied package, not a prediction of this sheet. No dynamics is derived; (X3) is supplied. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Measurements and falsifiers
Proposition (Observables).

• *(Closure-free.) For comoving emitter and observer on a flat FLRW host, the redshift drift is* $\dot{z} = (1+z)H_0 - H(z)$.

• *(Under* X_b*.)* $H(z)^2/H_0^2 = \Omega_{m0}(1+z)^3/(1-\Omega_X(z))$*, where* Ω_X *solves Theorem (a) with* $\Omega_X(0) = 1-\Omega_{m0}$*, and so* $\dot{z} = H_0[(1+z) - \sqrt{\Omega_{m0}(1+z)^3/(1-\Omega_X(z))}]$.

• *(Falsifiers.) The package is excluded on a redshift interval if the diagnostic* $\mathscr{B}$ *of Theorem is not constant there, or if no* $b > 0$ *reproduces the measured* $H(z)$ *or* $\dot{z}(z)$ *for the given* (H_0, Ω_{m0}).

Proof. (a) Successive wavefronts satisfy $\int_{t_e}^{t_0} dt/a = \text{const}$, so $dt_e/dt_0 = a(t_e)/a(t_0)$; differentiate $1+z = a(t_0)/a(t_e)$. (b) Theorem (b) with $a = 1/(1+z)$. (c) Theorem (d) and (b).

Status: proved. Part (a) is the closure-free identity of Big05-A v3 thm:Big05-drift(a) (source record; not a premise), re-proved here. Redshift drift as a measurable effect, including through strong-lensing time delays, is discussed by Helbig [Helbig2023]; a recent confrontation of holographic and Ricci dark energy with data reports tensions among data sets and a CMB-driven phantom regime for holographic dark energy (Naik [Naik2026]). Both are source records; not premises. Nothing is fitted here and no data set is used. Check DC-88. par *Non-claim.* No measurement is reported, no parameter is estimated, and the closure is not endorsed. This sheet does not predict dark matter, galaxy rotation curves, gravitational lensing or the primordial spectrum. Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened. par

Re-check of version-6 statements (version 7)

The version-6 check script (DC-70 to DC-78) was rerun from a backed-up copy, and its output is byte-identical to the delivered checks_Big02A_v6.json (sha256 912f38d58e47d54a566431a45e1da37f29316ef128bcbef7ab30d1d2bae122e4). The version-5, -4, -3 and -2 scripts (DC-1 to DC-69) were rerun from their read-only delivery copies, and their outputs are byte-identical to their delivered JSON files. The statements used in Sections and (Theorems (b),(d), (c), (b), (a)–(d), (c),(e), (a), , Lemma and Assumption) were re-read against the new proofs. No erratum was found. Two clarifications are recorded without changing any statement: Theorem lives on the comparison futures , not on DSS_H (PC7-1); and the range $(0,\infty)$ of Theorem (b) carries no energy condition, while Theorem (c) already gives the null-condition range (PC7-4).

What is deliberately not done (version 7)

• No dark-matter density, particle, clustering mechanism, stress tensor of dark capacity or dictionary $T_{\mathrm{dm}} = \Phi(\mathscr{C})$. Absolute vacuum energy stays open. The H-split stays a declaration. No local dynamical selection of ordinary, dark or vacuum pieces is derived.

• The optional closures of Big05-A are not premises, and the benchmark numbers printed in Big05-A are not measurements. The closure package X_{b} of Assumption is supplied, outside the ledger, and not endorsed.

• This sheet does not predict dark matter, galaxy rotation curves, gravitational lensing or the primordial spectrum. Nothing is fitted, no data set is used, no value of b, η or κ is supplied, and every number is illustrative.

• No Einstein dynamics, Page curve or emergence. The Friedmann equation enters only as the supplied hypothesis (X3).

• No McVittie host, no Host-B curvature sampling, no U6-sel prescription and no catalogue family is used as a premise (Section).

• No site write, push or registration; the wiring payload is wired: false. No discharge of O.1, XV-B, XVI-N, Gate 0 or ST4b. The open list is unchanged. Citations identify source records only; no proof transfer.

Status table

toprule Realization Certificate Status Permanent negative / not closed

midrule endhead I Redundancy Thm : Singleton via KL decoupling; promise form; no-promise unboundedness; encoder irrelevance proved (conditional on declared factorization; (c) on P_{η}) No ceiling without P_{η}; no holographic code; O1 open

II Horizon Thm : replacement, exact floor, placement indicator, same-temperature separation, pad proved Floor not a function of placement-blind data; no bath at T_{GH}; O6 open

III Junk Thm : zero-error embedding, flagged loss, m-cut, scrambled complement, floor chain proved Junk forces no floor; cut floor below horizon floor

IV Left the patch Thm : $\bar{J}$ slices, range of R_0, dark events, labels, Θ, analytic rigidity proved Dark status future-relative; no Page curve; no dynamics; absolute vacuum term open

Tools Lemmas –, , , Thm , Props , proved Sandwich not exact on every channel

I-R (v2) Thm , Props –: weight-t law $n - 4t$, operator sets, heterogeneous approximate Singleton, gauge ledger, CV sites, robust replacement, additive cost proved; (c) uses cited continuity; Prop (b) partial Dimension-dependent approximate term; no physical error model

II-R (v2) Thm , Thms , : approximate influence, general hosts, joint receivers, (L, m) cuts, same-temperature simplex, step proved; (d) no-seed $m \operatorname{nmid} d$ partial Causal relation must be supplied; not a Page curve

III-R (v2) Thm , Thm , Prop : convex bound, installed vs optimized residual error, exact envelope, partial access, harmonic sums, amplitude damping proved; (f) energy value open Entropy alone fixes no optimized error

IV-R (v2) Thm , Thm , Prop : moduli, regularity dichotomy, null-type ranges, affine junctions, flip region, double fibre proved; Gevrey clause conditional on cited closure No matter model or Einstein equation derived

Cross (v2) Thms , , , , Prop , Table proved (conditional where stated); table partial No conversion rates; no algebra constructed; no heat bill

Early host (v3) Def , Thms , , , , proved; Gevrey clause conditional on cited closure Prefix prescribed; labels external; κ unfilled; no Big-Bang derivation

Access/neg. (v3) Thm , Prop , Thm , Props , , Thm proved; ledger conditional Late label fixed only in calibrated countercase; no entropy bridge

Pack. (v3) Thm , Prop , Lem , Thm , Prop , Cor proved; value at $(3, 2, 2)$ open No conversion rates; formula $1 - f$ not universal

Energy (v3) Thm , Cor , Prop , Thms , , Props , proved; residual CV value bracketed Mean energy gives no

dimension; no physical noise

Corrections (v3) Props , proved Thm (a) equality example and Thm (b) direction corrected by addition

(L, m) (v4) Thms , , , Cor , Prop proved; $L \geq 3$ partial $e_{\text{seed}}(3, 2, 2) = (6 - 2\sqrt{2})/9$ exact; no $L \geq 3$ formula

Energy (v4) Lem , Thm (a),(b) proved; (c),(d) bracket Declared energy promise; no bath

Cost (v4) Thm , Prop proved; dimension form partial $(\varepsilon_A + \varepsilon_B)\ln K$ tight for Ξ

Early host (v4) Defs , , Thms , , proved Exponents and rate bounds declared; no dynamics

Re-check (v4) Section no erratum; five sharpenings Not an independent review

(L, m) (v5) Prop , Thms , , , Prop proved; LP and conic value conditional on cited HK theorem $L = 3$ criterion unconditional; surds for $L \geq 3$ numerical only

Observers (v5) Prop proved (value duplicates Prop) Correlated outputs need the joint witness

Energy (v5) Lems , , Thms , , , proved Exact variational value at every energy; $\mathscr{C}_n$, $n \geq 1$, as a certified limit

Coding (v5) Thm , Cor , Thm proved; dimension form open Bell form versus diamond made explicit

Geometry (v5) Prop , Thm , Prop , Thm proved Tail constant 1 (draft: 2); T_w/S_w not κ-free

Monoid (v5) Cor proved (merge) Four categories and an epoch tag

Re-check (v5) Section no erratum; one metric clarification Not an independent review

(L, m) (v6) Lems , , Prop , Thm , Prop , Thms , , , Cor proved, unconditional $L \geq 4$ middle range open (B2-O27)

Witnesses (v6) Cor (a) proved; (b) numerical Untwirled attainers: rank-one dual does not saturate

Early labels (v6) Prop , Thm proved $\mathscr{R}_{I/d}$ thermal only for scalar H_S; Gibbs-replacement error label-dependent otherwise

Re-check (v6) Section no erratum Not an independent review

Split fibre (v7) Lem , Thm , Prop proved No split selected; binary $\frac{1}{2}/\frac{1}{2}/1$, three-way $\frac{2}{3}$, ratio and vacuum term infinite; absolute vacuum term open

Host qualification (v7) Prop proved Package account-dependent; R_0, $\varepsilon_{\min}$ geometric functionals of the full future

Horizon alignment (v7) Prop , Rem , Cor , Prop proved No future, horizon or split selected; finite ledgers leave intervals

Coefficient blindness (v7) Def , Thm , Prop proved No κ; injectivity of Thm kept

Rank (v7) Prop proved Rank is not selection

Predictions (v7) Prop , Cor , Thms , , proved; B, D conditional on declared promises No future, band or κ supplied

Closure (v7) Ass , Thms , , Prop proved, conditional on the supplied package Outside the ledger; no fit; $b < 1$ rip is a property of the package

Re-check (v7) Section no erratum Not an independent review

bottomrule

Documentary cross-edges

Each of the following is a documentary separation, conditional on a source record; source record; not a premise.

Documentary record (I-C family). The classical-cut floor $1 - 1/d$ (I-C v2 thm:D; I-C-CERT v5 lem:floor, thm:dephase), the attainer $T_{d,k}$ (I-C-Ex-CERT v5 lem:attainer) and the dimension floor (I-C-Ex2-CERT v5 prop:v4-dim) are parallel to Lemma and Proposition (c). Source record; not a premise.

Documentary record (de Sitter certificate sheets). The Codazzi vacuity (V-DST-DSS-CERT v6 thm:VDC-dyn) and the warp and late-time separations (Red06-G-DSS-CERT v6 prop:RC-warp, prop:RC-late) parallel Theorem (d). Source record; not a premise.

Documentary record (Hawk07 family). Null invariance (Hawk07-C-Dyn-CERT v5 thm:v5-frw), Casimir injectivity (Hawk07-F-CERT v5 thm:s2-cas, cor:s2-cas-null) and the late-slice expansion separation

(Hawk07-C-CERT v5 prop:v5-expansion) parallel Theorem and Theorem . Source record; not a premise.

Documentary record (DS-OB-1 and PG-OB-1). Gibbons–Hawking label injectivity (DS-OB-1-CERT v5 thm:DS-coef6) parallels Theorem . The vacuum term not fixed by local geometry (thm:v5-vac) stays open. The replacement-recovery ceilings (PG-OB-1-CERT v5 thm:PG-exact6, cor:PG-ceiling6, thm:v5-PG-shared) parallel Theorem (e) and Theorem . Source record; not a premise.

Documentary record (Sheets cited by the t272u draft). XIV-K-ST v8 thm:fibre (fibre criterion), En02-A thm:Dprime (one-shot environment ledger), En02-C thm:C2, thm:C3 (stream recovery), Red06-G def:account, thm:main and Red10-A thm:fibre (closure fibre principle) were retrieved read-only from the site records and found under these labels. They are not used as premises. Source record; not a premise.

Remark (Latency is not a gate rate; t272u draft). A radial round trip from the centre to static radius $R < \ell$ takes at least $(2/H)\operatorname{artanh}(HR/c)$, which parallels Big01-A v5 prop:echo (source record; not a premise). This is a latency bound for a distributed task and not a bound on local gate rates. A rate bound needs a declared schedule: with a minimum layer duration τ_{tick} and at most p gates per layer, at most $p\lfloor T/\tau_{\mathrm{tick}}\rfloor$ gates complete in time T. The tick ceiling of Big01-A v5 thm:tick bounds hop counts $\lceil r/(cm)\rceil$, not ``exact target operations versus raw dimension".

Open items

toprule Item Statement Status after this sheet

midrule endhead B2-O1 A physical (not declared) finite Hilbert space for the patch open; negative certificate (Thm , Thm (d))

B2-O2 A physically identified noise model at T_{GH} open; label decides nothing (Thm (e))

B2-O3 Exact diamond error of amplitude damping and of a general channel by an elementary certificate open; computed value γ only (DC-6)

B2-O4 Subsystem bounds on $k + r$ not claimed; impure codes can exceed them [KS2007]

B2-O5 Amount of information emitted across a future-dependent horizon open; needs a state and channel

B2-O6 ``Cumulative" leakage, packing density and gate rates open; heuristic only

B2-O7 Absolute vacuum term open (DS-OB-1-CERT v5 thm:v5-vac, (source record; not a premise))

Ledger O.1, XVI-N, N-Ins.4, XV-B, H2, I3, J2, T3, U6-sel, Res02-A Q1–Q4 and A1–A2, Gate 0, ST4b unchanged (Section)

B2-O3 (v2) Exact diamond error by an elementary certificate amplitude damping closed by addition (Prop); general channel open

B2-O8 Side channel with $m\mathrm{nmid}\, d$, $m < d$, without a shared seed partial: bracket (Thm (d)); with a seed exact (Thm)

B2-O9 Entropy–error frontier for residual states closed by addition: installed envelope exact (Thm (b′)); optimized error constant

B2-O10 Exact energy-constrained floors for continuous-variable sites partial: admissible-test lower bounds (Thm (d)); equality open

B2-O11 Gevrey futures, $1 < s < \infty$ closed, conditional on cited Gevrey closure (Thm (b))

B2-O12 Type II_1/III lift finite-target lift proved (Thm); algebra for this host open

B2-O13 Energy–time ledger tied to the host open; conditional ledger with declared resources (Thm)

B2-O14 Residual bound controlled by Choi overlap with the code space (not block diagonal) open

B2-O15 Causal criteria for black-hole exteriors and conical products open; Thm (b) needs a supplied causal relation

B2-O16 Conversion rates between dark-capacity categories open; invariance only (Thm)

B2-O17 (v3) Dynamical derivation of $a(t)$ from a Big-Bang initial condition open; not supplied (Def)

B2-O18 (v3) Physical noise channel at the early radiation temperature open; labels fix no channel (Prop)

B2-O19 (v3) Identification of early entropy with late horizon entropy open; negative certificates (Thm , Prop)

B2-O10 (v3) Exact energy-constrained floors partial: cutoff-and-reset closed (Thm); residual bracket (Cor); general equality open

B2-O20 (v3) $F_{\max}(d, L, m)$ when $Lm > d$ and I_d is not a weighted sum of L rank-m projectors, e.g. $(3, 2, 2)$ partial: $e_{\text{seed}}(3, 2, 2) \in (\frac{1}{3}, \frac{6-2\sqrt{2}}{9}]$; seesaw evidence at the upper end

B2-O21 (v3) Criterion for $I_d = \sum_{x \le L} w_x P_x$ with rank-m projectors, beyond $L = 2$, $m \mid d$ and $L \ge d$ open

B2-O16 (v3) Conversion rates between typed certificates open; typed invariance only (Thm (d))

B2-O20 (v4) $F_{\max}(d, L, m)$ without projector decomposition closed for $L = 2$ (Thm); $(3, 2, 2)$ exact; for $L \ge 3$ see B2-O24

B2-O21 (v4) Weighted rank-m decompositions of I_d for $L \ge 3$ open; recast as a weighted tight fusion frame problem (Prop)

B2-O10 (v4) Exact energy-constrained floors partial: replacement exact for $v \le \frac{1}{3}$ (all M) and on $[\frac{1}{3}, v_2]$ ($M = 1$); $\mathscr{C}_n$ optimized for $q \le \frac{1}{3}$ (Thm)

B2-O22 (v4) Whether $\ln K - \ln \dim \mathscr{H}_C$ can grow like $\varepsilon \ln K$ open; only an $O(\varepsilon^2)$ dimension-free gain is constructed (Prop)

B2-O23 (v4) Replacement floor for $v > v_2$ ($M = 1$) and $v > \frac{1}{3}$ ($M \ge 2$) open; bracket (Thm (c),(d))

B2-O24 (v4) Closed form of the POVM maximum for $L \ge 3$ open; numerical evidence only (DC-46)

B2-O10 (v5) Exact energy-constrained floors replacement closed at every energy (Thm); $\mathscr{C}_n$ exact for $q \le q_{g_M(n+1)}$ and a certified limit otherwise (Thms ,)

B2-O21 (v5) Weighted rank-m decompositions for $L \ge 3$ closed for $L = 3$ (Thm); finite LP for every L, conditional on the cited HK theorem (Thm)

B2-O22 (v5) Dimension form of the coding cost open; upper allowance sharpened (Thm); allocation code small only in Bell form (Thm)

B2-O23 (v5) Replacement floor beyond v_2 and $\frac{1}{3}$ closed: exact two-parameter value (Thms ,)

B2-O24 (v5) Closed form of $F_{\max}$ for $L \ge 3$ partial: finite conic value (Thm); surds such as $Q_{5,3,2} = 5 + 2\sqrt{6}$ identified numerically only

B2-O25 (v5) Whether (N1),(N2) suffice for every (d, L, m) open; agreement with the Horn LP in all cases with $d \le 6$, $L \le 5$, and $(7, 3, m)$ (DC-55)

B2-O26 (v5) Elementary value of $\lim_K c_{n,K}(v)$, $n \ge 1$, beyond q_g open; certified bracket only (Thm)

B2-O24 (v6) Closed form of $F_{\max}$ for $L \ge 3$ closed for $L = 3$ (Thm) and, for every L, in the band $(L-1)m \le d \le Lm$ and the co-band $(L-1)d \le Lm$ (Thms ,); open otherwise (B2-O27)

B2-O27 (v6) $F_{\max}$ for $L \ge 4$, $Lm/(L-1) < d < (L-1)m$ open; closed at $(7, 4, 5)$ (Cor); POVM searches for $d \le 7$ reach either m/d or the certificate value (DC-74, evidence); whether meeting certificates and direct sums always match is open

bottomrule

Non-claims

This sheet does not derive a finite Hilbert space, a temperature, an entropy, a particle species, a clustering mechanism, Einstein dynamics, a derived metric, spacetime emergence, a Page curve, or quantized relational time without an external clock. The labels T_{GH}, S_{GH} and N_{GH} stay external labels with an unfilled coefficient slot. The four realizations are certified only as statements about declared finite registers and declared access channels. The background analogy (Section) is motivation and is not claimed. No result here is registered or written outward.

Citations identify source records only; no proof transfer. No compilation is asserted.

Standing ledger

No quantum coefficient is admitted. Host A and Host B stay separate; Host C is not created. O.1 stays NOT_CLOSED. XVI-N is not discharged; N-Ins.4 stays unresolved. XV-B is not opened. H2, I3, J2 and T3 stay unresolved. U6-sel is not moved. No Einstein dynamics, derived metric, spacetime emergence, Page curve, or quantized relational time without an external clock is claimed. The last reading bars stay open, and continuum acquisition stays a negative certificate. No Route-B successor is created. Res02-A Q1–Q4 and A1–A2 stay open. Gate 0 and ST4b stay NOT_SUPPLIED. Citations identify source records only; no proof transfer. No compilation is asserted.

Version 7 addendum. The optional closures of Big05-A (its Hypothesis hyp:Big05-closure and the closure of Section) are not premises, and the benchmark numbers of Big05-A are not measurements. No local dynamical selection of ordinary, dark or vacuum pieces is derived; dark capacity is constant on split fibres (Theorem). The standing ledger above is unchanged.

Authorship. The title page names Ngo Cheung. Authorship requires confirmation. Citation does not by itself assign authorship. No compilation is asserted.

appendix

Plan corrections (t271u)

The plan is kept verbatim and read-only as PLAN_t271u.md (sha256 fbe198aafda60b70dfe51dc2f13a7059c4d5486f69de968d4cd6d30ad09b1052). Each correction gives the plan sentence, the source-record fact or the local result, and the replacement wording. Ledger-unsafe wording is rephrased in PC17–PC21. toprule No. Plan sentence Correction

midrule endhead PC1 Reorganization monoid of thm:D, acting by host-invariant unitaries and classical record operations The monoid is Big01-A v5 thm:D4. It adjoins declared qubits and metadata; it does not apply unitaries. thm:D is the fibre-supremum ceiling.

PC2 Tick ceiling: exact target operations scale with lattice constants, never with raw dimension Big01-A v5 thm:tick gives $\delta \le m$ and hop counts $\lceil r/(cm) \rceil$; it says nothing about operations versus dimension (Remark).

PC3 Capacity $\le \log N - \Omega(d_{\mathrm{code}})$ from the monoid and commutation False as a derivation. The correct statement is Lemma : Knill–Laflamme decoupling plus subadditivity on a declared factorization. The N-form needs P_{η} (Theorem (c)). Without it, K is unbounded (Theorem (d)).

PC4 Redundancy suppressing errors below the shredder floor cannot cancel the horizon floor True in the form of Theorem (e): an encoder before a replacement is irrelevant.

PC5 thm:B concerns information that interacted with the horizon Big01-A v5 thm:B concerns a located source with $u \le 0$ (on or beyond the horizon). It is a statement about placement, not interaction (Theorem (a)).

PC6 Residual correlations are permanent; no local unitary can purify them (thm:B) thm:B says nothing about purification. The replacement output is a product with the reference (Theorem (b)).

PC7 Strengthen exactness to every channel False: amplitude damping (Proposition). Exactness holds on the classes of Proposition . Exactness over every decoder after replacement is already Big01-A v5 thm:Bexact (source record; not a premise).

PC8 The horizon floor is constant on fibres of any map ignoring causal structure False: $\varepsilon_{\min}$ is *not* a function of placement-blind data (two-point error $\frac{1}{2}(1 - 1/d^2)$, Theorem (d)). The floor value is κ-independent, which is the opposite of the coefficient certificate.

PC9 Floor $1 - k/d$ when the complement is traced out (the classical-record floor) Tracing out a complement is undefined. An isometric code has error 0 (Theorem (a)). $1 - k/d$ is the flagged-cut floor (Lemma); the classical-record floor is the case $k = 1$.

PC10 A scrambled complement tightens the bound to the horizon floor False: error $1 - 1/(1 + (d-k)^2) < 1 - 1/d^2$

(Theorem (e)). ``Host-invariant channel'' is undefined.

PC11 Error correction cannot go below the horizon floor set by temperature and code distance False in direction: the cut floor lies *below* the horizon floor, by $(kd - 1)/d^2$. No floor depends on temperature or distance (Theorem (f), Theorem (e)).

PC12 Historical accounting fixed R_0, the instantaneous radius R_0 is fixed by the prescribed future, and it is the event-horizon radius at the junction (Theorem (a),(b)).

PC13 Residual correlations differ after the junction Undefined. It is replaced by the dark-event separation, R_e, Θ_{tt} and $N^{(\kappa)}(K)$ (Theorem (c),(d)).

PC14 Two-point minimax unbounded, ``exactly as in'' prop:Dyncert For $N^{(\kappa)}(K)$ and the proxies: unbounded (Theorem (d)). For $\varepsilon_{\min}$ it is bounded, at least $\frac{1}{2}(1 - 1/d^2)$. prop:Dyncert has the finite minimax $\frac{3}{2}|K_1^2 - K_2^2|$.

PC15 All four components satisfy the shared features Not universal (Section).

PC16 The logical reduced state is maximally mixed independently of the dark component True only for the optimal decoder after a replacement. In general, see the one-time pad (Theorem (f)).

PC17 ``entropy bounded by the named horizon area'' Rephrased: the entropy of a declared register state is at most $\ln d$; it is related to $S^{(\kappa)}$ only under P_η.

PC18 ``a scalar Hamiltonian at the named temperature saturates the floor'' Rephrased: Gibbs replacement attains $1 - 1/d^2$ at every supplied β iff the Hamiltonian is scalar; the named temperature plays no role (Theorem (e)).

PC19 ``additional thermal reservoir'' Rephrased: a declared unused sector; no reservoir is derived.

PC20 ``total von Neumann entropy of the patch'' Rephrased: the entropy of the declared register state, given by the ledger .

PC21 ``bulk-to-boundary map'' Rephrased: the redundancy of a holographic encoding [ADH2015]. It is heuristic and not constructed.

PC22 Couples more strongly to the bath, raising the noise Unproved and refused (B2-O2, B2-O6).

PC23 prop:Lp: the model sentence is norm-relative Consistent with Big01-A v5 prop:Lp (source record; not a premise); it is not used here.

bottomrule

Relation to the user draft (t272u)

The user supplied a draft of this sheet (file attached_t272u.tex, sha256 194e672f7987b94b8d29c8150a77ce52c574249acf21063499390a4467b457ff), which is kept read-only. We checked each claim against our own proofs and checks, and each citation against the site or local records. Verdicts: *adopted* (verified and used); *corrected*; *not adopted*. toprule No. Draft item Verdict

midrule endhead DR1 Structure: background and scope; host and labels; finite-channel certificates; Realizations I–IV Adopted. A mandated analogy background section is added.

DR2 def:dark, access-relative dark capacity Adopted (Definition).

DR3 prop:fibre, prop:coefficient Adopted; derivatives verified (DC-1); Lemma , Theorem .

DR4 ass:capacity, prop:reorganization Adopted (Assumption).

DR5 lem:sandwich ($Y \pm J \geq 0$) Correct, but in general it rests on Watrous weak duality [Watrous2009]; the draft's elementary proof covers only $\mathrm{Tr}_{\mathrm{out}} J = 0$, as the draft says. It is replaced by the elementary Lemma .

DR6 prop:replacement, cor:replacementrecovery Verified; adopted as Proposition (a).

DR7 def:mcut, lem:accesshierarchy, attainer Λ_m Verified analytically and by semidefinite programs (DC-13); adopted (Lemma).

DR8 cor:gap, prop:temperature Verified; adopted (Theorem (e), DC-16).

DR9 thm:redundancy Dimension count adopted. The Singleton proof goes through recovery; the local

KL-decoupling proof is preferred (Lemma). The subsystem logical-factor bound is proved here through gauge fixing, a strengthening of the draft's caution. Bounds on $k + r$ are not claimed.

DR10 thm:horizon, pad eq:pad Adopted (Theorem (a),(f); DC-15).

DR11 thm:junk, eq:junkentropy Verified (DC-14); adopted (Theorem (a)–(d),). Part (a) refutes the plan's literal junk floor.

DR12 thm:futures, cor:areafibre, prop:dynamics Verified with $\tau = 1$ (DC-8, DC-9); adopted (Theorem).

DR13 Latency, analogy section, status table, shared-features negatives Adopted (Remark , Sections ,).

DR14 Plan-audit appendix (15 items) Consistent with Appendix , which is finer (23 items).

DR15 Embedded SymPy checker Extracted read-only and run: it completes with ``All displayed symbolic assertions completed."

DR16 Empty verb || Corrected: the title page names Ngo Cheung, and authorship requires confirmation.

DR17 Standing boundaries (abbreviated ledger) Corrected: the standing ledger is included verbatim (Section). The draft omits N-Ins.4, the reading bars, Route-B, Res02-A, Gate 0 and ST4b.

DR18 Bibliography: Big01-A ``catalogue identifiers" b3d61121, 34f8f68a Not hashes of the source. The Big01-A version 5 source has sha256 e1160613ea8e02b37fb45954fe677d89f5fa7cca90062d42b68973b03a2bff1d, equal to the site record.

DR19 Cited sheets XIV-K-ST v8, En02-A/B/C, Red06-G, Red10-A, I-C v2 Retrieved read-only from the site; every cited label exists (thm:fibre, thm:Dprime, thm:C2, thm:C3, def:account, thm:main, def:cut, thm:D). Version annotations: XIV-K-ST is version 8 and I-C is version 2, as the draft says. The site records of En02-A/B/C, Red06-G and Red10-A carry no version line, so the draft's ``version 1" for them is recorded as not verified.

DR20 External references GH1977, Bousso2000, CLPW2023, ADH2015, Watrous2009, KL1997, Rains1999, KS2007, HSR2003, HO2013 Bibliographic data verified against Crossref and arXiv (titles, volumes and pages).

DR21 Stray non- text after verb || A file defect (ignored by LaTeX); not adopted.

bottomrule

Labelled computer-assisted checks

The script checks_Big02A.py is deterministic (seed 20271023, PYTHONHASHSEED=0, SCS with tolerance 10^{-9} , values rounded to six digits). It writes one JSON object, and two runs are byte-identical. Every check passed. Semidefinite values are numerical evidence and are not used as proofs. toprule Check Content and result

midrule endhead DC-1 SymPy κ-derivatives of T, S and N: nonzero with the stated signs; injective; the floors have derivative 0.

DC-2 $[[5,1,3]]$, $[[4,2,2]]$, $[[6,4,2]]$: KL condition below the distance (violation 0), decoupling error $< 10^{-15}$, Singleton attained.

DC-3 Replacement: t^{*} = primal overlap $= 1/S_{\sigma}$ and SDP $= 1 - 1/S_{\sigma}$ ($d = 2, 3, 4$). Encoder composite: SDP 0.756309 = formula.

DC-4 Pinchings with blocks $[1,1]$, $[1,1,1]$, $[2,2]$, $[2,1]$, $[3,2]$, $[2,2,2]$: SDP $= 1 - 1/m$. Isotropic $(d,k) = (3,2), (4,3), (5,2), (4,2), (5,4)$: SDP $= 1 - k/d = 1 - t^{*}$.

DC-5 Scrambled complement $(d,k) = (3,2), (4,2), (5,2), (5,3)$: SDP $0.5, 0.8, 0.9, 0.8 = 1 - 1/(1 + (d-k)^{2})$, all below $1 - 1/d^{2}$.

DC-6 Amplitude damping $\gamma = 0.1, 0.3, 0.5, 0.75$: $t^{*} = 0$, SDP $= \gamma$, below the elementary upper bound.

DC-7 Gibbs replacement: $\operatorname{Tr} g_{\beta}^{-1} = Z_{\beta} Z_{-\beta}$; floor attained iff scalar.

DC-8 Futures at $H_{1} = 2$, $c = 1$: $R_{0}(K)$ for $K = 0.05, \ldots, 8$ equals 7.68454, 1.797246, 1.059996, 0.68925, 0.5, 0.398836, 0.33861. It is strictly decreasing and satisfies the lower bound, and the dark event is at $|\mathbf{x}| = 0.544043$.

DC-9 $G_{tt} = 3\dot{a}^2/a^2$; $\Theta_{tt} = 3(K^2 - H_1^2)$; $\ln N^{(\kappa)}(K) \to \infty$ as $K \to 0$.

DC-10 Floor chain in exact rationals, $d = 2, \ldots, 8$ (35 cases).

DC-11 $[[n, n-2, 2]]$ for $n = 2, 4, 6, 8$: KL weight-1 violation 0. ($n = 2$ gives $K = 1$.)

DC-12 Horizon-distance ray reaches $\mathbf{x} = 0$ only in the limit.

DC-13 Λ_m for $2 \le d \le 5$, $1 \le m \le d$: Choi identity, SDP $= 1 - m/d$, $t^* = m/d$.

DC-14 Flagged loss $k = 2, 3$, $p \in \{0, 0.25, 0.6, 1\}$: SDP $= p(1 - 1/k^2)$.

DC-15 One-time pad $d = 2, 3$: marginals maximally mixed, recovery error $< 10^{-12}$.

DC-16 Zero-Hamiltonian thermal operations at $d = 3$: errors 0, 2/3, 8/9.

DC-17 Perturbed $[[3, 1, 2]]_3$ codes, $\eta = 0, 0.02, 0.05, 0.1, 0.2$, Petz recovery: errors 0, 0.011556, 0.050667, 0.162428, 0.2251; every $I(R:A) \le 2\varepsilon \ln K + g(\varepsilon)$ and hold.

DC-18 $((2, 3))$ qutrit threshold scheme: single-share Choi minus replacement 3.3×10^{-16}; two-share Petz Choi minus identity 6.7×10^{-16}.

DC-19 Residual channels $(d, k) = (3, 1), (4, 2), (5, 2)$: SDP $= 1 - 1/(1 + \mathrm{Tr}\,\tau^{-1}) = 1 - t^*$; maximally mixed 0.8 ($j = 2$), 0.9 ($j = 3$); sampled installed frontier for $j = 3$.

DC-20 R_0 modulus bounds and sharpness (residual 0); bump L^1 bound; null-type range $R_0 \ge c/H_1$; shell 4-volume closed form = quadrature $= 0.501819$; multi-junction radii 20.248831, 2.249117, 0.749574, 0.372254, 0.261566.

DC-21 Block side channels $[1, 1, 1]$, $[2, 2]$, $[2, 1]$, $[3, 3]$, $[2, 2, 2]$, $[3, 1]$, $[1, 1, 1, 1]$: SDP 0.666667, 0.875, 0.8, 0.944444, 0.916667, 0.9, $0.75 = 1 - 1/\sum k^2$; flagged loss with side records 0.2625, 0.6125; thermal mixtures; approximate influence 0.675, 0.45 (tight).

DC-22 $[[5, 1, 3]]$: KL violation 0 at weight ≤ 2, $K = 2$, $n - 4t = 1$; SymPy $G(k, k) = -2\dot{H}$.

DC-23 (L, m) cuts: 240 random cuts below $\min\{m/d, Lm^2/d^2\}$; seeded attainers (SDP) 0.777778, 0.5, 0.8125, 0.75, 0.777778, $0.75 = 1 - f$.

DC-24 Weyl-syndrome law, $d = 2, 3$, random priors and transcripts: joint decoder SDP = formula (0.444179, 0.223219, 0.661414, 0.563135).

DC-25 Same-temperature simplex $T_{p,q}$, $d = 2, 3$, ten points: SDP $= 1 - p - q/d - (1 - p - q)/d^2$.

DC-26 Harmonic direct sums (four block patterns) and a product certificate: SDP = formula.

DC-27 Residual channels $(k, j) = (1, 2), (1, 3), (2, 2)$ with maximally mixed, random and pure residual states: optimized SDP $= 1 - 1/(1 + j^2)$; installed envelope $j = 3, 4$ against 20000 sampled level-set spectra (sampled maximum below and within 10^{-3}).

DC-28 Amplitude damping: $Y \pm J \ge 0$ and $\mathrm{Tr}_{\mathrm{out}}\, Y \le 2\gamma I$ in exact arithmetic, $\gamma \in \{1/10, 3/10, 1/2, 3/4, 1\}$.

DC-29 Fock counting and Bell mean number ($M \le 6$, $n \le 9$, and symbolic $M = 3$); Gibbs minimum equals s_{osc} (SymPy).

DC-30 Weighted modulus; local-discontinuity sequence ($R_0 = 1$ exactly for $n = 1, 2, 3$); bounded-Hubble radii in $(c/H_1, c/h_*)$, decreasing in τ; affine law residual $< 10^{-20}$; window 4-volume closed form = quadrature.

DC-31 Heterogeneous approximate Singleton and approximate decoupling on perturbed $[[3, 1, 2]]_3$ codes.

L17–L26 The symbolic listing of the t275u draft (its DC-17 to DC-26), extracted read-only and run once: all assertions completed (Appendix). Renamed to avoid a clash with the numbering above.

DC-32 Radiation prefix: $c\int_t^0 \mathrm{d}s/a_{\mathrm{r}} = B(1 - z)$ (SymPy); arrival time and deadline formulas against quadrature of a smooth radiation-to-de Sitter transition (three sources, agreement $< 10^{-7}$).

DC-33 Label dictionary: κ-derivatives, aT and S constant, $TS = 4UV_c/(3a)$, calibration from T or S, (U, κ) rescaling, joint Jacobian $-3/4$, apparent-horizon derivatives (SymPy, exact).

DC-34 Fixed-tail fibres with $H_0 = 2$, $H_1 = \frac{1}{4}$: free class $R_0 = 0.007021$ and 53.32 ($D = 4$); deadline $\tau = \frac{1}{2}$ gives $4.490 < c\tau + D = 4.5$; three nonincreasing profiles (germ deviation $\le 3.5 \times 10^{-10}$) give $R_0 = 3.9587$, 0.5060 and 1.8248, all in $(B, D) = (0.5, 4)$, the last above the deadline endpoint 1.7876; $\dot{H}(0) = -2H_0^2$.

DC-35 Horizon formulas on the prefix (SymPy); sharp modulus: equality for constant profiles; 300 adversarial pairs ($h = 0.5$, $\delta = 0.3$) give at most $0.6556 \le 0.75$, versus the version-2 constant 3.0.

DC-36 Flip four-volume primitive (SymPy); instance $19049\pi/215040 = 0.278293$ exact and by 30-digit quadrature of the defining integral; the two draft normalizations agree; no label symbol occurs.

DC-37 Cross-epoch access SDPs: correlated joint replacement 0.75; envelope $(1-\lambda)\mathscr{R} + \lambda\mathrm{id}$, $d = 2, 3$: distance λa_d, error $(1-\lambda)a_d$; unflagged $T_{p,m}$ ($d = 3$, $m = 2$, $p = 0, 0.4, 1$): installed = optimized = formula.

DC-38 Errors $0, \frac{2}{3}, \frac{8}{9}$ for identity, dephasing, replacement ($d = 3$); zero-Hamiltonian implementations; late-label minimax and countercase; FLRW Einstein tensor with c: $G_{tt} = 3H^2$, G_{ij}, $(G + \Lambda g)(k, k) = -2\dot{H}$; inverse-source continuity identity (SymPy).

DC-39 Spectator invariance of ε^* for replacement, dephasing, amplitude damping (SDP); padded $[[5, 1, 3]]$: KL violation 0 for weight ≤ 2 on six sites; padding cost $+\ln 2$; tensor of isotropic channels: error $1 - t_1 t_2$.

DC-40 Sign-lemma identities and witness traces (SymPy, exact); isotropic SDP $= 1 - t$; attainers $F_e = f$ for seven (d, L, m), including $m \mathrm{nmid}\, d$ with $L \ge d$; $(3, 2, 2)$: $F_e = (3 + 2\sqrt{2})/9$, twirled SDP 0.352397, seesaw best 0.647603 (and 0.5 on $(4, 2, 2)$); replacement witness PSD, $t_\sigma = 0.096774$, invariant under $\otimes \mathrm{id}_2$.

DC-41 Energy-constrained SDP for cutoff-and-reset, $n = 0, 1$, $q = 0.1, 0.5, 1$: values 0.354138, 0.809017, 1, equal to $F(q)$ and below $\sqrt{q} + q/2$; trace norm (SymPy); Fock counts; draft numbers $\frac{1}{3}, \frac{8}{9}, \frac{5}{9}$; residual threshold; finite residual 0.8.

DC-42 Information defect additive under tensor products (random isometries); entropy counterexample (SymPy); erasure interpolation SDP ($d = 2$, five (L, p)); damping mixtures $p\gamma$; gauge balance identity.

DC-43 Residual entropy chain on 21 spectra: installed $\ge$ uniform $\ge$ entropy form; the version-2 direction fails on all 21.

DC-1–31 Rerun of the version-2 script: output byte-identical to the version-2 JSON.

T1-32–T1-38 The t279u_1 draft has no executable listing; its analytic checks (its DC-32 to DC-38, renamed) are covered by DC-34, DC-36, DC-39, DC-40 and DC-41 (map in Appendix).

T2-L32–T2-L40 The listing of the t279u_2 draft (its DC-32 to DC-40), extracted read-only and run once: all assertions completed. Renamed to avoid a clash.

DC-44 Ky Fan branch bound: 240 random instrument trials, 0 violations (largest ratio 0.726147); branch seesaw equals $\|Z\|_{(m)}^2$; the attaining Kraus pair is exact.

DC-45 Two-label law: symbolic derivative and value $3 + 2\sqrt{2}$; table $d = 2, \ldots, 7$; full seesaws $(3, 2, 2)$ 0.647603, $(4, 2, 3)$ 0.728553, $(5, 2, 3)$ 0.586274.

DC-46 $L \ge 3$ POVM search (evidence): $(5, 4, 2)$ and $(6, 4, 4)$ reach m/d; $(5, 3, 2)$ 0.395959, $(5, 3, 3)$ 0.597128, $(4, 3, 3)$ 0.743686, $(7, 3, 3)$ 0.424448.

DC-47 Energy floor: truncated joint SDP matches $F(v)$ and $3v(1-v)$; $v_2 = 0.436528$; polynomial identity, KKT threshold, lower-witness slacks, certificate supremum $= \alpha$; reduced inequality maximum 0.999544.

DC-48 $\mathscr{C}_n$ with optimized decoder: 0.354138, 0.632183, 0.512311, equal to $F(q)$.

DC-49 Flag code ($K = 2, \ldots, 5$): $I(R:A) = 2a \ln K$, $S(C)$, $\varepsilon_A = a(1 - K^{-2})$ to 10^{-9}; allocation code, e.g. $K = 20$,

$c = 18$: $\varepsilon_A = 0.354138 = F(0.1)$, gain 0.051293.

DC-50 General-w labels: Jacobian $-3w/(1+w)$, κ-exponents, constancy of S_w and $a^{3w}T_w$, the U identity (SymPy, exact).

DC-51 Power-law flip volume against quadrature, $\gamma \in \{0, \frac{1}{2}, 1, 2, 3\}$; reduction to version 3 at $\gamma = 2$; instance 0.278293.

DC-52 Rate-bounded ranges: $\overline{R} = 1.639882$ ($\gamma_{\max} = 3$), 1.5 ($\gamma_{\max} = 2$), 3.609763; deadline infima; admissible profiles strictly inside; deceleration partial integrals diverge.

DC-53 Re-check of version-3 constants; errata found: 0; two-label ledger example.

DC-1–43 (v4 rerun) The version-3 script rerun in version 4; output byte-identical to the version-3 JSON.

DC-54 All-L isotropic witnesses (Thm (b)): exact for $(3, \frac{2}{3})$ and $(2, \frac{1}{2})$, both Choi signs; numerical at the conic values of $(5, 3, 2)$ and $(4, 3, 3)$.

DC-55 Horn index sets by exact Schur arithmetic (Weyl for $d = 2$; the 12 Horn inequalities for $d = 3$); weight LP in 48 cases ($d \le 6$, $L \le 5$; $(7, 3, m)$): feasibility equals the rank conditions (N1),(N2) in every case; $L = 3$ classification.

DC-56 Conic value: $L = 2$ law for $(3, 2, 2)$, $(4, 2, 3)$, $(5, 2, 3)$; DC-46 values 0.395959, 0.597128, 0.743686 reproduced; linearization gap $\le 10^{-8}$; POVM search attains each value; surds $5 + 2\sqrt{6}$, $7 + 2\sqrt{6}$, $8 + 4\sqrt{3}$, $13 + 4\sqrt{3}$ (numerical).

DC-57 Correlated joint replacement: 24/25 by SDP, marginals $I_2/2$, product value 15/16 rejected; $1 - 1/\operatorname{Tr}\sigma^{-1}$ on six random states.

DC-58 All-energy formula with reconstructed primal (boundary shells): gap $\le 1.4 \cdot 10^{-8}$ for $M = 1$ (10 energies) and $M = 2$ (8 energies); version-4 SDP values and brackets; fresh truncated SDPs; convexity of A_M.

DC-59 Thresholds $q_1 = \frac{1}{3}$, $q_2 = \frac{4}{7}$, $q_3 = \frac{9}{13}$; threshold equivalence; witness slack over 900 random outputs; failure beyond q_g.

DC-60 Cutoff recovery SDPs $c_{n,K}$ ($K \le 5$) with the allowance $F(\nu/(K+1))$; monotone in K; $n = 0$ gaps 0, 0.047811, 0.108620, 0.039621 at $\nu = 0.2, 0.5, 1, 2$.

DC-61 Allocation code: half-diamond $1 - 1/(1+r^2)$ for six (c, r); an explicit decoder attains it at $(1, 2)$ and $(2, 3)$; Bell-form optimum below $F(r/K)$.

DC-62 Three-junction transfer law; power-law coefficients for $\gamma \in \{0, \frac{1}{2}, 1, 2, 3\}$ to 10^{-9}; midpoint minimax in $\ell^\infty, \ell^2, \ell^1$.

DC-63 Benchmark volume $9293\pi/26880 + 35\pi/9 = 341479\pi/80640$ (SymPy), the version-4 formula, quadrature 13.303422.

DC-64 Label exponents $\pm 3w/(1+w)$; $T_w S_w$ κ-free but U-dependent; $T_w/S_w \propto \kappa^{6w/(1+w)}$ (SymPy).

DC-65 Spectator invariance of the optimized recovery SDP (amplitude damping, 0.242131 both ways); zero appended Hamiltonian.

DC-66 $H_K(\varepsilon) \le 2\varepsilon \ln K + g(\varepsilon)$; isotropic entropies; flag and allocation codes satisfy Thm .

DC-67 Trine$\otimes I_m$, complement blocks, Naimark move $(4, 4, 3) \to (8, 4, 3)$; $L = 3$ ratios $\{1, \frac{3}{2}, 2, 3\}$ for $d \le 12$.

DC-68 Window-plus-tail bound on 60 random pairs (largest ratio 0.350160); tail constant 1 approached.

DC-69 Re-check of version-4 constants; errata found: 0; one metric clarification.

DC-44–53 (v5 rerun) The version-4 script rerun in version 5; output byte-identical to the version-4 JSON.

DC-70 Meeting inequalities (I1)–(I3) on 12 cases: 150 random and 100 diagonal POVMs and three maximizing

searches per case never exceed D, and the largest left side equals D in every case; eight general-position examples verified by exact rational ranks.

DC-71 Band theorem: symbolic identity for $md - d^2 F_{\max}$, $L = 2$ equals Theorem ; constructions attain the value in 10 cases; POVM searches (8 cases) agree with it.

DC-72 Co-band theorem: symbolic difference identity; constructions attain the value in 9 cases; POVM searches (7 cases) agree with it.

DC-73 Three labels: closed form, construction and search agree for all $d \le 7$; continuity at $d = 2m, \frac{3}{2}m$; exact surd identities; version-5 conic values (DC-56, 10 of 11 rows) and version-4 search values (DC-46, 6 of 8 rows) agree with the closed forms; the remaining rows lie in the open middle range and equal m/d.

DC-74 $(7, 4, 5)$: certificate = construction $= 18 + 12\sqrt{2}$; $L = 4, 5$ middle cases with $d \le 7$ (evidence).

DC-75 Witnesses Y_t, Z_t, H_t exact at four closed values, both signs and orders; spectator $s = 2$; untwirled attainers: $t^* = 0$, errors 0.666667, 0.5, 0.333333.

DC-76 Thermal operations for $d = 2, 3$, three temperatures, scalar and non-scalar H_S: commutation, channel identity, Gibbs preservation, SDP errors.

DC-77 Label sweep: $e(\kappa)$ strictly decreasing with range in $(1 - 1/d^2, 1)$; midpoint-rule supremum $\frac{1}{2d^2}$ approached.

DC-78 Citation labels for items 3 and 6 in the read-only corpus; thm:th-cert is an energy-gain injectivity theorem; errata found: 0.

DC-1–69 (v6 rerun) The version-5, -4, -3 and -2 scripts rerun in version 6; outputs byte-identical to their JSON files.

DC-79 Split fibres: SymPy Einstein tensors of DSS_H and of flat FLRW; T_{tot}, simplex and proportional splits conserved; indicators, ratio, three-way minimax $\frac{2}{3}$; flagged-loss and idle-qubit account changes; mutation (a split-coupled certificate) detected.

DC-80 $R_0(K)$ for 9 values of K; NEC sign iff $K \le H_1$; $R_0 < c/H_1$ iff $K > H_1$; power-tail family a_p; finite status ledgers (random, with the boundary case); mutation (claimed bound $R_0 \ge c/H_1$ for all K) fails as expected.

DC-81 Exact adaptive experiments (3 rounds, quantum memory, stopping): transcript laws equal across κ; two-point and packing bounds; relative loss; label identification (positive control); $\mathscr{R}_{\sigma_\kappa}$ and XOR examples; mutation (κ-dependent kernel) detected.

DC-82 Factorization: certificates on accounts with varied (σ, Λ, χ) constant; vacuum-slot shift conserved; Einstein-region rank one and kernel (SymPy, with quadratic tensors vanishing on de Sitter).

DC-83 Null rays on de Sitter, power-law and rip futures: $r < \chi_e$ reached, $r = \chi_e$ not; exact minimax $\frac{1}{2}a_d$.

DC-84 Theorem B: 200 random band profiles within the bounds; constant profiles attain; ramp profiles approach; $1.25\, c/H$; mutation ($\varepsilon_+ \ge 1$) unbounded.

DC-85 Theorem C and the drift identity: SymPy formulas for de Sitter and power law; unique crossing; N_{access} rate.

DC-86 Theorem D: $[[4, 2, 2]]$ and $[[5, 1, 3]]$ erasure correction by the KL condition; $\ln K = (n - 2r)\ln q$; $n - r$ weaker; R_{crit}, $C_\kappa (c/H)^2 = \eta S^{(\kappa)}/k_B$; $\partial_\kappa R_{\mathrm{crit}} > 0$.

DC-87 Closure: SymPy derivation of w_X, $\mathscr{B} \equiv b$, Ω_X' and ε; mutation ($\mathscr{B}$ with b varying) detected.

DC-88 Closure numerics for $b = 0.7, 1, 1.5$: $\Omega_X$uparrow1, $R = bc/(H\sqrt{\Omega_X})$ equals the horizon integral, power-law exponent, finite rip time; $H(z)$ and drift formulas.

DC-89 Labels in the site records (and a fabricated control); Big05-A site record = local version 3 plus five bibliography items and one blank line; the abstract sentence at lines 98–99; Crossref and arXiv metadata; equations in the Li and Cui–Zhang texts; no tracking parameters in the source.

DC-90 Insertion only: every version-6 line retained in order; 208 version-6 labels kept; no duplicate labels; no numerical κ in version-7 blocks; ledger verbatim in the abstract and at the end; required sentences present.

DC-1–78 (v7 rerun) The version-6, -5, -4, -3 and -2 scripts rerun in version 7; outputs byte-identical to their JSON files.

bottomrule

Version 2 checks. DC-17 to DC-31 are produced by checks_Big02A_v2.py, which reproduces DC-1 to DC-16 unchanged and adds two separate generators (seeds 20271024 and 20271025). Two runs are byte-identical, and every check passed. As in version 1, SDP values are numerical evidence and are not used as premises.

Version 3 checks. DC-32 to DC-43 are produced by checks_Big02A_v3.py (seed 20271026, SCS). Two runs are byte-identical, and every check passed. The version-2 script was rerun, and its output is byte-identical to the version-2 JSON. As before, SDP and quadrature values are numerical evidence and are not used as premises.

Version 4 checks. DC-44 to DC-53 are produced by checks_Big02A_v4.py (seed 20281023, CLARABEL). Two runs are byte-identical, and every check passed. The version-3 script (DC-32 to DC-43) and the version-2 script (DC-1 to DC-31) were rerun, and their outputs are byte-identical to the delivered JSON files. SDP, seesaw and quadrature values are numerical evidence and are not used as premises.

Version 5 checks. DC-54 to DC-69 are produced by checks_Big02A_v5.py (seed 20281024, CLARABEL, and SCS for the two-mode SDPs). Two runs are byte-identical, and every check passed. The version-4 script (DC-44 to DC-53), the version-3 script (DC-32 to DC-43) and the version-2 script (DC-1 to DC-31) were rerun, and their outputs are byte-identical to the delivered JSON files. SDP, LP, conic, seesaw and quadrature values are numerical evidence and are not used as premises. The Horn–Klyachko theorem used by DC-55 and DC-56 is cited, not checked.

Version 6 checks. DC-70 to DC-78 are produced by checks_Big02A_v6.py (seed 20281025, CLARABEL). Two runs are byte-identical, and every check passed. The version-5 script (DC-54 to DC-69), the version-4 script (DC-44 to DC-53), the version-3 script (DC-32 to DC-43) and the version-2 script (DC-1 to DC-31) were rerun, and their outputs are byte-identical to the delivered JSON files. POVM searches, SDP values and random tests are numerical evidence and are not used as premises.

Version 7 checks. DC-79 to DC-90 are produced by checks_Big02A_v7.py (seed 20281026, SymPy and NumPy/SciPy quadrature). Two runs are byte-identical, and every check passed, including the negative controls and mutations listed in the table. The version-6 script (DC-70 to DC-78) and the version-5, -4, -3 and -2 scripts (DC-1 to DC-69) were rerun, and their outputs are byte-identical to the delivered JSON files. Quadrature and ODE values are numerical evidence and are not used as premises.

Provenance and sha256 of inputs

The read-only input copies are listed below with their sha256 values (paths relative to /workspace/Big02/inputs). The frozen trees XVI-P, XVI-Q and XVI-R hash to ec965c15ae25cd3bb9cc2b3a134e966b19a1803c78d1fe5bf7554da2a8b3aaa6, 0df9a0d10409d31ac0c2d0a47197e3f6e03c7ae258ec5fb775137f66cb0493c4 and 57187bc6825f5c6ae88633b7279c6c0be57f7c4918fdbc525da4aaf958331edb before and after this work. Only read-only GET requests to https://enter.grok.me/api/manuscript/ were made. toprule File sha256

midrule endhead Big01-A-v5.pdf e18147ba7f1c8614ccd1795fe50f4b91bf588788b0f24701a3af061f947e028d

Big01-A-v5.tex e1160613ea8e02b37fb45954fe677d89f5fa7cca90062d42b68973b03a2bff1d

DS-OB-1-CERT-v5.tex a0805f8eb081a250f78e8074b9de5986ccd23106749df9e0fe5fac2f71e157f9

En08-ST-DSS-CERT-v6.tex 7d821d99ced3a9d265d5437f5c1466d8f524a09884730d13662278c31b1da086

Hawk07-A-CERT-v5.tex 9db43c8772a84fcf2fcf8d3674b86781ab407b93de05d370740e89f3f00b7add

Hawk07-B-CERT-v5.tex 42086292e1d72716506f90e51522e847546c07c6ca6e4fb1ddfe8ffb78331034

Hawk07-C-CERT-v5.tex a4b14674959f0c02010e9174d1663721d4edf3e6dfe04af39c4b5a9925e10796

Hawk07-C-Dyn-CERT-v5.tex cdee6fc6fbeffdd59d939c5d82fd63ca2c8714989ee14f0f2ff628d5e3be01fa

Hawk07-D-CERT-v5.tex 219f5a91b501c186f3d135482de038db06ab5c732c6b6e5fbb1358eaa18d41aa

Hawk07-E-CERT-v5.tex 632507ba74893e5a90425eb2c3659a263f41d91a62b6904462aafa6689820986

Hawk07-F-CERT-v5.tex 2cfa6282f30e25ac57188298f2e16d3b05a66fff4440204e6f11acce352f5bc3

Hawk07-G-CERT-v5.tex dcba12f3a037812b9d4ad39a80e601d834edc9261ea376a0e0837d0073862e4b

Hawk07-H-CERT-v5.tex 74a70015a03906e76cac8fde88a082da9789ea6e863d987be1dbba67f08bb5dd

I-C-CERT-v5.tex 56c30a1a434ce3cec589929093d7764a97330fbc27748afdf8e6f5bc82b984bf

I-C-Ex-CERT-v5.tex ba3babcc99d2b98ee9118fffc57d3e441dd1e7b4beecd49bab449b4be277b176

I-C-Ex2-CERT-v5.tex b147e794d9836bae2c7622bbdc0e07ccb6006adb3f755b3037ae0904525dd5b1

I-C-Th-CERT-v5.tex caa290f8dad7b6f56809947eb4b9e5a8a2f77fd56642add01de0265224a85c63

MANIFEST-Big01-A-v5.sha256 450e8a622e0c8038ae889ab197ae207351807a2cd4cc6ad80a28fd2a39687c03

MANIFEST-CertPlan-S1v5.sha256 c58c83168eae7ec065b73852e3da0a9692e7562b4f39eeb5d47406fbd8fe2d8b

MANIFEST-CertPlan-S2v4-OB.sha256
95076ed004a8da1ed6a099f6e69b183e15f04852e045df8fae020188ebde95b8

MANIFEST-CertPlan-S2v5-Hawk07.sha256
b79e90d164a95e8b1147f526ca56af50bda0f63650b23977c43fcae6c2bf4cc9

MANIFEST-CertPlan-S2v5-OB.sha256
1f1f58e304f89d81d63c3199358011862c354ecebd9c4e36dfeaf770199585d2

MANIFEST-DSS-CERT-v6.sha256 af40ca7c8ab153d318740ae4ba3b37dd46d83f3089eaad365c0a5da909acd490

PG-OB-1-CERT-v4.tex 3dc921457bd781606e9e673231a8c832be104435950913521fdfb2ed737fc1c1

PG-OB-1-CERT-v5.tex c96ea4631de51c635521ca42fd24337728591676ee9019a24f3ad9fcdcdeb8a2

Red06-G-DSS-CERT-v6.tex c3f04faab2c76fb661054f2c1393bd71f484a1bfdfeeace52e5c612a34fc4599

V-DST-DSS-CERT-v6.tex 710c656210aab6dd708cd9f830ab7bf59df592f0887ab168b1e7fa7be2df95f9

VC-OB-1-CERT-v5.tex d3a93a3afb0e8803d18f230ce824f78c19253f6a4bc841f8ceef763ce14f7cac

rebuild_Big01-A-v5.sh d7b5889261505bf21e993947db6b2babc81cee49f9280491d413f949f0504e32

t272u/attached_t272u.tex 194e672f7987b94b8d29c8150a77ce52c574249acf21063499390a4467b457ff

t272u/background_t272u.md 9daee2379583fdfe349bc5d2d28709f125b2d8fd49f38e729dbf924730583c6a

site/En02-A-site.tex e153d13285543526ae99364413f1b72f37570af79b48af1f2b1e55fc5abc519d

site/En02-B-site.tex 6000bdc72ab6cb433d2b6083bed7d0212678b91b09b7c73251d4a976bcd881b4

site/En02-C-site.tex bd25c58222e82472eb2852972fb7ed7052d0161371dd08036079618dd54c66bb

site/I-C-site.tex 6684c95253a6b61d5690ad3374c6a38779c11d569fcc5d4c45a6bea639c5d187

site/Red06-G-site.tex 6e311fe162757351de51ed9227296ddfb3b97d8e82d98640841053e7725e12d2

site/Red10-A-site.tex 717a574a6333e5bdf799a458fb06e49eb780ba4e12f0d16ac360af62a55846f7

site/XIV-K-ST-site.tex be24a0c8c443368bbece73e4241e2a673ded5bcda16b560c75edf429e5e9db23

bottomrule

New cross-sheet edges (not registered)

Big02-A → Big01-A v5 (thm:A, thm:B, thm:Bsharp, thm:Bexact, prop:Bblocks, thm:D, thm:D4, thm:fibre, thm:Hcert, thm:Bthermal, thm:Bdephase, prop:shredsharp, thm:tick, prop:echo, thm:Efuture, prop:Esmooth, prop:Eanalytic, prop:Dyncert, prop:Lp); → I-C v2 and I-C-CERT / I-C-Ex-CERT / I-C-Ex2-CERT v5; → Hawk07-A/C/C-Dyn/F-CERT v5; → En08-ST-DSS, Red06-G-DSS, V-DST-DSS CERT v6; → DS-OB-1 and PG-OB-1 CERT v5; → XIV-K-ST v8, En02-A, En02-C, Red06-G, Red10-A. Each edge is documentary: source record; not a premise. None is registered.

Plan audit, version 2 (t273u), reconciled with t275u

The plan is kept verbatim and read-only as PLAN_t273u.md (sha256 b44b0a574e1363a12668c6690ad0c8c75d6f3a6cd74b771bfb1e7c25dae58fd8). Each row gives the plan item, the verdict (adopted, corrected or open) and, where the t275u draft bears on it, the reconciliation (Appendix). toprule No. Plan item Verdict and reconciliation

midrule endhead T1 ``All new results use the primal–dual sandwich'' Corrected: Theorem (c) uses a cited continuity bound, and Proposition uses a non-rank-one certificate.

T2 Weight-t errors give $\ln K \le (n-2t)\ln q$ Corrected to $(n-4t)\ln q$; $(n-2t)$ holds for known erasures (Theorem (a)). The draft agrees.

T3 I-C resource laws ``proved for the conical host'' Corrected: they are finite cut statements (source record; not a premise).

T4 ``The same subadditivity step used in Big01-A's reorganization monoid'' Corrected: Big01-A v5 thm:D4 uses no subadditivity. The draft agrees.

T5 Gauge entropy ledger Adopted (Proposition); correlated form from the draft.

T6 Approximate correctability: ``additive $O(\varepsilon\log(1/\varepsilon))$''; ``tighter bounds used in the I-C-Ex2 checks'' Corrected: the term $2\varepsilon\ln K$ is dimension-dependent (); I-C-Ex2 has no continuity bounds. The draft agrees and supplies the heterogeneous form.

T7 CV sites; ``energy-constrained statements in the Hawk07 family'' Adopted in part (Propositions , Theorem). No energy-constrained statement was found in the Hawk07 sheets. A fixed number of modes is not a finite dimension.

T8 No code improves on replacement Adopted (Proposition), with the draft's upper bound.

T9 ``Multiplicative cost $2(\delta-1)\ln q$'' Corrected: additive in $\ln K$; the factor $q^{-2(\delta-1)}$ is multiplicative in K (Proposition). The draft agrees.

T10 Hawk07-B/C exact dual and hard-ceiling pattern Labels verified (Appendix).

T11 Approximate influence floor $1-1/d^2-\varepsilon$ Adopted with normalization stated (half-diamond ε; full-diamond $\eta/2$); tight (Theorem (a)).

T12 ``Same continuity technique as Theorem I'' Corrected: only the triangle inequality and contraction are needed.

T13 Larger metric classes, ``FLRW with a supplied stress tensor'' Adopted conditional on a supplied causal relation; a stress tensor would be dynamics and is not used. Causal criteria for black holes and conical products: open (B2-O15).

T14 Warp family ``in V-DST-DSS-CERT v6'' Corrected: the warp family is Red06-G-DSS-CERT v6 prop:RC-warp and Hawk07-C-Dyn-CERT v6 prop:v5-warp; V-DST says the slice warp is not an operation on its family.

T15 Multiple observers Adopted and strengthened (joint replacement; threshold counterexample for marginals).

T16 Side channel ``interpolates to the flagged $m$-cut floor''; ``already verified for the m-cut'' Corrected: it interpolates from the horizon floor to the shredder floor; DC-13 concerns Λ_m only. With a shared seed the interpolation is exact (Theorem); without it, $m\mathrm{nmid}\, d$ is partial.

T17 Least error not a function of placement-blind data Adopted (version 1 Theorem).

T18 ``Partial dephasing already in DC-16'' Corrected: DC-16 has none. The same-temperature simplex (Theorem (e), from the draft) gives every error.

T19 En08/Red06 ``lattice witnesses and geodesic incompleteness'' Corrected attribution: lattice reorganizations are En08 (thm:EC-reorg); incompleteness is Red06-G-DSS (prop:RC-geod, cor:RC-geodS).

T20 OpenAI notes ``that refuse a derived temperature or a Page curve'' Corrected: the catalogue record refuses a Page-curve reading (rem:physics); no temperature refusal was found.

T21 General residual channel with ``controlled overlap'' Reformulated as the flagged convex bound and harmonic

direct sums; the overlap-controlled version is open (B2-O14).

T22 Entropy–error trade-off from ``majorization" Corrected: entropy direction reversed (maximal entropy gives least installed error), the installed envelope is exact (from the draft), and optimized recovery does not depend on entropy.

T23 Partial adaptive junk access interpolates to zero Corrected: zero needs quantum access; classical records stop at $p(1 - 1/k)$. The draft's syndrome law is a different, exact model.

T24 CV analogue with mean-photon-number floors Partial: admissible-test floors (Theorem (d)); equality open (B2-O10).

T25 Floor chain promoted from DC-10 Adopted (Theorem (d)).

T26 Monoid cannot reduce residual error Adopted as processing monotonicity and access degradation (Theorems (e),); arbitrary host-preserving declarations can reduce it.

T27 V-DST ``junk forces no additional curvature term" Heuristic, not claimed.

T28 ``Intermediate classes fill the gap" Corrected: dichotomy quasi-analytic (rigid) versus not (full range); Gevrey $s > 1$ proved conditional on cited closure.

T29 Modulus of continuity of R_0 Adopted (Theorem (a), Theorem); local regularity alone gives none (Proposition).

T30 A null-energy future ``shrinks" the range Made precise: $[c/H_1, \infty)$, still unbounded; separation persists. Bounded-Hubble and supplied-equation variants from the draft.

T31 ``Each junction radius independent of all earlier data" Corrected: not a function of earlier data, but affinely tied to later radii (Theorem (d)). The draft states the affine law.

T32 Dark-event measure; ``ray calculation already in DC-8" Adopted (Theorem (e)); the ray calculation is DC-12, not DC-8.

T33 Coefficient certificate for $N^{(\kappa)}(K)$ Adopted (Theorem (f)).

T34 Separation from a Page-curve reading Adopted (Theorem (f), Proposition).

T35 Red06-G-DSS ``$C^{1,1}$-inextendibility" Corrected: listed there only as a parent record (thm:R-C11, Theorem 27.4).

T36 Resource theory with costs ``already present in Big01-A" Partial: category table and access degradation proved; conversion rates open (B2-O16).

T37 Type II_1/III lift Finite-target lift proved (Theorem); algebra open (B2-O12).

T38 Energy ledger ``already checked in Hawk07 and I-C-Th" Corrected: I-C-Th has thm:v3-allU and prop:th-vc only. A conditional ledger with declared resources is proved (Theorem); a host-tied energy–time ledger is open.

T39 Robustness subsections Adopted (Sections –).

T40 Expanded checks Adopted (DC-17 to DC-31; Appendix).

T41 Emergence-page note Adopted verbatim, not registered (Section).

T42 Single sheet versus companions Adopted: one sheet, version 1 retained.

T43 Big01-A v5 labels Verified.

T44 DS-OB-1 v5 labels Verified.

T45 VC-OB-1 v5 labels Verified.

T46 I-C v5–v6 labels Verified; thm:v6-generic supports Proposition , and prop:v6-mixpinch parallels the mixtures.

T47 ``Hawk07 v5" in the appendix list Corrected: the v6 sheets are cited (Hawk07-B also v5), as instructed.

bottomrule

Relation to the user draft (t275u)

The user supplied a draft addendum, ``Big02-A: Version 2 Addendum" (file attached_t275u.tex, sha256

c2174538df86e876de98092b397dd4c35f79b42061c0df1b983c529f6572d449), which is kept read-only. Each theorem was re-proved or checked here. The draft's symbolic listing was extracted read-only (sha256 896e2f25803d40ab96d35fde0c87fe3482cea394ece8c2b49091502da337ac28) and run once; all assertions completed. Its numbering DC-17 to DC-26 is renamed L17 to L26 here. Sanitization: the code listing is not reproduced; bibliography entries were checked against Crossref or arXiv (one published title corrected); attributions follow this sheet's standard (``authorship requires confirmation''); the XIV-I and XIV-J records were retrieved read-only and their stated version is not asserted. toprule No. Draft item Verdict

midrule endhead D1 Conventions and certificate interfaces Adopted; identical to Section .

D2 thm:operatorKL Adopted; merged into Theorem (a),(b) with the subsystem clause.

D3 General operator set remark Adopted: a containment hypothesis is required (Theorem (b)).

D4 lem:continuity Adopted as the cited bound [Winter2016]; not re-proved.

D5 thm:approxSingleton Adopted: heterogeneous two-error form and approximate decoupling, now Theorem (c); check DC-31.

D6 Promise evaluation with $s_{\mathrm{GH}}^{(\kappa)}$ Adopted in substance only as a conditional substitution into version 1 Theorem (c); not restated. It does not determine κ.

D7 prop:gauge Adopted into Proposition .

D8 cor:encodedreplacement Adopted into Proposition .

D9 thm:network Adopted into Theorem (a)–(c). Corrected: hosts such as black-hole exteriors need a supplied causal relation (B2-O15).

D10 cor:influence Adopted into Theorem (a).

D11 thm:Lm Adopted (Theorem); check DC-23. It closes the seeded case of the side channel.

D12 thm:syndrome Adopted (Theorem); check DC-24.

D13 thm:thermalSimplex Adopted into Theorem (e); check DC-25.

D14 thm:directsum Adopted (Theorem); check DC-26.

D15 thm:convex Adopted into Theorem (a).

D16 thm:residualspectrum Adopted into Theorem (b); check DC-27. It corrects this sheet's earlier framing of the frontier as a recovery law.

D17 thm:entropycurve Adopted as Theorem (b′); check DC-27 and L21. Closes B2-O9.

D18 Entropy insufficiency examples Adopted (Theorem (b)).

D19 prop:amplitude Adopted (Proposition); checks DC-28 and L22.

D20 Choi overlap remark Adopted (status note after Proposition).

D21 thm:energy Adopted (Theorem); (e) uses a cited relative-entropy fact; check DC-29 and L23.

D22 thm:Gevrey Adopted into Theorem (b), conditional on the cited Gevrey closure [Rodino1993].

D23 Local Sobolev remark Adopted; only local classes are claimed.

D24 thm:modulus Adopted (Theorem); check DC-30.

D25 prop:localdiscontinuity Adopted (Proposition), with the proof based on .

D26 thm:NEC Adopted into Theorem (c); no Einstein equation is supplied or derived.

D27 thm:junctions Adopted into Theorem (d); check DC-30 and L24.

D28 thm:darkvolume Adopted into Theorem (e); checks DC-20, DC-30, L25.

D29 Margin robustness paragraph Adopted in substance; the indicator is discontinuous at the horizon boundary.

D30 prop:latecoefficient Adopted into Theorem (f).

D31 thm:resource Adopted (Theorem).

D32 thm:jointledger Adopted, conditional on declared resources (Theorem); ζ is not identified with any quantum

coefficient.

D33 thm:algebra Adopted (Theorem).

D34 prop:noPage Adopted (Proposition).

D35 Plan disposition table Reconciled row by row in Appendix ; no conflict remains.

D36 Documentary edges Adopted after label verification (Appendix). XIV-I and XIV-J: boundary records only.

D37 Symbolic listing Run read-only as L17–L26; passed.

D38 Release boundary and emergence sentence Adopted; the sentence matches the plan text verbatim.

bottomrule

Documentary edges, version 2 (not registered)

Every label below was found in the read-only local copy named (Appendix). Each edge is a parallel or a boundary. Citations identify source records only; no proof transfer.

Documentary record (Big01-A v5). thm:D4 (declared-data monoid, distinct from access degradation, Theorem); prop:Bblocks (Theorem (a),(d)); prop:Dyncert (Theorem (c)); prop:Eanalytic (Theorem (b)); prop:echo and thm:tick (Theorem). Source record; not a premise.

Documentary record (I-C-CERT, I-C-Ex-CERT, I-C-Ex2-CERT, I-C-Th-CERT v6). I-C-CERT v6 lem:cert-sandwich, thm:replacement, lem:floor, thm:dephase, thm:thermalsep, thm:v5-exact, thm:v6-generic, thm:v6-singular (Theorems , , , Proposition); I-C-Ex-CERT v6 lem:schmidt, lem:attainer, thm:kdreorg, thm:cert-every, prop:v6-mixpinch (Theorems (a), (e)); I-C-Ex2-CERT v6 prop:v4-dim, prop:v5-pinch (Theorems ,); I-C-Th-CERT v6 thm:th-cert, prop:th-horizon, prop:th-vc, thm:v3-allU (boundary: no heat bill follows from Theorem). Source record; not a premise.

Documentary record (Hawk07-B-CERT and Hawk07-C-Dyn-CERT v6). Hawk07-B-CERT v6 thm:s2-exact, cor:s2-every, prop:s2-ceiling, cor:s2-ceiling, cor:v2-spectator, prop:v6-ceil (Theorem , Proposition); Hawk07-C-Dyn-CERT v6 prop:s2-vac, prop:v5-warp, thm:v5-frw (Theorem (c)). No wormhole or evaporation channel is transferred. Source record; not a premise.

Documentary record (DSS-CERT v6 sheets). Red06-G-DSS-CERT v6 thm:RC-coef, prop:RC-dyn, prop:RC-warp, prop:RC-bdry (partial there), prop:RC-geod, cor:RC-geodS; the parent thm:R-C11 is listed there only as a record. En08-ST-DSS-CERT v6 thm:EC-coef, thm:EC-reorg, lem:EC-replacement, prop:EC-regfloor, prop:EC-assisted (register recovery with shared randomness parallels the seeded attainers of Theorem). V-DST-DSS-CERT v6 thm:VDC-dyn. These parallel Theorems (b) and ; no lattice schedule, field or operator domain is derived from the horizon modulus. Source record; not a premise.

Documentary record (OB-CERT v5 sheets). DS-OB-1-CERT v5 thm:DS-coef6, thm:v5-vac; VC-OB-1-CERT v5 prop:s2-vac, thm:s2-sep, cor:v4-VC-fibre, thm:v5-vac; PG-OB-1-CERT v5 thm:PG-exact6, cor:PG-ceiling6, thm:v5-PG-shared. They parallel Theorem (c),(f) and Proposition ; the vacuum term stays open. Source record; not a premise.

Documentary record (Catalogue and boundary records). OpenAI rem:physics (Page-curve refusal; Remark). XIV-I and XIV-J [XIVIJ] (manuscript records retrieved read-only): boundary only; none of this sheet's finite channels supplies their joint characteristic map, and J2 stays unresolved. [CLPW2023]: external observer algebra (Theorem). Source record; not a premise.

Provenance and sha256, version 2

Paths are relative to /workspace/Big02v2. The version-1 delivery in /workspace/Big02/deliver was hashed before and after this work and is unchanged; its files are copied read-only to inputs/v1. The version-5 and DSS v6 inputs are those of Appendix , copied read-only to inputs/v5. Site records were retrieved by read-only GET from https://enter.grok.me/api/manuscript/ on 2026-10-23. The frozen trees XVI-P, XVI-Q and XVI-R hash to the values of Appendix before and after. toprule File sha256

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logs/t275u_listing.log 339673ac134dde3e50f39eeebed90d014930d16352c06052f9b3973463d9849f
bottomrule

Plan audit, version 3 (t276u)

The plan is kept verbatim and read-only as PLAN_t276u.md (sha256 in Appendix). Verdicts: *adopted* (proved here as planned), *corrected* (the planned sentence is false or mis-cited, and the correct form is proved or recorded), *duplicate* (already in version 2 and not restated), *open*. The column ``t279u'' gives the drafts' own disposition where they made one; differences are discussed in Appendix . toprule No. Plan item Verdict t279u
midrule endhead U1 S 1 architecture (freeze v2; insert; heuristic marking; DC-32 onward; edges; authorship)

Adopted. Version-2 lines retained verbatim (assembly check); DC numbering single (DC-32 to DC-43). same

U2 2.1 early host on $[t_i, t_f]$ with radiation then late de Sitter phase Adopted as Definition and Theorem . Corrected: a finite window fixes the causal relation inside it, but not $J^-(\gamma)$; the continuation must be supplied. Citations corrected: Codazzi-class vacuity is in V-DST-DSS-CERT (26 occurrences), not Red06-G-DSS-CERT (0); Red06-G-DSS-CERT has $C^{1,1}$ and incompleteness statements; ``geodesic incompleteness'' was not found in V-DST-DSS-CERT v6. same (attribution corrected in its edges)

U3 2.2 labels $(T_{\mathrm{rad}}, S_{\mathrm{rad}})$ with coefficient certificate Adopted with an explicit dictionary (U, α, V_c, κ), Theorem . Clarified: once an equation of state and adiabaticity are declared, $S \propto a^3 T^3$ is constant in t, so it is a function of the geometry up to one constant; the constant depends on U, κ. A calibrated label reveals κ; (T, S) together determine (U, κ). same, without the joint (T, S) clause

U4 2.3 causal access across epochs Adopted, Theorem (a), for the *joint* channel. same

U5 2.4 junction accounting: late R_0 not fixed by early geometry; NEC modulus Corrected. With H_1 fixed and eternal the settled late radius is c/H_1. What varies is the early-junction radius: $(0, \infty)$; under $\dot{H} \le 0$, $(c/H_0, c/H_1)$; with a deadline, a smaller open interval (Theorem). With the terminal rate free, the NEC range is $(c/H_0, \infty)$, open at c/H_0 (the version-2 closed endpoint concerned a de Sitter prefix). The version-2 modulus is under tail hypotheses, not NEC; its sharpness example is corrected (Proposition). same

U6 2.5(i) one early temperature, errors $0, 1 - 1/d, 1 - 1/d^2$ Adopted, Proposition , for a visible source. same

U7 2.5(ii) no early rule reproduces the late GH label Corrected: true for an unfixed terminal rate; false when H_1 is fixed and κ is supplied or calibrated (Theorem (c)). same

U8 2.5(iii) $G + \Lambda g$ conservation selects no matter model Adopted: the identity holds on any prescribed FLRW host and is vacuous there (Proposition). same

U9 2.6 status, three open items, heuristic background Adopted: B2-O17 to B2-O19; Section . same

U10 S 3 I-R: resource-monoid action with additive $O(\varepsilon \log(1/\varepsilon))$ cost Corrected. The additive object is the information defect Ξ (Theorem); its allowance is $(\varepsilon_A + \varepsilon_B)\ln K + \frac{1}{2}(g(\varepsilon_A) + g(\varepsilon_B))$; a dimension-free cost is refuted (part (d)). The $\sqrt{\varepsilon}$ of version 2 is the trace-distance decoupling, a different quantity; both are right. same

U11 CV version under an energy cutoff; cite energy-constrained remarks in I-C-Th-CERT v6 and Hawk07-B-CERT v6 Duplicate of Theorem (a), plus the cutoff-site ledger, Theorem (e). Citation corrected: neither sheet contains an energy-constrained diamond-norm statement (searched ``energy-constrained'', ``Shirokov''); the external references [Shirokov2018,WinterECD] are used. partly

U12 Gauge-entropy ledger as an exact identity Adopted: Proposition ; version-2 Proposition kept. same

U13 II-R interpolating family as an exact attainer Duplicate of Theorem (a); strengthened to the unflagged channel, Theorem (c). new there

U14 Tightness of the influence form Duplicate of Theorem (a); recorded as the envelope Theorem (b). new there

U15 Dual exactness on the joint channel (I-C-Ex2, Hawk07-B) Adopted: Corollary ; citations documentary. same

U16 III-R limited syndrome access as a convex combination between zero error and no access Corrected: with $L < d^2$ classes even perfect delivery is not zero error; exact law Theorem . same

U17 Amplitude damping in the sandwich family Corrected: the rank-one sandwich has $t^* = 0$; damping belongs to the two-sided certificate (Proposition); mixtures in Proposition . same

U18 CV residual under mean photon number Adopted in corrected form: exact cutoff value and residual bracket (Theorem , Corollary); mean energy gives no dimension. same

U19 IV-R explicit modulus under tail hypotheses Duplicate (Theorems (a),); sharp form added. same

U20 Affine tying of junctions Duplicate (Theorem (d)); radiation coefficient z noted. same

U21 NEC range $[c/H_1, \infty)$ does not remove the floor Duplicate for the de Sitter prefix (Theorem (c)); for the

radiation prefix the range is open, see U5. same

U22 Early-universe epoch as a fifth category Corrected: an application of category II, not an independent category (Proposition). same

U23 Type II_1/III lift documentary Duplicate (Theorem , Remark). same

U24 Joint ledger extended by early labels Adopted conditionally, Theorem . same

U25 DC-32 to DC-40 Adopted and renumbered: DC-32 to DC-43, mapped in Appendix ; all executed and passed. specified, not executed

U26 Expanded plan audit; documentary edges; emergence sentence Adopted: this appendix, Appendix , Section (not registered). same

U27 S 5 non-claims Adopted, Section . same

U28 Cited labels and versions thm:DS-coef6 is defined in DS-OB-1-CERT v5. Hawk07 v6 and I-C v6 exist. DSS-CERT v6 exists; v7 is the newest delivered and re-defines the cited DSS labels; v8 was in progress and not read. thm:kdreorg is defined in I-C-Ex-CERT v6 and I-C-Ex2-CERT v6 (mentioned only in I-C-CERT v6). — bottomrule

Plan audit, version 3 (t277u)

The plan is kept verbatim and read-only as PLAN_t277u.md. toprule No. Plan item Verdict t279u

midrule endhead W1 S 3 monoid of thm:D4 acting on the four categories; ``no reorganization reduces a certified floor'' Corrected. thm:D4 adjoins declared qubits and metadata; it acts on channels only through a spectator lift (Definition , Theorem). The sentence holds for the lifted action and for access degradation, but not for general declaration changes, which can lower the floors attached to declared channels and codes (Proposition). The II and IV floors are invariant under every host-preserving declaration change. Cost ``$2t\ln q$'' corrected to $4t\ln q$ for unknown weight-t errors ($2r\ln q$ for erasures). same; category-wise clause new here

W2 S 3 component labels All cited labels are defined in the read records; thm:cert-every, prop:v6-mixpinch and thm:kdreorg are in I-C-Ex-CERT v6 (the last also in I-C-Ex2-CERT v6), not in I-C-CERT v6. —

W3 S 4 least error ``exactly'' $1-\min\{m/d, Lm^2/d^2\}$ Corrected. The seeded optimum is $1-F_{\max}$ (Theorem (b)). It equals the formula iff $Lm \le d$ or I_d is a weighted sum of L rank-m projectors (covering $m \mid d$ and $L \ge d$). It fails for $L=2$, $d<2m<2d$, e.g. $(3,2,2)$ (Proposition). Without a seed and with $m$nmid d, B2-O8 is unchanged. exactness for $Lm \le d$, $m \mid d$; $(3,2,2)$

W4 S 4 dual $\sigma\otimes I - \Xi^{-1}\Omega$ and $(1-t)\Omega$ Corrected in scope. With $\Omega = |\Omega\rangle\langle\Omega|$ (the Hawk07-B notation), the block dual is the correct replacement witness Y_σ; the sign is correct for $C = J(T) - W_d$. It is not a witness for the (L,m) attainers, whose witnesses are Y_t, Z_t, H_t (Theorem (b), Lemma). Multi-observer: joint replacement witness and tensor extension, Corollary . same

W5 S 5 floor $1-m/d_{\text{eff}}$ from a mean-photon promise; $O(\varepsilon\log(1/\varepsilon))$ term Corrected. A mean-energy promise does not truncate dimension; only a hard support does, giving exactly $1-m/D$ (Proposition (a)). Mean energy gives test floors at $E \ge \overline{E}_P$ ((b)), the exact cutoff-and-reset value (Theorem) and the residual bracket (Corollary). The continuity term is $(\varepsilon_A+\varepsilon_B)\ln K + \frac{1}{2}(g+g)$, or $2\sqrt{\varepsilon}$ for decoupling, not $O(\varepsilon\log(1/\varepsilon))$. same

W6 S 6 integral $\int \mathrm{d}t\, a^3 \int \mathrm{d}^3x$; R_0 ``independent of the entire early geometry''; NEC $[c/H_1, \infty)$; DC-8 parameters Corrected: invariant four-volume carries c ($\sqrt{-g} = c\,a^3$); closed form on the radiation prefix (Theorem , Remark). The fibre statement is correct, but the volume depends on early geometry, and under $H \le 0$ the range is $(c/H_0, \cdot)$, tied to the junction rate. DC-8 is an exponential prefix, so a radiation instance is specified instead. same

W7 Packaging, edges, ledger, emergence sentence Adopted; Section ; edges in Appendix . same

W8 Checks (monoid, PSD witness, energy ordering, volume; rerun of the existing series) Adopted: DC-39, DC-40, DC-41, DC-36; DC-1 to DC-31 rerun byte-identical. specified, not executed

bottomrule

Relation to the user drafts (t279u)

The user supplied two drafts, kept read-only: ``Big02-A: New Results for the Version-3 Interface'' (attached_t279u_1.tex) and ``Big02-A: Version 3 New-Results Insert'' (attached_t279u_2.tex); sha256 in Appendix . Each theorem was re-proved here or checked symbolically or numerically. The listing of t279u_2 was extracted read-only and run once; all of its assertions completed. Sanitization: each draft's ledger paraphrase is replaced by the verbatim ledger (Section); the authorship line is the one on the title page; no draft hash is used; no non- text is adopted (each draft's commented URL annotations are dropped); citations are source records only.

Draft t279u_1 (addresses t277u)

toprule No. Draft item Verdict

midrule endhead F1 Conventions (W_d operator, ω_d vector) Adopted (Section).

F2 thm:monoid Adopted and merged into Theorem ; the category-wise clause (e) is added here.

F3 prop:monoid-negative Adopted as Proposition ; its degradation half duplicates Theorem (a).

F4 Remark on typed costs; distance normalization $4t\ln q$ Adopted, Theorem (b),(d); $t_{12} = t_1 t_2$ checked (DC-39).

F5 lem:sign Adopted, Lemma ; exact check DC-40.

F6 thm:hybrid(a),(b) Adopted, Theorem (a),(b); witnesses verified exactly.

F7 thm:hybrid(c): exact iff $Lm \le d$ or $m \mid d$ Adopted and strengthened: exactness holds iff $Lm \le d$ or $I_d = \sum w_x P_x$ (Theorem (d)); this adds $L \ge d$ with $m \nmid d$, e.g. $(3,3,2)$ and $(5,5,2)$ (DC-40).

F8 prop:strict $(3,2,2)$ Adopted; the proof was re-done, adding the step that the decoders in a branch are proportional. Generalized to all $L = 2$, $d < 2m < 2d$ (Proposition). Seesaw evidence that the draft's construction may be optimal; open.

F9 cor:joint Adopted, Corollary , with the plan's notation reconciled.

F10 thm:energy(a) Duplicate of Lemma on $P_{\mathcal{H}}$ and Theorem ; recorded as Proposition (a).

F11 thm:energy(b)–(d) Adopted; generalizes Theorem (d) to any finite-rank P; the infinite-support example is added.

F12 prop:soft Superseded: its cutoff bound $\sqrt{q} + q/2$ duplicates Theorem (c) and is replaced by the exact value of t279u_2 (Theorem); its approximate Singleton duplicates Theorem (c).

F13 Remark: entropy bound, $E = 0$ Duplicate of Theorem (e); the $E = 0$ clause adopted in Proposition (c).

F14 thm:radfibre Adopted into Theorem (d),(e) (free terminal rate); the version with fixed H_1 comes from t279u_2.

F15 thm:radvolume and evaluated instance Adopted; it coincides with t279u_2 after $H_0 = 1/(2\tau)$, and one statement is kept (Theorem). The instance $19049\pi/215040$ was verified exactly and by quadrature (DC-36).

F16 cor:radlabels Duplicate of Theorem (d) and Theorem ; the ``volume depends on early geometry'' clause is adopted.

F17 Its DC-32 to DC-38 (analytic, not executed) Renamed T1-32 to T1-38 and covered by executed checks (Appendix).

F18 Plan disposition table Agrees with W1 and W3–W6, except W3, where this sheet's exactness criterion is stronger.

F19 Documentary edges Adopted with corrections: ``XIV-H'' is cited only as mentioned within the XIV-I/J records; the ``technical XV-B'' sentence is replaced by the ledger; thm:s2-exact is defined in both Hawk07-B-CERT v6 and PG-OB-1-CERT v5, and the two locators are kept distinct.

F20 Ledger, verification boundary Replaced by the verbatim ledger. Its ``no execution'' statement is superseded: checks were executed here.

bottomrule

Draft t279u_2 (addresses t276u)

toprule No. Draft item Verdict

midrule endhead G1 Radiation-type prefix, host Adopted, Definition .

G2 thm:causal, cor:finitewindow Adopted, Theorem ; DC-32.

G3 Radiation-label dictionary; thm:radcoefficient Adopted, Definition and Theorem ; the joint (T, S) determination is added here.

G4 Apparent-horizon remark Adopted, Theorem (e).

G5 thm:fixedtail Adopted, Theorem (a)–(c),(e); verified, DC-34.

G6 Remark: correction of the NEC endpoint Adopted in substance. The draft writes the version-2 interval as $[c/H_0, \infty)$; version 2 wrote $[c/H_1, \infty)$ with H_1 the de Sitter-past rate. Same content, different notation.

G7 prop:null Duplicate of Theorem (c); the Λ clause is merged into Proposition .

G8 thm:radiationhorizon Adopted, Theorem (a),(b).

G9 thm:sharpmodulus and the version-2 correction Adopted, Theorem (c) and Proposition ; DC-35.

G10 thm:radvolume Adopted (merged with F15), Theorem .

G11 Multiple-epoch remark Duplicate of Theorem (d).

G12 thm:crossloss Adopted, Theorem (a).

G13 prop:influencebudget Adopted, Theorem (b) (duplicate in substance of Theorem (a)).

G14 thm:interpolation Adopted, Theorem (c); it strengthens the example in Theorem (a) by removing the flag. DC-37.

G15 prop:threeerrors Adopted, Proposition .

G16 thm:lateidentification Adopted, Theorem .

G17 prop:conservation and remark Adopted, Proposition ; DC-38.

G18 thm:defectmonoid Adopted, Theorem (a)–(c).

G19 prop:dimensioncounter Adopted, Theorem (d).

G20 cor:cutoffsites Adopted, Theorem (e).

G21 prop:gaugebalance Adopted, Proposition .

G22 thm:syndromeerasure Adopted, Theorem ; DC-42.

G23 prop:dampingmixture Adopted, Proposition .

G24 thm:exactcutoff Adopted, Theorem ; energy-constrained SDP, DC-41.

G25 cor:CVresidual Adopted, Corollary .

G26 prop:entropycorrection Adopted, Proposition ; DC-43.

G27 prop:epochtag Adopted, Proposition .

G28 thm:earlyledger and remarks Adopted as conditional, Theorem ; the promise requires a supplied κ.

G29 Algebraic-lift remark Duplicate of Theorem .

G30 Plan disposition (Appendix A of the draft) Agrees with U2–U24, except as noted in the comparison below.

G31 DC-32 to DC-40 listing Renamed T2-L32 to T2-L40; run read-only; all assertions completed.

G32 Documentary edges with corrected attributions Adopted (Codazzi attribution, slice-warp caveat), Appendix .

G33 Status of physical identifications; emergence sentence Adopted: B2-O17 to B2-O19; sentence in Section .

G34 Bibliography The lecture-note record for the radiation template is adopted as [TongCosmology]; the others are already in the bibliography.

bottomrule

Comparison of dispositions

Where the drafts and this sheet disposition the same plan item, they agree on every verdict except the following. (i) W3: the drafts state exactness for $Lm \le d$ and $m \mid d$; this sheet proves the exact criterion, which adds $L \ge d$, and closes $L = 2$. (ii) W1: this sheet adds that II and IV floors are invariant under every host-preserving declaration change. (iii) U3: this sheet adds that $(T_{\mathrm{r}}, S_{\mathrm{r}})$ jointly determine (U, κ), and that under a declared equation of state and adiabaticity S_{r} is constant up to a U, κ-dependent factor. (iv) U25, W8: the drafts record their checks as specified but not executed; here every check was executed and passed. (v) U11: the drafts do not note that I-C-Th-CERT v6 and Hawk07-B-CERT v6 contain no energy-constrained statement. (vi) G6: notation only. Where the two drafts overlap each other (the radiation-prefix flip volume, the free-rate and fixed-rate fibres, the cutoff bound), one statement is kept: the volume of t279u_2 in the normalization H_0, both fibre ranges in Theorem , and the exact cutoff value of t279u_2.

Single check numbering, version 3

toprule This sheet Plan t276u target t279u_1 analytic check t279u_2 listing

midrule endhead DC-32 causal access (2.1, 2.3) — —

DC-33 coefficient derivatives — T2-L33

DC-34 early fibre; null range T1-37 T2-L32

DC-35 modulus — T2-L35

DC-36 dark four-volume T1-36 T2-L36

DC-37 multi-epoch access SDP — T2-L34

DC-38 three negative propositions T1-38 T2-L40

DC-39 (t277u S 3 monoid) T1-32 —

DC-40 (t277u S 4 duals) T1-33, T1-34 —

DC-41 (t277u S 5 energy) T1-35 T2-L39

DC-42 — — T2-L37

DC-43 residual envelope — T2-L38

bottomrule

Documentary edges, version 3 (not registered)

Every row is a documentary relation; source record; not a premise. Citations identify source records only; no proof transfer. Versions are those read in this run. toprule Record Relation

midrule endhead Big01-A v5 thm:D4, prop:shredsharp, thm:tick, prop:echo, prop:Esmooth, prop:Eanalytic, prop:Dyncert Declared-data monoid (lifted in Definition); latency; smooth-future and analytic fibres paralleled by Theorem ; dynamics vacuity paralleled by Proposition . Source record; not a premise.

I-C-CERT v6 lem:cert-sandwich, thm:replacement, thm:v5-exact, thm:v6-generic Dual conventions and replacement exactness, parallel to Lemma . Source record; not a premise.

I-C-Ex-CERT v6 lem:schmidt, lem:attainer, thm:cert-every, prop:v6-mixpinch, thm:kdreorg Access floors and covariant attainers, parallel to Theorem . Source record; not a premise.

I-C-Ex2-CERT v6 prop:v4-dim, prop:v5-pinch, thm:kdreorg Dimension floors, parallel to Proposition . Source record; not a premise.

I-C-Th-CERT v6 thm:th-cert, prop:th-horizon, prop:th-vc, thm:v3-allU Separation of access, reset and thermal bills, parallel to Proposition . No energy-constrained statement is found there. Source record; not a premise.

Hawk07-B-CERT v6 thm:s2-exact, cor:s2-every, prop:s2-ceiling, cor:s2-ceiling, cor:v2-spectator, prop:v6-ceil Replacement block dual (Corollary), spectators (Theorem), hard ceilings. Source record; not a premise.

Hawk07-C-Dyn-CERT v6 prop:s2-vac, prop:v5-warp, thm:v5-frw, prop:v6-frw Prescribed warp and FLRW families, parallel to Definition . Source record; not a premise.

V-DST-DSS-CERT v6/v7 thm:VDC-dyn, prop:VDC-frw (v7) Codazzi-class dynamics vacuity on prescribed backgrounds, parallel to Proposition . Source record; not a premise.

Red06-G-DSS-CERT v6/v7 prop:RC-warp, prop:RC-geod, cor:RC-geodS; parent record thm:R-C11 (mentioned, not defined there) Geometric comparison and $C^{1,1}$ statements; no inextendibility theorem is transferred to the radiation prefix, and no Codazzi vacuity is attributed to this sheet. Source record; not a premise.

En08-ST-DSS-CERT v6/v7 thm:EC-reorg, lem:EC-replacement, prop:EC-assisted Register recovery with shared randomness, parallel to Theorem (b). Source record; not a premise.

PG-OB-1-CERT v5 thm:PG-exact6, cor:PG-ceiling6, thm:v5-PG-shared; its own thm:s2-exact Exact dual pattern and Page-curve non-identification. Source record; not a premise.

DS-OB-1-CERT v5 thm:DS-coef6 Coefficient injectivity, parallel to Theorem ; calibrated labels need the data-map distinction of Theorem . Source record; not a premise.

VC-OB-1-CERT v5 cor:v4-VC-fibre, thm:v5-vac, prop:s2-vac Vacuum-term fibre; a conserved tensor selects no entropy-transfer model. Source record; not a premise.

OpenAI rem:physics Refusal boundary (Page curve, islands, Einstein dynamics) [OpenAIcat]. Source record; not a premise.

XIV-I, XIV-J [XIVIJ] Analytic boundary only; no characteristic map or admission is supplied (XIV-H appears only as a mention inside these records). Source record; not a premise.

CLPW2023 [CLPW2023] External observer-algebra construction; no Type II_1/III algebra is derived here. Source record; not a premise.

bottomrule

Provenance and sha256, version 3

Paths are relative to /workspace/Big02v3. The version-2 delivery in /workspace/Big02v2/deliver and the version-1 tree were hashed before and after this work and are unchanged; the version-2 files are copied read-only to inputs/v2. The companion sheets are copied read-only from the delivered trees (DSS-CERT v7 from /workspace/DSS/deliver). The frozen trees XVI-P, XVI-Q and XVI-R hash the same before and after. toprule File sha256

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bottomrule

Derived agenda and audit, version 4 (t280u)

No plan accompanied request t280u, ``Work further and upgrade and expand for Big02A towards v4''. The agenda was derived from the open, partial and refuted items of version 3, and was written as PLAN_v4_derived.md before any version-4 material was built (sha256 in Appendix). toprule Target Source in version 3 Verdict Where

midrule endhead P1 B2-O20: $e_{\mathrm{seed}}(3,2,2)$ bracket (Prop) proved Thm : $(6-2\sqrt{2})/9$ exact

P2 B2-O10, Thm (f): energy floors partial Thm : exact for $v \le \frac{1}{3}$ (all M) and on $[\frac{1}{3}, v_2]$ ($M = 1$); bracket beyond (B2-O23)

P3 Thm (d): dimension-free cost refuted as a route proved for Ξ and $\ln K - S(C)$; partial for $\ln \dim \mathscr{H}_C$ Thm , Prop (B2-O22)

P4 B2-O20, B2-O21: (L, m) exactness for $d < 2m < 2d$ and general L proved for $L = 2$; partial for $L \ge 3$ Thms , , , Prop (B2-O24)

P5 Section : radiation-only labels and volume; null-condition ranges proved Thms , ,

P6 Re-check of version-3 constants no erratum Section

— B2-O16 to B2-O19 open (unchanged) Section

bottomrule

Check map, version 4. DC-44 → Thm ; DC-45 → Thms , ; DC-46 → Prop ; DC-47 → Lem , Thm (a)–(d); DC-48 → Thm (a) ($\mathscr{C}_n$); DC-49 → Thm , Prop ; DC-50 → Thm ; DC-51 → Thm ; DC-52 → Thm ; DC-53 → Section , Cor . Numerical values are evidence and are not used as premises.

Documentary edges, version 4 (not registered)

Every row is a documentary relation; source record; not a premise. Citations identify source records only; no proof transfer. The labels were verified in the version-3 run, and no new record was read. toprule Record Relation

midrule endhead I-C-Ex-CERT v6 lem:schmidt, lem:attainer Schmidt-rank floors, of which the Ky Fan branch bound (Theorem) is a spectral refinement with general Z. Source record; not a premise.

I-C-Th-CERT v6 thm:th-cert, prop:th-vc Separation of access and thermal bills, parallel to the non-claim of Theorem : an optimal decoder output is not a bath. Source record; not a premise.

Hawk07-C-Dyn-CERT v6 prop:v5-warp, thm:v5-frw Prescribed FLRW families, parallel to the power-law prefixes of Definition . Source record; not a premise.

DS-OB-1-CERT v5 thm:DS-coef6 Coefficient injectivity, parallel to Theorem (a),(c). Source record; not a premise.

Big01-A v5 prop:Esmooth Smooth-future fibres, parallel to the rate-bounded fibres of Theorem . Source record; not a premise.

Ky Fan [KyFan1949]; Casazza et al. [Casazza2011] Ky Fan maximum principle (used as a standard fact in Theorem); fusion-frame construction literature (Proposition). Source record; not a premise.

bottomrule

Provenance and sha256, version 4

Paths are relative to /workspace/Big02v4. The version-3 delivery in /workspace/Big02v3/deliver is copied read-only to inputs/v3, and the version-2 check script and JSON to inputs/v2. The trees /workspace/Big02, /workspace/Big02v2 and /workspace/Big02v3 were hashed before and after this work and are unchanged. The frozen trees XVI-P, XVI-Q and XVI-R hash the same before and after. toprule File sha256

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bottomrule

Plan audit, version 5 (t283u)

The plan t283u is stored read-only as PLAN_t283u.md (sha256 in Appendix). It is binding, and every label and claim it cites was checked against the read-only sheets. Verdicts: *adopted* (implemented as stated), *corrected* (implemented after a correction), *open* (not achieved; the gap is stated), *duplicate* (already in versions 1–4). toprule Item Plan statement Verdict Where / correction

midrule endhead PC-0 Title ``Expansion plan for Big02-A version 4" corrected The plan describes version 4 as done; it is the plan for version 5.

1.1 Exact dual witnesses for every L duplicate Theorem (b) with $t = Q_{d,L,m}/d^2$ (Remark); DC-54.

1.1a Pattern cited from I-C-CERT v6 lem:cert-sandwich, thm:replacement, thm:cert-every corrected thm:cert-every is not in I-C-CERT v6; it is in I-C-Ex-CERT v6. The other labels exist.

1.1b Hawk07-B-CERT v6 thm:s2-exact, cor:s2-every adopted Labels exist; documentary only (Appendix).

1.2 Projector-sum criterion for general L adopted (conditional) $L = 3$ unconditional (Theorem); every L by a finite LP resting on the cited Horn–Klyachko theorem (Theorem); rank conditions (Proposition). Sufficiency of the rank conditions is open (B2-O25).

1.2a ``Fusion frame of the type cited from Casazza et al." corrected [Casazza2011] treats equal-weight tight fusion frames; the weighted case is the multi-summand Horn problem [BLR2015].

1.2b ``When it fails the Ky Fan value is the tight substitute" adopted (conditional) Theorem : finite conic value; numerical surds only (B2-O24).

1.3 Multi-observer witness tensors corrected Valid for an identity factor or independent payloads; false for correlated joint outputs (Proposition).

2.1 Dual certificate beyond the proved energy intervals adopted Theorems , : exact value with matching primal and dual for every v and M.

2.1a Hawk07-B-CERT v6 ``energy-constrained remarks" corrected No such remark exists in Hawk07-B-CERT v6, Hawk07-B-CERT v5 or I-C-Th-CERT v6 (search: none); no edge is drawn.

2.1b Winter 2016 and Shirokov 2018 ``already in the bibliography" adopted [Winter2016], [Shirokov2018] and [WinterECD] are present.

2.2 Cutoff-and-reset optimal for all v (one mode) or exact gap corrected Optimal only for $q \le q_g$ (Theorem); for $n = 0$ the exact gap is $F(\min\{1, v\}) - \mathfrak{r}_M(v)$; for $n \ge 1$ a certified limit (Theorem ; B2-O26).

2.3 Effective dimension under a stronger promise duplicate Proposition (a),(c) (Remark).

2.3a ``Theorem III" floor $1 - m/d_{\text{eff}}$ adopted Lemma on the supported subspace.

3.1 Matching lower bound for the dimension form open Not achieved (B2-O22). The allocation code is small only in Bell form (Theorem).

3.2 Approximate Singleton under diamond distance duplicate, sharpened Theorem (c), sharpened by Theorem and Corollary .

4.1 Multiple junctions; ``each junction radius is independent of all earlier data" duplicate, corrected Theorem (d); radii are tied by the transfer law. New: Proposition .

4.2 Moduli under Gevrey or $C^{k,\alpha}$ tails adopted Theorem (with tail control); without it, Theorem (b).

4.3 Radiation-to-de Sitter four-volume ``on the explicit family of the DC checks" corrected No such two-epoch family exists in the version-3/4 checks; a new family is specified (Proposition).

4.4 ``Ratios of early labels are κ-free" corrected Theorem : $T_w/S_w \propto \kappa^{2p}$.

5.1 Monoid on ``five categories" corrected Four categories and an epoch tag (Proposition); Corollary .

5.2 Monoid preserves the new values adopted Corollary ; DC-65.

6.1 DC-54 onward adopted DC-54 to DC-69, executed (Section).

6.2 Documentary edges incl. ``the OpenAI catalogue" adopted Appendix ; the record is the OpenAI Research Catalogue [OpenAIcat].

6.3 Emergence-page sentence adopted Recorded, not registered (Section).

7 Non-claims adopted Section ; B2-O16 to B2-O19 unchanged.

bottomrule

Check map, version 5. DC-54 → Remark ; DC-55 → Theorems , ; DC-56 → Theorem , Proposition ; DC-57 → Proposition ; DC-58 → Theorems , ; DC-59 → Theorem ; DC-60 → Theorem ; DC-61 → Theorem ; DC-62 → Proposition ; DC-63 → Proposition ; DC-64 → Theorem ; DC-65 → Corollary ; DC-66 → Theorem ; DC-67 →

Proposition , Theorem ; DC-68 → Theorem ; DC-69 → Section . Numerical values are evidence and are not premises.

The user draft t285u: per-item verdicts

The draft attached_t285u.tex is stored read-only (sha256 in Appendix). Every item was re-proved or checked here. Where it overlaps the plan, one statement is kept in its stronger verified form. Its listed ``DC-54 onward'' were specifications, not execution reports; they were implemented and run in checks_Big02A_v5.py. Its standing-ledger paragraph (which mentions N-Ins.4, ST4b and Gate 0) and its author line were not adopted: the version-1 ledger and the frozen title page are kept verbatim. Its draft bibliography entry for version 4 and its source comments were not copied. toprule Draft item Verdict Where / reason

midrule endhead def:spectralpolytope, (HK) input adopted Definition ; HK cited [KnutsonTao2001,Fulton2000,BLR2015], not reproved.

thm:projectorLP adopted (conditional) Theorem ; DC-55 executes the LP ($d \le 7$).

cor:projector-obstructions corrected (a),(b) are Proposition (b),(c); (c),(d) are proved unconditionally by Theorem , whereas the draft's proof of (d) cited a ``rank-span condition'' that does not exclude $(5,3,2)$ by itself.

thm:generalSOCP adopted (conditional) Theorem ; DC-56 reproduces the $L = 2$ law and the DC-46 values.

rem (closed form) adopted Status of Theorem ; surds are numerical (B2-O24).

prop:conicdual adopted Proposition ; a linearization certificate is computed in DC-56.

thm:allLdual duplicate Theorem (b); Remark ; DC-54.

prop:jointobservers duplicate, adopted example Value: Proposition (a); joint channel: Theorem (a); the correlated example is new (Proposition ; DC-57).

lem:diagonal-energy adopted Lemma .

lem:scalarFenchel adopted Lemma .

thm:allenergy adopted Theorem ; the proof checks, with the truncation step made explicit; DC-58 (gap $\le 2 \cdot 10^{-8}$).

thm:energy-primal adopted Theorem ; a boundary shell is generic and is reconstructed in DC-58.

thm:degeneracyinterval adopted Theorem ; DC-59.

thm:cutofflimit adopted Theorem ; DC-60.

prop:finite-energy-SDP adopted Inside Theorem .

prop:hardpromise duplicate Proposition ; Remark .

thm:BellSingleton adopted Theorem ; DC-66.

cor:weightform adopted Corollary .

thm:allocation-diamond adopted Theorem ; DC-61. It clarifies the metric of Proposition (c); not an erratum.

rem (dimension growth) adopted Remark ; B2-O22 open.

thm:multijunction duplicate, adopted in part Transfer law and scalar minimax: Theorem (d). New parts: Proposition ; DC-62.

thm:regularitymodulus corrected Tail constant 2 → 1 (sharp); parts (c),(d) duplicate Theorem (a),(b). Theorem ; DC-68.

prop:newvolume adopted, last sentence open Proposition ; DC-63. The smoothing-convergence sentence needs unspecified uniform bounds and is not adopted.

thm:labelinvariants adopted Theorem ; DC-64.

SCM, thm:monoid duplicate, merged Theorem , Proposition ; Corollary ; DC-65.

Disposition table superseded Appendix and this table.

DC-54+ specification and listing executed DC-54 to DC-69; the listing's identities are included.

Edge ``nuADaM01-A contents were not available'' corrected Version 1 was read from /workspace/nuADaM01/deliver; its labels exist (Appendix).

Standing ledger, author line not adopted Kept verbatim from versions 1–4 (verb |Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; XVI-N is not discharged; XV-B is not opened.|, frozen title page).

bottomrule

Documentary edges, version 5 (not registered)

Every row is a documentary relation; source record; not a premise. Citations identify source records only; no proof transfers. The labels were verified in the read-only copies (version-3 corpus, and inputs/nuADaM01v1). toprule Record Relation

midrule endhead I-C-CERT v6 lem:cert-sandwich, thm:replacement; I-C-Ex-CERT v6 thm:cert-every Replacement and sandwich certificates, parallel to the witnesses of Theorem (b) and Proposition . Source record; not a premise.

I-C-Ex-CERT v6 lem:schmidt, lem:attainer, prop:v6-mixpinch; I-C-Ex2-CERT v6 prop:v5-pinch Schmidt-rank and pinching floors, parallel to the decomposition criteria of Section . Source record; not a premise.

Hawk07-B-CERT v6 thm:s2-exact, cor:s2-every Complete-access replacement pattern for finite channels; it contains no energy-constrained statement. Source record; not a premise.

I-C-Th-CERT v6 thm:th-cert, prop:th-horizon, prop:th-vc Access and thermal bills stay separate; no oscillator statement is imported into Section . Source record; not a premise.

En08-ST-DSS-CERT v6 lem:EC-replacement, prop:EC-assisted Replacement and assisted-recovery comparisons on supplied hosts. Source record; not a premise.

PG-OB-1-CERT v5 thm:PG-exact6 Finite recovery and Page-curve separation; the energy program is not an evaporation channel. Source record; not a premise.

Big01-A v5 thm:D4, prop:Esmooth, prop:Eanalytic, prop:Dyncert Spectator declaration monoid and future-fibre parallels (Corollary , Proposition). Source record; not a premise.

Hawk07-C-Dyn-CERT v6 thm:v5-frw; V-DST-DSS-CERT v6 thm:VDC-dyn Prescribed warp and FLRW families; no matter model follows. Source record; not a premise.

DS-OB-1-CERT v5 thm:DS-coef6; VC-OB-1-CERT v5 thm:VCc-fibre Coefficient injectivity and fibre logic, parallel to Theorem . Source record; not a premise.

nuADaM01-A v1 thm:fibre, thm:volume, prop:entry, thm:dark, prop:cutoff Two-epoch fibre, flip volume, horizon-entry classification and hard-cutoff dimension, parallel to Propositions , and Remark . Proposition is self-contained and verifies nothing in nuADaM01-A. Version 2 of nuADaM01-A is not read or cited. Source record; not a premise.

OpenAI Research Catalogue [OpenAIcat] Boundary record only; no catalogue entry supplies a premise. Source record; not a premise.

bottomrule

Provenance and sha256, version 5

Paths are relative to /workspace/Big02v5. The version-4 delivery in /workspace/Big02v4/deliver is copied read-only to inputs/v4, and nuADaM01-A version 1 (/workspace/nuADaM01/deliver) to inputs/nuADaM01v1. The trees /workspace/Big02, /workspace/Big02v2, /workspace/Big02v3 and /workspace/Big02v4 were hashed before and after this work and are unchanged. The frozen trees XVI-P, XVI-Q and XVI-R hash the same before and after. toprule File sha256

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logs/rerun/v2.json bb3712bcc6cce65fccdf65d57009fda9316289ecd3b4d051fdf4ab257ff4b771

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bottomrule

Plan audit, version 6 (t287u)

The plan t287u is stored read-only as PLAN_t287u.md (sha256 in Appendix). Every label and claim it cites for items 3 and 6 was checked against read-only copies. Verdicts: *adopted* (implemented as stated), *corrected* (implemented after a correction), *open* (not achieved; the gap is stated), *duplicate* (already in versions 1–5). toprule Item Plan statement Verdict Where / correction

midrule endhead PC-1 Numbering of the five items recorded The items are numbered 2 to 6; there is no item 1.

2, 4, 5 En08 symmetric-subspace dual; Red06-G proper-time bound; V-DST dimension count placed in DSS-CERT DSS-sheet items; not treated here.

3.1 $d^2F_{\max} = \max \sum_x \|\sqrt{E}_x\|^2_{(m)}$ duplicate Theorem .

3.2 Block dual from the Ky Fan maximizer, feasible and saturating under both Choi conventions, via the sandwich lemma corrected With a seed, the exact witnesses are those of Theorem (b), checked under both conventions by Lemma (Remark); the new parts are the closed t (Corollaries , (a)). Without a seed, the rank-one witness built on an attainer does not saturate: $\Omega \notin \operatorname{ran} J$, so $t^* = 0$ (Corollary (b), numerical). The genuinely new dual object is the

meeting-subspace certificate (Lemma , Theorem), which makes the values of B2-O24 unconditional.

3.3 Min formula exact iff I_d is a weighted sum of L rank-m projectors duplicate Theorem (c),(d). The criterion concerns $Lm > d$; for $Lm \le d$ the value Lm^2/d^2 always holds.

3.4 ``Otherwise the Ky Fan value is the tight substitute" adopted Duplicate of Theorem as a statement. New: closed forms in the band, the co-band and for $L = 3$ (Theorems , ,). Open in the middle range for $L \ge 4$ (B2-O27).

3.5 Dual tensors across observers with no path duplicate Corollary and Proposition (false for correlated joint outputs).

3.6 ``Fusion-frame characterization from Casazza et al." corrected As in the version-5 audit row 1.2a: [Casazza2011] is construction literature for equal-weight tight fusion frames, not a characterization of weighted projector sums.

3.7 $(3, 2, 2)$ counterexample recovered duplicate Theorem ; also Theorem with $L = 2$, $k = 1$.

3.8 Components: I-C-CERT v6, I-C-Ex-CERT v6, Hawk07-B-CERT v6, En08-ST-DSS-CERT v6 adopted Labels verified (DC-78); thm:cert-every is in I-C-Ex-CERT v6. En08-ST-DSS-CERT v6 prop:EC-assisted is consistent with Lemma at $L = 1$. Documentary only (Appendix).

3.9 Checks: primal = dual for $L \ge 3$; classification; observers adopted DC-71 to DC-75; observers are covered by DC-57 (version 5, rerun).

3.10 Non-claim adopted Each result in Section .

6.1 One early label compatible with errors 0, $1 - 1/d$, $1 - 1/d^2$ duplicate Proposition ; nuADaM01-A v3 thm:negid(a).

6.2 ``All three are thermal operations at every $\beta > 0$ when the Hamiltonian is scalar" corrected True for scalar H_S. For general H_S: identity always; dephasing iff H_S is diagonal; replacement iff the output is γ_β (Proposition).

6.3 Early label not a function of admitted data duplicate Theorems , (a); nuADaM01-A v3 thm:coefrad.

6.4 Channels and errors exhibited in Big01-A v5 thm:Bthermal, thm:Bdephase adopted Labels verified (DC-78); used in Proposition .

6.4a … and in I-C-Th-CERT v6 thm:th-cert corrected thm:th-cert is the injectivity of the energy gain (a thermal bill), not a channel construction; cited only as a documentary edge.

6.5 ``The fibre lemma shows the late error is independent of the early label" corrected True for recovery errors and the undecoded errors of the identity and dephasing. False for Gibbs replacement at the label temperature when H_S is not scalar: minimax $1/(2d^2)$ (Theorem (b)).

6.6 ``Early radiation Hamiltonian" corrected Not defined in any cited sheet; read as a supplied H_S (Section).

6.7 Checks; non-claim adopted DC-76, DC-77; non-claims after Proposition and Theorem .

P-1 Documentary-edge paragraph adopted Appendix .

P-2 Update open items; no gate discharged adopted B2-O24 (v6), B2-O27; no gate discharged.

P-3 Emergence-page sentence recorded Section ; not registered.

bottomrule

Check map, version 6. DC-70 → Lemmas , , Proposition ; DC-71 → Theorem ; DC-72 → Theorem ; DC-73 → Theorem , Corollary ; DC-74 → Corollary , B2-O27; DC-75 → Corollary ; DC-76 → Proposition ; DC-77 → Theorem ; DC-78 → Appendices , and Section . Numerical values are evidence and are not premises.

Documentary edges, version 6 (not registered)

Every row is a documentary relation; source record; not a premise. Citations identify source records only; no proof transfers. The labels were verified in read-only copies: inputs/corpus and inputs/nuADaM01v3, a copy of /workspace/nuADaM01v3/deliver. toprule Record Relation

midrule endhead I-C-CERT v6 lem:cert-sandwich, thm:replacement; I-C-Ex-CERT v6 thm:cert-every,

lem:schmidt, lem:attainer Sandwich and attainer certificates, parallel to Corollary . Source record; not a premise.

Hawk07-B-CERT v6 thm:s2-exact, cor:s2-every Block dual pattern for replacement channels. Source record; not a premise.

En08-ST-DSS-CERT v6 prop:EC-assisted Assisted-register optimum $1 - m^2/d^2$, the case $L = 1$ of Lemma . Source record; not a premise.

Big01-A v5 thm:Bthermal, thm:Bdephase Replacement and dephasing as thermal operations; used in Proposition . Source record; not a premise.

I-C-Th-CERT v6 thm:th-cert Energy-gain injectivity; the thermal bill stays separate from the access errors of Theorem . Source record; not a premise.

nuADaM01-A v3 thm:negid, thm:coefrad, lem:fibre Negative identifications, radiation coefficient certificate and fibre lemma, parallel to Theorem . Source record; not a premise.

bottomrule

Provenance and sha256, version 6

Paths are relative to /workspace/Big02v6. The version-5 delivery in /workspace/Big02v5/deliver is copied read-only to inputs/v5. The version-4 check output is copied to inputs/v4json, nuADaM01-A version 3 (/workspace/nuADaM01v3/deliver) to inputs/nuADaM01v3, and the corpus sheets listed below to inputs/corpus. The trees /workspace/Big02, /workspace/Big02v2, /workspace/Big02v3, /workspace/Big02v4 and /workspace/Big02v5 were hashed before and after this work and are unchanged. The frozen trees XVI-P, XVI-Q and XVI-R hash the same before and after. toprule File sha256

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bottomrule

ifnum c @section>26 A@Alph{ numexpr c @section-26} else @Alph{ c @section} fi appendix.section

Plan audit, version 7 (t333u)

The plan t333u is stored read-only as inputs/PLAN_t333u.md (sha256 in Appendix). Every label it cites was re-read in the site records retrieved read-only on 25 October 2026 (Appendix). Verdicts: *adopted*, *corrected*, *duplicate* (already in versions 1–6), *recorded* (a fact about the inputs). toprule Item Plan statement Verdict Where / correction

midrule endhead PC7-1 Version-6 summary: Realization IV ``on DSS_H'' corrected Theorem lives on the flat FLRW comparison futures with a de Sitter past; Theorems and (b) give the DSS_H and general-host floors.

PC7-2 Big05-A versions available on the site recorded The site record of Big05-A is version 3: the local version-3 source (inputs/local/Big05-A-v3.tex) plus five inserted bibliography items and one blank line (DC-89). It is cited as a site record with both hashes.

PC7-3 Big05-A flagship gives error $\frac{1}{2}$ and infinite ratio error corrected These risks are in Big05-A v3 thm:Big05-fibre(e); the flagship refers to them.

PC7-4 ``Big05-A's range $[c/H_{\mathrm{E}}, \infty]$" vs ``Big02's $(0, \infty)$" corrected The Big05-A range assumes the NEC ($H \le 0$); the Big02 range has no energy condition; Theorem (c) already gives the null-condition range (Proposition).

PC7-5 ``Dark capacity is not a host functional" (Big05-A) corrected The sentence is in the version-1 paragraph of the Big05-A abstract (lines 98–99 of the site record) and is proved as prop:Big05-dark(a) in its section sec:dark. It is true of the package, not of each entry: R_0 and $\varepsilon_{\min}$ are functionals of the full prescribed metric (Proposition). Not-a-host-functional and split blindness are different statements, proved separately.

PC7-6 Big04-A-coeff v5 thm:C-generic: readouts identify κ recorded Identification needs $M \ge M^*(m)$ coefficient-sensitive readouts; this is the regime excluded by Definition .

PC7-7 Big03-A v2 prop:R-floors: floors have κ-derivative zero corrected True for the listed floors; part (b) of the cited proposition is a κ-dependent family under a declared tie.

PC7-8 All four certificates are coefficient-blind corrected The promise form of Theorem (c) carries a supplied κ and is excluded (Lemma).

PC7-9 thm:curvature-sampling in En04-A corrected The label is in En04-B (version 4), not in En04-A; the magic-fibre result is En04-A v3 thm:magic-fibre.

PC7-10 Separation from a McVittie comparison via I-D prop:v5-u6 and Hawk05-B thm:page corrected Neither record mentions McVittie: prop:v5-u6 tabulates tested prescriptions for Γ_1, Γ_2 (U6-sel), and thm:page is a Haar-average entropy history. McVittie comparisons occur only in Big05-A. Recorded as a non-claim.

PC7-11 OpenAI Families 003 and 265 adopted rem:oai-003 (quasi-Riemann, $\mathrm{Re}\, s > 7/8$) and rem:oai-265 (2D gapped area law) exist; neither concerns dark capacity; the catalogue states that a gapped area law is not a Page curve.

PC7-12 Preserve the injectivity of Theorem adopted Theorem (e): one exact label value identifies κ.

PC7-13 Fibre lemma gives error $\ge \frac{1}{2}$ corrected Lemma (b) gives $\frac{1}{2}|G_1 - G_2|$; for indicators the minimax is $\frac{1}{2}$ (real or randomized) and 1 (deterministic Boolean), Theorem (c).

PC7-14 Open list: ``continuum acquisition" open corrected In the ledger, continuum acquisition stays a negative certificate. The open list is unchanged.

PC7-15 Ledger ``carried verbatim" (short list) corrected The full version-6 ledger is carried verbatim, in the abstract and at the end.

1 Split-fibre theorem adopted Lemma , Theorem , Proposition (DC-79, DC-82).

2 Comparison with Big05-A adopted Proposition , Corollary (DC-80).

3 Coefficient-blind certificate adopted Definition , Theorem , Proposition (DC-81).

4 Documentary separations adopted Section .

5 Structure, non-claim, edges adopted Sections –, ; Appendix .

6 Open list unchanged adopted No row added to Section .

bottomrule

The extrapolations t334u: per-item verdicts

The input t334u is stored as inputs/PLAN_t334u.md. Each item was re-proved locally (Section). Links were stripped of tracking parameters. toprule Item Statement Verdict Where / correction

midrule endhead X-1 Interface $e_{\min} = a_d 1[r \ge \chi_e]$ held Proposition , extended to futures with $t_{\mathrm{end}} < \infty$; the boundary $r = \chi_e$ is dark (DC-83).

X-2 Theorem A: past-only error $\ge \frac{1}{2} a_d$ held (duplicate) Corollary of Theorem and Lemma ; the exact-minimax form is kept (DC-83).

X-3 Theorem B: hypotheses ``future complete", $0 \le \varepsilon_- \le \varepsilon \le \varepsilon_+ < 1$ corrected Completeness is a consequence of $\varepsilon_- \ge 0$; only $a \to \infty$ at the future end is needed (Theorem (b)).

X-4 Theorem B: bounds ``attained by constant-ε profiles'' corrected Attained iff $\varepsilon \equiv \varepsilon_{\pm}$, which requires the present ε to equal the bound; otherwise approached, not attained (Theorem (c)); illustrative $1.25\,c/H$ checked (DC-84).

X-5 Theorem C: $\dot{\chi}_e$, $\dot{R}_e$, single crossing, de Sitter and power-law times, N_{access} held Theorem , Proposition (DC-85).

X-6 Theorem D: $R_{\text{crit}} = \sqrt{(\ln K + 2r\ln q)/C_\kappa}$ held Theorem ; $n - 2r$ is the quantum Singleton bound and is attained by $[[4,2,2]]$ and $[[5,1,3]]$ (DC-86).

X-6a Pre-check: ``Theorem D likely wrong $(n - r)$'' rejected $n - r$ is valid but weaker (Theorem (b)).

X-6b C_κ against Assumption corrected Equal only at $R_e = c/H$; for other R_e the promise is new (Theorem (c)). κ stays symbolic.

X-7 Closure $u_X = 3b^2c^4/(8\pi G R_e^2)$ recorded Supplied package outside the ledger (Assumption); Li's holographic dark energy [Li2004] is prior work.

X-8 Theorem E: w_X and $\mathscr{B} \equiv b$ held, sharpened The Friedmann equation is not needed (Theorem); matches Li eq. (9) in the dominated limit and Cui–Zhang eq. (4) (DC-87, DC-89).

X-9 Theorem F: Ω_X' and the three futures held Theorem ; matches Li eq. (18). The $b < 1$ future is incomplete for every $b \in (0,1)$ and every initial $\Omega_X \in (0,1)$; $b \geq 1$ futures are complete; the power laws are asymptotic (DC-87, DC-88).

X-10 Boundary condition $R_e/a \to 0$ held Automatically satisfied by the closed solution (Theorem (f), Proposition).

X-11 $H(z)^2/H_0^2 = \Omega_{m0}(1+z)^3/(1-\Omega_X)$; $\dot{z} = (1+z)H_0 - H(z)$ held Proposition (DC-88).

X-12 Forecasting: fit b, H_0, Ω_{m0} not done No fits by rule; the falsifiers are stated (Proposition (c)).

X-13 MNRAS 519(2) 2769 for the drift formula corrected Helbig, ``Redshift drift and strong gravitational lensing'' [Helbig2023]; its abstract concerns measuring drift through strong-lensing time delays. Cited only for drift as a measurable effect; the formula is proved here.

X-14 MNRAS 547(3) stag365: ``substantial viability problems'' corrected Naik [Naik2026]: the abstract reports that the Ricci model is observationally non-viable and, for holographic dark energy, tensions among data sets and a CMB-driven phantom regime. Cited in those terms.

X-15 PRD 15, 2738; hep-th/0403127; EPJC 74, 2849 held Gibbons–Hawking [GH1977], Li [Li2004], Cui–Zhang [CuiZhang2014]: metadata (Crossref, arXiv) and equations checked (DC-89).

X-16 Three-level table; non-claims adopted Table ; Proposition non-claim.

bottomrule

The user draft t335u: per-item verdicts

The draft ``Big02-A: Version-7 New Findings'' is stored read-only as inputs/findings_t335u.tex (sha256 in Appendix). Every result was re-proved; where it overlaps the plan t333u or another version-7 result, only the stronger form is kept. Draft labels are mapped in Appendix . toprule Item Draft statement Verdict Where / correction

midrule endhead T5-1 Fixed-account split fibre; every split of vacuum form held, generalized Definition : general splits, with the vacuum-form class used where needed.

T5-2 Retained package (CI)–(CIV) of the draft corrected Lemma . The flagged-cut floor of Theorem (d) is $1 - m/k$ (k the logical dimension), not $1 - m/d$.

T5-3 Fixed-account split-fibre non-selection (i)–(iv) held Merged into Theorem (a)–(c),(e); three-way, vacuum-slot, coefficient and adaptive parts added (DC-79, DC-82).

T5-4 Explicit simplex fibres; three-way error $\frac{2}{3}$ held Proposition (a) with a DSS_H realization; $\frac{2}{3}$ in Theorem (d) (DC-79).

T5-5 Remark on admissibility held Status text after Proposition .

T5-6 Account dependence vs geometric dependence held Proposition (a)–(c); flagged-loss values 0, $1 - 1/k^2$ verified in Theorem (c).

T5-7 Plan qualification (quote) adopted Status text after Proposition ; PC7-5.

T5-8 Two horizon ranges (i)–(iv) held Proposition (a),(b),(d),(e). The draft's attribution of the geometric/energy-condition distinction to thm:IVR and prop:v3-conservation was verified ($G(k,k) = -2\dot{H}$ in both).

T5-8a Power-tail family a_p held (duplicate realization) Remark ; Theorem (c) already attains the range (DC-80).

T5-9 Corollary (i)–(iii) held Corollary (a)–(c); (iii) holds for every $r > 0$ in $\mathscr{F}$; (a) relaxed from attained to approached endpoints.

T5-9a Corollary (iv): finite status ledgers do not identify R_0 corrected Every nonempty joint-status fibre has positive length, except in $\mathscr{F}_{\text{downarrow}}$ when a dark event has threshold exactly c/H_1; that ledger forces $R_0 = c/H_1$ (Corollary (d), DC-80).

T5-10 Horizon drift identity held Proposition , extended to $t_{\text{end}} < \infty$; used by Theorems , , (DC-85).

T5-11 Non-selection qualification adopted Non-claim after Corollary .

T5-12 Operational coefficient blindness adopted Definition ; it replaces the weaker plan definition.

T5-13 Adaptive experiments (i)–(vi) held Merged into Theorem (a)–(c),(f); (iv) relaxed to infimum and supremum; relative loss and positive identification added (DC-81).

T5-14 Remark on labels and promises adopted Status text after Theorem .

T5-15 Blindness repair: $\mathscr{R}_{\sigma_\kappa}$ and the XOR example held Proposition (DC-81).

T5-16 Big04-A-coeff orbit qualification held Verified against prop:C-orbit(d),(f) and thm:C-generic.

T5-17 Rank does not become selection; Einstein-region map held Proposition ; (c) matches DS-OB-1-fibre v5 thm:F-einstein(a) exactly and is re-checked symbolically (DC-82).

T5-18 Documentary table corrected Section . The phrase ``explicit curvature compression'' for En04-B was not found in the record; the record's own content (neck modes, arc carrier, $\mathcal{R}_B$ and g_B unchanged) is used.

T5-19 Corrections 1–7 adopted Propositions , (e), , ; Definition ; Theorem (c)–(e).

T5-20 Status additions; ``version-6 checks neither repeated nor altered'' corrected Rows added to Section . The version-6 checks were rerun, byte-identical (Section).

T5-21 Permanent non-claims; ledger verbatim adopted Section ; the draft's ledger equals Section verbatim (DC-90).

T5-22 ``Online retrieval did not provide complete copies of Big05-A, Big04-A-coeff, DS-OB-1-fibre'' superseded Complete site records were retrieved read-only (Appendix), and every draft label was checked (DC-89).

T5-23 Draft bibliography corrected cor:C-nogo, prop:R-refinement and thm:F-rank exist. Hawk05-B is called ``version 1'', but the site record states no version, so none is cited. The draft's title page has an empty author; this sheet names Ngo Cheung (authorship requires confirmation). No draft bibliography item is copied.

bottomrule

Label and citation verification, version 7

Each record was fetched read-only by GET from https://enter.grok.me/api/manuscript/<ID> on 25 October 2026; the sha256 of each response is in Appendix . DC-89 checks every row and a fabricated control label (thm:Big05-nonexistent), which is correctly reported missing. toprule Record Labels Content checked

midrule endhead Big05-A (v3) thm:Big05-flagship, thm:Big05-fibre, thm:Big05-futures, thm:Big05-drift, prop:Big05-predsplit, prop:Big05-dark, def:Big05-split, def:Big05-dominance, hyp:Big05-closure, thm:Big05-bare futures range under $\dot{H} \le 0$; drift (a); closure $\dot{H} = -\frac{n}{2}(H^2 - H_\infty^2)$ (a different closure)

Big04-A-coeff (v5) thm:C-adaptive, prop:C-orbit, thm:C-generic, thm:C-worstcase, cor:C-nogo orbit scope (d), (f); $M^{*}(m)$

Big03-A (v2) thm:R-unsupplied, prop:R-vac-sep, prop:R-floors, prop:R-refinement floors; part (b) of prop:R-floors

DS-OB-1-fibre (v5) thm:F-einstein, thm:F-generic, cor:F-ranknotsel, thm:F-rank Einstein-region rank one and kernel

En04-A (v3), En04-B (v4) thm:magic-fibre; thm:curvature-sampling label locations (PC7-9)

I-D (v5); Hawk05-B prop:v5-u6; thm:page no McVittie (PC7-10)

OpenAI rem:oai-003, rem:oai-265 families exist (PC7-11)

midrule [GH1977] doi:10.1103/PhysRevD.15.2738 Crossref title, volume, pages

[Li2004] doi:10.1016/j.physletb.2004.10.014; arXiv:hep-th/0403127 Crossref; arXiv journal reference; eqs. (9), (18)

[CuiZhang2014] doi:10.1140/epjc/s10052-014-2849-8; arXiv:1402.1829 Crossref; $\Omega_{de}^{'}$ and eq. (4)

[Helbig2023] doi:10.1093/mnras/stac3623 Crossref: MNRAS 519(2) 2769–2772; abstract

[Naik2026] doi:10.1093/mnras/stag365 Crossref: MNRAS 547(3) stag365, 2026; abstract

bottomrule

Crosswalk from version 6 and from the draft labels

Every version-6 line is retained verbatim and in order (DC-90). The 208 version-6 labels are unchanged. New material: toprule Source Version-7 label Relation to version 6

midrule endhead t333u 1; t335u def:v7-split def:v7-accounts, lem:v7-factor uses Definitions ,

t333u 1; t335u thm:v7-not-host = prop:Big02-v7-flagship thm:v7-splitblind applies Lemma

t335u prop:v7-split-witness prop:v7-witness uses Theorem (d)

t335u prop:v7-host-qualification prop:v7-nothost uses Theorems –

t333u 2; t335u prop:v7-future-alignment prop:v7-horizon, rem:v7-powerfamily restates Theorems (b), (c)

t335u cor:v7-future-ledger cor:v7-ledger extends Theorem (b),(c)

t335u prop:v7-drift prop:v7-drift new

t333u 3; t335u def:v7-blind, prop:v7-adaptive def:v7-blind, thm:v7-coefblind preserves Theorem

t335u prop:v7-blindness-repair prop:v7-blindrepair uses Theorem (a)

t335u prop:v7-ranknotsel prop:v7-ranknotsel new

t334u 1, A prop:v7-interface, cor:v7-A extends Theorem (a),(c)

t334u B, C, D thm:v7-B, thm:v7-C, thm:v7-D Theorem extends Assumption

t334u closure, E, F, S 7 ass:v7-closure, thm:v7-E, thm:v7-F, prop:v7-measure outside the ledger

bottomrule

Documentary edges, version 7 (not registered)

Every row is a documentary separation; source record; not a premise. The wiring payload deliver/wiring_Big02-A-v7.json has wired: false and is not sent anywhere. toprule Record Relation

midrule endhead Big05-A v3 Split blindness, the not-a-host-functional qualification and the NEC range (Theorem , Propositions ,). Its closures are not premises; its benchmark numbers are not measurements. Source record; not a premise.

Big04-A-coeff v5 Coefficient-blind adaptive experiments (Theorem). Source record; not a premise.

Big03-A v2 Unsupplied coefficient, vacuum separation, floors (Theorem , Theorem (f)). Source record; not a premise.

DS-OB-1-fibre v5 Rank is not selection (Proposition). Source record; not a premise.

En04-A v3, En04-B v4, I-D v5, Hawk05-B, OpenAI Separations of Section . Source record; not a premise.

bottomrule

Check map, version 7. DC-79 → Theorem , Propositions , ; DC-80 → Proposition , Remark , Corollary ; DC-81 → Theorem , Proposition ; DC-82 → Lemma , Theorem (f), Proposition ; DC-83 → Proposition , Corollary ; DC-84 → Theorem ; DC-85 → Theorem , Proposition ; DC-86 → Theorem ; DC-87 → Theorems , (a); DC-88 → Theorem (b)–(f), Proposition ; DC-89 → Appendix ; DC-90 → insertion-only retention and the ledger.

Provenance and sha256, version 7

Paths are relative to /workspace/NewSheets333/Big02-A. The version-6 delivery /workspace/Big02v6/deliver is copied read-only to inputs/v6-deliver; the Big05-A version-3 source /workspace/NewSheets331/Big05-A/deliver/Big05-A-v3.tex to inputs/local; the site records, fetched read-only by GET on 25 October 2026, to inputs/api; and Crossref, arXiv and text extracts of the cited papers to inputs/refs. The trees /workspace/Big02v4, /workspace/Big02v5, /workspace/Big02v6, /workspace/NewSheets315, /workspace/NewSheets319 and /workspace/NewSheets331 were hashed before and after this work and are unchanged, and the frozen trees XVI-P, XVI-Q and XVI-R hash the same before and after. toprule File sha256

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src/v7_preamble.tex 55b88b0b2207ef389a95e1f488af96f3d9217d99640bd12764321af9f8461257

src/v7_pred.tex d5569c520bcff342bcaf59d50d9b8dda6017afd2ec12abdd8081ea451a544f65

src/v7_recheck.tex 434988b8c7b38c97bfb37b492c5523704ac00965782965798ceb089dc5713314

src/v7_rows_checks.tex 57d12229cf83295c802780c68a3b01dab8d0d1932b21b818608e39ca80ff6d09

src/v7_rows_status.tex bad25117149b0f45405d7d0338db190ba9dc4af3fb1cb83df8a08ea71fbb5bb6

src/v7_split.tex 95dec53dea3d23c8f232601adca44a7bbedde08ea9f3e010295fab9667b4ec35

checks_Big02A_v7.py 6d84f38adbda7e8e5bfe302a84d30838835c6658fb82ff7d0a1f9367c83cd743

deliver/checks_Big02A_v7.json cffa340e11db0bc674fd0a1b650dbd1e96dcf9df8cf44820ec253ba3a23f0f81

bottomrule

Standing ledger, version 7 (verbatim)

No quantum coefficient is admitted. Host A and Host B stay separate; Host C is not created. O.1 stays NOT_CLOSED. XVI-N is not discharged; N-Ins.4 stays unresolved. XV-B is not opened. H2, I3, J2 and T3 stay unresolved. U6-sel is not moved. No Einstein dynamics, derived metric, spacetime emergence, Page curve, or quantized relational time without an external clock is claimed. The last reading bars stay open, and continuum acquisition stays a negative certificate. No Route-B successor is created. Res02-A Q1–Q4 and A1–A2 stay open. Gate 0 and ST4b stay NOT_SUPPLIED. Citations identify source records only; no proof transfer. No compilation is asserted.

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